sonal variations in the temperature structure.

There is no more reason to believe that water vapor is homogeneously distributed in the high-latitude mesosphere than to believe that such a situation exists in the troposphere. The meridional circulation shown in Fig. 2 represents the average motion in the stratosphere and mesosphere, but this circulation is subject to frequent and dynamic changes, and, therefore, any assumptions of steady-state flow or homogeneous composition are unrealistic. Thus the low temperatures at the mesopause may not produce noctilucent clouds if there is insufficient water vapor available. Questions of the amount of water vapor available and the magnitude of the vertical velocity necessary to transport water vapor to the mesopause must remain unanswered for the present, since no *in situ* measurements have been made to confirm or to refute the estimates and extrapolations that have been published. Figure 3 shows that Hesstvedt estimated the mixing ratio at the mesopause to be 1 g kg^{-1}, which corresponds to a frost point of 172°K at 82 km (*14*). Hesstvedt later revised this value downward to the order of 10^{-2} g kg^{-1} (*12*), which is in reasonable agreement with Paton's estimate (*15*) of 6×10^{-3} g kg^{-1} (6×10^{-3} g kg^{-1} corresponds to a frost point of 150°K at 80 km). Charlson used our mesopause temperature measured at Kronogard (130°K) as a conservative approximation to develop a steady-state model for noctilucent clouds (*16*). Thus, estimates of the content of water vapor at the mesopause have grown smaller as the estimates and observations of mesopause temperatures have become lower.

Figure 3 also shows a comparison of the frost points for various mixing ratios and a curve which has been extrapolated from measurements at altitudes reached by balloon-borne instrumentation, with the average temperature profile for the five soundings made during displays of noctilucent clouds. Curves *A*, *B*, and *C* were computed for constant mixing ratios with the use of the pressure profile derived from the average of 15 measured temperature profiles at high latitudes in summer. Curve *A* gives the frost points for a mixing ratio of 10^{-3} g kg^{-1}, curve *B* for 10^{-2} g kg^{-1}, and curve *C* for 10^{-1} g kg^{-1}. Badinov *et al.* (*17*) have extrapolated values for frost points from measurements made by various techniques at balloon altitudes of 28 to 30 km, and these are given by curve *D*. These values are based on a mixing ratio measured in the stratosphere which they believe stabilizes with height to a value on the order of 10^{-3} g kg^{-1}. The average temperature profile for the five soundings of noctilucent clouds is given by curve *E* in Fig. 3. The mesopause temperature of 143°K corresponds to a saturation mixing ratio of approximately 1.3×10^{-3} g kg^{-1}. It must be remembered, however, that this is an average value of the mixing ratio and that both cooler and warmer temperatures, corresponding to lower and higher saturation mixing ratios, have been measured during the cloud displays.

Since no significant difference in the observed temperature was noted between the soundings conducted in the presence of noctilucent clouds and those conducted in the absence of the clouds, we conclude that a mesopause temperature of less than 150°K is a necessary but not sufficient condition for the existence of noctilucent clouds. Variability of the content of water vapor at the mesopause is also believed to be a key factor in the occurrence of these clouds. In view of the circulation that is implied by the seasonal differences in temperature in the mesosphere, we believe that small amounts of water vapor are transported by this circulation into the mesosphere during the summer at high latitudes. It is at these latitudes that saturation or supersaturation occurs in the narrow layer at the mesopause where these extremely cold temperatures are observed. We believe further that dust particles, probably originating from incoming meteors, serve as sublimation nuclei in this saturated region and grow to sufficient size to scatter sunlight, thus producing noctilucent clouds.

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2 May 1967

Wake Collapse in a Stratified Fluid

Abstract. *A two-dimensional model is used to obtain quantitative data on characteristics of turbulently mixed wakes of bodies submerged in stratified fluids (more dense below than above). The time between turbulent mixing and maximum expansion of the wake before vertical collapse starts is 0.44* T, *where* T *is the local Väisälä-Brunt period. Time after mixing for maximum rate of horizontal spreading is about 2.0* T. *The average Väisälä-Brunt period for the oçeans and atmosphere is discussed. It is predicted that the wake collapse phenomenon is not unusual in these environments. The characteristic time for the most active phase of vertical wake collapse should be between a few minutes to several tens of minutes. Qualitative observations of aircraft vapor trails tend to confirm that the phenomenon does occur at full scale.*

Bodies such as a model submarine traveling in a tank filled with water having homogeneous density, or an aircraft flying in air of uniform density, leave behind turbulent wakes that are due to mixing in the boundary layer and mixing by the propellers or jet engines. These turbulent wakes expand to have an irregular conical shape. Frequently, exhaust trails of jet aircraft illustrate this type of turbulent mixed wake when taking off.

