Reports

Radio Method for the Precise Measurement of the Rotation Period of the Earth

Abstract. *Radio interferometry with independent high-precision clocks, without a high-frequency communication channel between the stations, is now a possibility. It allows the stations to be as far apart as the earth can accommodate. This then makes the radio band from 10- to 60-centimeters wavelength the best frequency range for high-precision angular measurements, since the variability of the atmosphere is less disturbing there than in the optical band.*

Radio astronomy measurements show that small-angle radio sources exist, probably smaller than 1/100 second of arc. Radio interferometry can then be used to determine the variations in the speed of rotation of the earth to a precision possibly 20 times better than present optical techniques, with angular precision better than to 1/100 second of arc. A new range of geophysical effects can then be studied (*1*). The proposed technique also involves a number of other high-precision determinations. Radio-frequency oscillators can be kept indefinitely in precise synchronism, simultaneity to better than nanosecond precision is therefore available at the two sites. Tidal deformations of the solid earth and movements of the instantaneous axis or rotation both come within the range of measureable effects.

Optical methods for the determination of the variations of the length of the day are limited in their precision by the variability of the atmospheric refraction. The aperture of optical instruments that it is useful in this regard is given by that dimension at which the variable refraction causes phase differences of as much as $\pi/2$: in practice this is about 30 cm. Corresponding accuracies of single measurements are then limited to what is also the diffraction limitation of such an instrument, which is approximately 0.4 seconds of arc. Although improvements in technique have reduced other sources of error, the atmospheric limitation has been the basic one for over 200 years.

Radio techniques seemed inappropriate since the long wavelengths implied that very large apertures would be needed before optical precision could be approached. Such apertures, at any rate as interferometer baselines, are now a possibility. The atmospheric limitation, however, is very much less severe. It would appear that a very substantial improvement in precision, by a factor of 10 or more, is now attainable with modern techniques that were developed for other purposes.

To facilitate the description of the radio precision transit measurement, we shall first assume that precise timing and phase can be distributed to the two instruments of an interferometer, however far apart, and that the central lobe of the interference pattern can always be recognized. We shall later discuss these problems in detail.

Even if optical phase could be distributed to two distant sites without error, the unknown phase delay of the two rays through the atmosphere, unknown to many whole cycles, would still impose the essential limitation. The delay times for radio waves are similar to optical delays, and the phase uncertainty is therefore much smaller. There is particular interest for our purposes in that range of wavelengths at which the uncertain atmospheric delay usually does not amount to as much as $\pi/2$. In that range, an interferometer of long baseline becomes useful, for then the precision obtainable will continue to increase as the baseline is lengthened; the uncertainty in angle remains a quantity less than λ/d, where λ is the wavelength and d the interferometer spacing.

Our assumptions concerning atmospheric variability appear indeed to be satisfied at a wavelength of 30 cm (1000 Mhz), and possibly even at 10 cm. If we had an interferometer baseline across the United States (say 3000 km from east to west) the interference pattern would have a fringewidth of 10^{-7} radians, or 0.02 second of arc at 30 cm. This makes clear that it is interesting to see whether the assumptions made can be satisfied, and whether, in fact, radio interferometry can supersede optical techniques for high-precision angular measurements.

The first requirement is, of course, that there should be sources in the sky whose angular diameter is as small or smaller than the precision required. This does not mean that the entire angular size of a radio source needs to be as small as the required precision, but only that there should be recognizable structure as small within it. Information from radio lunar occultations and from the interplanetary scintillations of radio sources, as well as other types of information (*2*), suggest strongly that this requirement is in fact satisfied.

With such sources, and with the availability of precisely synchronized radio-frequency oscillators which we are assuming for the moment, it becomes possible to record at each station of an interferometer an intermediate frequency signal. These records can subsequently be correlated, and if precise timing is also recorded and used in the correlation operation, interference fringes will be observed in the result.

The next requirement for discussion is that of phase being available correctly at the two sites. Conventionally this has been done in radio interferometry, with a cable distributing the local oscillator signal to both sites, and another cable bringing the intermediate frequency signals back together. Over distances of more than a few kilometers this clearly becomes impracticable. Another method has been the distribution of the local oscillator signal over a radio link. With distances of more than 100 or 200 km this becomes not only expensive, but also inaccurate with respect to phase (even if a self-correcting round-trip loop is used, and the only atmospheric variations of significance are then those that take place within a single round-type period).