The oceans and the atmosphere seldom are of homogeneous density (iso-

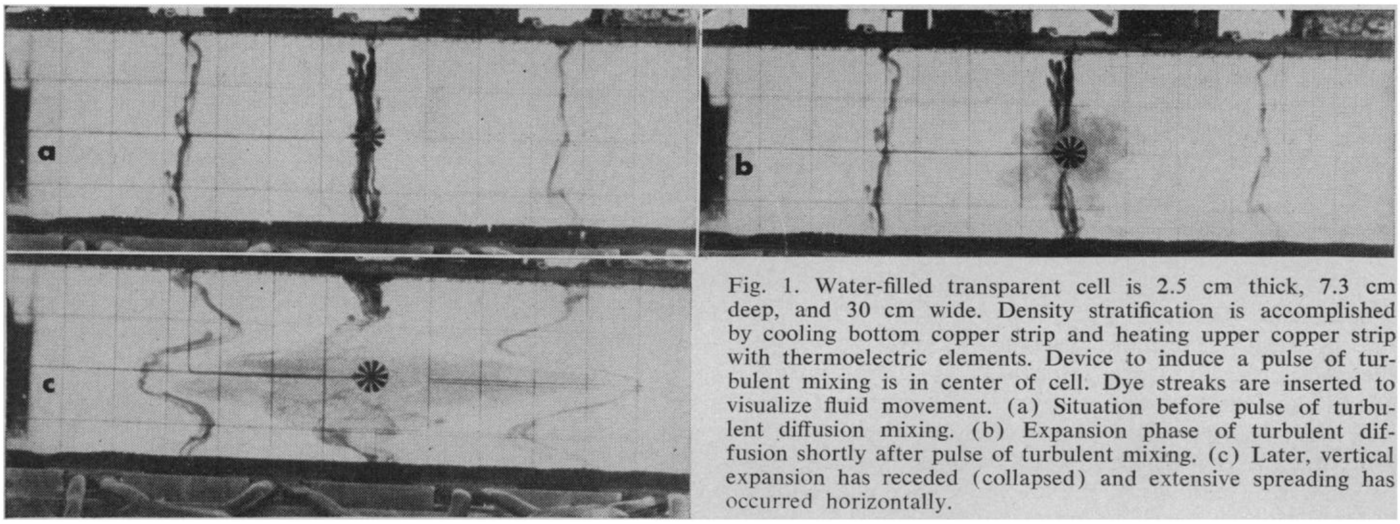

Fig. 1. Water-filled transparent cell is 2.5 cm thick, 7.3 cm deep, and 30 cm wide. Density stratification is accomplished by cooling bottom copper strip and heating upper copper strip with thermoelectric elements. Device to induce a pulse of turbulent mixing is in center of cell. Dye streaks are inserted to visualize fluid movement. (a) Situation before pulse of turbulent diffusion mixing. (b) Expansion phase of turbulent diffusion shortly after pulse of turbulent mixing. (c) Later, vertical expansion has receded (collapsed) and extensive spreading has occurred horizontally.

pycnic), particularly in the vertical direction. It is not unusual for the oceans and atmosphere, except relatively near the surface, to be stably stratified vertically. Stable stratification means that the fluid below is everywhere more dense than the fluid directly above. The wake of a body traveling in a stratified fluid is strikingly different than that for the isopycnic case.

One may imagine a relatively large element of fluid through which a body has passed perpendicularly to the center. In the isopycnic case, the fluid in the turbulent wake a short time after passage will attain a more or less round shape, and the diameter will continue to increase with time after passage due to eddy diffusion. If the fluid is stratified, the turbulent wake at first will also diffuse outward in a more or less circular form. However, this expansion will not continue indefinitely, and a characteristic time later the wake will start to decrease vertically (collapse) and spread out horizontally.