Fortunately, independent clock interferometry is now a possibility. Hydrogen maser oscillators exist with a precision of frequency approaching one part in 10^{14}. Thus, at 1000 Mhz, a phase precision can be maintained for 10^{14} cycles or about 1 day. There remain, however, the requirements of recognition of a particular interference fringe, say the central one, on each successive day, and of stabilizing the phase relation between the two independent oscillators for long periods of time so that this same, identified fringe does not continue to shift its direction over periods longer than a day. If clock precisions of one part in 10^{16} or better could be obtained, the second requirement would disappear, at least until long period variations in the speed of rotation of the earth are examined.

The recognition of a particular fringe repeatedly on successive sources or on the same source on successive days can be achieved in a number of ways. The use of a very wide radio-frequency and an intermediate frequency bandwidth would be one possibility. The deep fringes would then be limited to a number equal to the ratio of the radio-frequency to the bandwidth. If a limitation to ten fringes were desired in order to pick out the central one it would be necessary to record a 100-Mhz signal for radio frequency of 1000 Mhz. This is technically difficult. A technically much simpler method is the use of two or more narrow band channels that may all fit within the overall radio frequency bandwidth. In such a system one may then correlate the records for each frequency separately, and the different sets of fringes so obtained will be of slightly different width. Beats between them can be obtained, and they will now define individual primary fringes; only now there will be ambiguities. In practice it would not be difficult to remove all these by other considerations (for example, that too great a change in the rotation speed of the earth would be implied).

Accumulation of clock errors has to be prevented; otherwise a particular identified fringe in the interferometer pattern will still drift continuously. Small-angle radio sources can be used for this purpose.

For simplicity in considering the nature of the method, let us suppose that there were available a small-angle radio source located precisely at the north celestial pole. Suppose further that the radio instruments were situated at such a latitude that they could view the 90° from the pole to the equator, and that all around an equatorial belt there were many small-angle sources whose transit could be observed. The discussion is easiest in terms of a real-time intermediate frequency channel (although in practice this will not be needed). The antennas could have an observing program according to which they are turned at the celestial pole at intervals sufficiently frequent for the possible clock error to amount to a small phase angle only, say once per hour for a hydrogen maser that could maintain phase for a day. The clock error would then be deducible from the fractional fringewidth shift that had taken place, and sign and magnitude could be measured with a switching system that introduced a phase advance and retardation. For example a servo loop could be designed that would operate on the phase of one of the clocks only, and that would continuously correct this, so as to maintain the center of a particular fringe pointing at this celestial pole source.

It is worth noting that this provides a method for maintaining precise radio frequency synchronism for two clocks anywhere on earth, at frequencies as high as 1000 Mhz, or having simultaneity available to a precision much better than one nanosecond (just as a source position can be determined to a fraction of a fringewidth).

With such periodic corrections one could then be certain that, at no time, the clock errors can amount to a substantial phase angle. Now in turn, with the instrument so maintained, one observes the transit of equatorial sources. If there was just a single one, then an uncertainty would arise when the variation of the length of the day had amounted to a substantial fraction of a fringewidth: it would no longer be clear in which fringe the source is located at a particular time, and a method of identifying a particular fringe would then be needed. If, on the other hand, there are enough radio sources to be observed so that the phase variation due to variations in the rotation speed of the earth can never amount to a large phase angle between any one and the next, then an account can be kept of the fringes, and the uncertainty can be avoided. There would then be no need for a method of identification of a fringe. Of course there is then no knowledge available of the position of the sources to this high precision, but only of variations arising from the rotation speed fluctuations, or possibly from proper motion of the sources. In the example chosen, such variations might be seen if they amounted to an angle of 0.02 second of arc, or even less if a fraction of a fringewidth can be recognized despite atmospheric variation. (For studies of proper motion of radio sources there would be no restriction on the length of the time interval during which the motion must have taken place.)

In practice, if accuracies of this order are obtainable there will be numerous other quantities that need to be taken into consideration. A correction for the atmospheric phase delay, resulting from the variable amount of overlying atmosphere could be made with the measured atmospheric pressure. With the pole observation correcting the phase, this will not be necessary, as it will already be included in the adjustment to the clock. The gradient in the atmospheric pressure in the north-south direction will have an effect, since it will cause a different amount of atmosphere to be traversed for the polar as for the equatorial source. An approximate correction for this gradient can be made.

Since the total atmospheric delay only amounts to an additional pathlength of 3 meters or 10 wavelengths for the example chosen, such a correction can easily be sufficiently accurate. The ionospheric phase advance is possibly a greater problem, especially at dawn or dusk. The nighttime steadiness is however usually adequate in the middle latitudes, and it may be that accurate observations would have to be restricted to that period.