The dramatic vertical wake collapse is due to the forming of an unstable region of isopycnic fluid, due to turbulent mixing, in a fluid that everywhere else has a vertical density gradient. Gravity forces enter to overcome the vertical turbulent momentum forces and vertical wake collapse occurs in order to restore density equilibrium within the fluid. In the vertical direction, it is like projecting a parcel of mixed fluid upward into a less dense region and projecting downward a parcel of mixed fluid into a more dense region. Under such conditions, it may be assumed that a characteristic time for vertical wake collapse to occur is a function of the acceleration of gravity, g; the density of the fluid, ρ; and the vertical density gradient, $d\rho/dz$. These variables are contained in the expression for Väisälä-Brunt period T

$$T = \frac{2\pi}{60\left(\frac{g}{\rho}\,\frac{d\rho}{dz}\right)^{1/2}} \text{ min/cycle}$$

Conceptually, Eckart (*1*) describes T as the period which a discrete parcel of fluid will oscillate about its equilibrium position in a stratified fluid if the parcel is lifted or depressed from its equilibrium position and let go.

Figure 1a shows a transparent laboratory cell 2.5 cm thick, 7.3 cm high, and 30 cm wide. The cell is completely filled with water that can be stratified to various degrees by the amount of cooling applied to the bottom and by the amount of heating applied to the top. For convenience of control, this was done by the polarity and the amount of current applied to commercial thermoelectric devices attached to copper strips forming the bottom and the top of the cell. The three irregular dye streaks in the stratified water were introduced through small holes in the upper copper strip by a long hypodermic needle and syringe. In the center of the center dye streak is a 1.3-cm diameter device that looks like a propeller but will be called a mixer. In Fig. 1a, the mixer is stationary. Figure 1b represents the situation in the cell a short time later. In between the two pictures, the mixer was vibrated forward and backward (not rotated) ten times in 2 seconds. This brief action introduced turbulent mixing simulating the passage of a body perpendicularly through the center of the cell. The irregular circular outline of the dye in the center shows the initial eddy diffusion of the turbulent mixed fluid within the body of the stratified fluid. Note that the left and right dye streaks are similar in both Fig. 1, a and b. Figure 1c shows the situation in the cell considerably later, after wake collapse has taken place. After the events shown in Fig. 1b, the approximately circular dye pattern started to flatten vertically and spread horizontally. This action is made visible by the distortion of the left and right vertical dye streaks that are shown in Fig. 1c.

Figure 1 shows selected frames from a 16-mm cinefilm. A series of six films were taken with different vertical temperature gradients at mixer depth. A small thermistor was lowered through a hole in the upper copper plate to

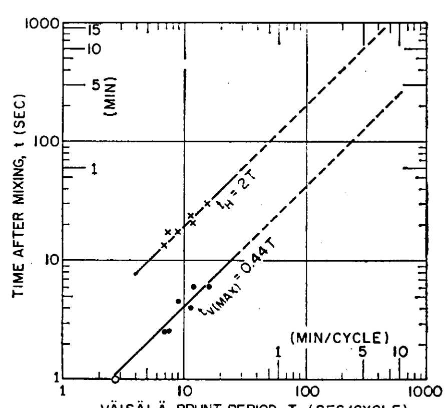

Fig. 2. Time after turbulent mixing for maximum expansion of vertical wake to occur (lower line) and time after mixing for the maximum rate of horizontal wake spreading (upper line) as a function of Väisälä-Brunt period at mixer depth. The solid circle points are for the experiments described. The open circle point is from Schooley and Stewart (*2*).

measure the vertical temperature profile from which the density gradients could be determined. This in turn made it possible to calculate the Väisälä-Brunt period for each of the six experiments. Knowing the frame rate of the cinefilm, it was possible to estimate the time for the turbulent "wake" to reach its maximum diameter after the relatively short mixing interval, and also to determine the time that the rate of horizontal spreading became maximum (Fig. 2). The experiments show two characteristic times related to the wake collapse phenomenon. The maximum vertical expansion phase occurred at about 0.44 times the Väisälä-Brunt period of the fluid. After this, vertical wake collapse started, and at about 2.0 times the Väisälä-Brunt period the maximum rate of horizontal spreading occurred. The small experimental cell did not permit studying the long time characteristics of the horizontal spreading phase due to side wall effects.

The ratio of the maximum diameter of wake expansion before collapse to the mixer diameter was also studied as a function of Väisälä-Brunt period at mixer depth. This study indicated that the maximum vertical wake diameter is proportional to approximately the cube root of the Väisälä-Brunt period. This relation appears to be reasonable because the turbulence of a fixed amount of mixing will certainly diffuse a bit further before vertical collapse occurs when the density gradient is low (Väisälä-Brunt period long). Therefore, the amplitude of the wake collapse phenomenon will be somewhat greater for long Väisälä-Brunt periods than for short periods. However, the various phases of the phenomenon will proceed faster when the Väiälä-Brunt period is short.

Eckart (*1*) has given average values of Väisälä-Brunt periods for the oceans and atmosphere of the world as a function of depth and height, respectively. Adaptation of his data indicates that T for the ocean varies between about 8 to 35 min/cycle for depths from 1 km to near the surface. Near the surface, diurnal and seasonal variations of the T become important. For the atmosphere between a height of 100 km and near the surface, T varies from about 4 to 10 min/cycle. Again near the surface, T is quite variable.

Although the experiments were done on a very small scale, they likely have significant geophysical implications. The experiments predict that the wake collapse phenomenon is not unusual in connection with bodies traveling in the oceans or in the atmosphere. The characteristic time for the most active part of the vertical wake collapse phenomenon is predicted to occur on the order of the Väisälä-Brunt period at the location where the body is operating. The contrails (vapor trails) left by aircraft often spread very extensively horizontally in a way that leads to the feeling that wake collapse actually does occur. Pilots have told me that the contrails spread much more horizontally than they do vertically. This also lends credence to the wake collapse theory.

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12 April 1967; revised 7 June 1967

Resonance Rotation of Venus

Abstract. *Combination of two types of radar data shows the orbital plane and equator of Venus to be included by less than 2 degrees, and the sidereal rotation period to be 243.09 ± 0.18 days (retrograde)—remarkably close to the 243.16-day period for which the spin would be in resonance with the relative orbital motions of Earth and Venus. In this resonance, Venus would make, on average, four axial rotations as seen by an Earth observer between successive close approaches of the two planets. Estimates of the instantaneous spin period, accurate within about 0.01 day, would provide important information on the difference of Venus's equatorial moments of inertia, on their orientation, and on the magnitude of the tidal torque exerted on Venus by the sun.*

Radar observations during the past few years have for the first time enabled reliable determination of the rotation rate of Venus. In 1961, measurements made at its close approach to Earth (inferior conjunction) indicated that the spin angular velocity of Venus was very low (*1, 2*); the possibility of the rotation being retrograde was also pointed out (*3*). Radar data from the 1962 inferior conjunction, obtained with more sensitive equipment, established that the rotation was retrograde (*4*), with a sidereal period of 250 ± 40 days (*5–7*). Soon thereafter it was noticed (*8*) that this period was close to 243 days, for which Venus would present the same "face" to Earth at each inferior conjunction. Reduction of the observations from the close approaches of 1964 and 1966 yielded estimates ranging from about 242 to 246 days, with the quoted errors being generally consistent with this spread (*9–12*).

Each of these determinations of spin period has involved one of two methods (*13*): In the first (*5, 6*), the total radar echo from the planet is analyzed in frequency, and spectral regions containing unusually high power ("features") are noted; by following the the spectral positions of these features as a function of time, the spin vector can be estimated. In the second method (*6, 9*), the echo from a particular annulus, or ring, on the planet is isolated and its bandwidth is measured. Since this bandwidth is proportional to the magnitude of the sum of the planet's inertial spin vector and the apparent spin vector (due to the relative orbital motion of planet and observer), a series of such measurements enables the inertial vector to be estimated unambiguously. The first method has the advantage that, independently of the improvement attributable to statistics, the accuracy of the determination of period increases in direct proportion to the time interval over which the observations extend. A disadvantage stems from the somewhat subjective nature of the necessary identification of features, from separate spectra, as being associated with the same physical region on the planet. The identification is especially difficult in general because of the strong dependence of the radar reflectivity of a region on the aspect from which it is viewed, and because of the present limitations on achievable signal-to-noise ratios and resolution. For Venus this difficulty is markedly reduced since, as mentioned earlier, the planet presents almost the same aspect at each close approach to Earth. Although completely objective in applica-