The solid earth tides are clearly significant to this precision. A rise and fall of one site of 10 cm would be equivalent to a change in the angle to a source of 0.007 second, a significant amount for the accuracy obtainable by repeated measurements. To the extent to which the magnitude of the tides are known, this can be allowed for. Beyond that, there will be a term in all the observations dependent on this effect: but since the temporal variations of the exciting function are known precisely, most of the unknown part will be removable in long-period observations. In turn, the height of the earth tide at the observing stations may become known to a better accuracy (by a measurement which is in effect the determination of the difference in the distance to some very distant source—a few centimeters in 10^{10} light years).

In the preceding discussion, the existence of a polar radio source was assumed. There is no need for a source in precisely that position, and the required information can be obtained equally from one or from a small number of sources displaced somewhat from the pole. On the other hand, the variations in the direction of the instantaneous axis of rotation of the earth will produce an error in the form of an apparent diurnal variation in the speed of rotation of the earth (since the fringes are stabilized relative to a direction that now has a diurnal motion). Again, from many observations spread out over the day, some of these errors can be deduced. The magnitude of day-to-day changes in the direction of the pole is likely to be enough to enter into these considerations. The best solution obtainable from all the data would include a determination of this effect.

The system discussed here is, of course, elaborate. Nevertheless, it may be worthwhile setting it up, not only for the more precise measurements of the motion of the earth around its center of mass, but also for the determination of distortions of the surface and other geophysical effects, and possibly also for the determination of proper motion in radio sources. Very precise measurements are often used not only to determine the magnitude of known or expected effects, but also to search for new ones.

Note added in proof. The technique of independent radio interferometry has been successfully demonstrated by at least two groups, a combined Canadian group (*3*) and a group from the National Radioastronomy Observatory and Cornell University (*4*). The results indicate that there are a large number of radio sources with structures smaller than 0.01 second of arc.

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References and Notes

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Implications for Geophysics of the Precise Measurement of the Earth's Rotation

Abstract. *A radio interferometer could yield an error on the order of 10^{-9} second at the semidiurnal frequency. With errors of this magnitude, yearly changes in the rate at which the earth's rotation is slowing down could be determined. The proposed interferometer could also yield significant improvements in the determination of the Love number* k *and its variation with frequency, and in the changes in angular momentum of the atmosphere for periods greater than 1 week.*

The determination of the length of day depends both on the precision of astronomic observations and on the time standard by which the length is fixed. Precision of the time standard depends on two things: the precision in the clock rate and the precision with which it can be read. As a consequence of the development of hydrogen maser oscillators and accompanying electronics the precision in time standard can now approach one part in 10^{14}. The limit in determination of the length of day for periods shorter than about a year lies in "reading" the rate of rotation; that is, in astronomical observation determining the right-ascension transits of predetermined stars across the meridian (*1*). Gold (*2*) outlines a scheme reducing the error in fixing the variable rate of rotation, $m_3(t)$.

The probable error $\mathcal{E}$ of a time sight from one night's observation is about 5 msec, as calculated with conventional astronomy (*3*). Gold suggests that with a two-point interferometer variation in the rate of rotation may be determined to a precision corresponding to an angular displacement of 10^{-7} radians, for an observation of a radio source lying near the equatorial plane. The precision of a single determination would thus be about 1 msec or even less, if a fraction of a fringe width can be recognized.

The mean square error of observation of N sources is $\mathcal{E}^2/N$. Values of the rotation rate can then be obtained at intervals of M days where N radio sources have been observed in the M days. For maximum geophysical information, M should be ¼ or less, since this sampling rate permits the determination of fluctuations in the time whose period exceeds ½ day. In principle, such high-frequency terms could also be obtained from nighttime optical astronomical observations, but this would require the combination of observations from the different observatories as well as a marked improvement in precision. There would remain errors associated with the differing instrumental techniques of the various observatories. In the case of the radio interferometer, a single installation is sufficient, although care must be exercised to remove properly the day-night fluctuation due to ionospheric conditions.

The spectral density of the error in the rate of rotation is

$$(2\pi f)^2\,(2N/N)\Gamma\mathcal{E}^2 = (2\pi f)^2(M/N)\;0.2\text{ second} \quad \text{for } \mathcal{E} \approx 1\text{ msec} \qquad (1)$$

where f is the frequency under consideration and Γ is the length of day. The error spectrum of the suggested radio interferometric measurements would be at least a factor of 25 less than that of modern optical techniques. At the semidiurnal frequency, the error spectrum is 1×10^{-9} second for four radio sources observed during a 6-hour period.

The spectrum of variations of the rate of rotation is composed of two parts. There are spectral lines centered at annual, semiannual, monthly, fortnightly, diurnal, and semidiurnal periods, and there is a continuum. The lines are in part due to the tidal alteration of the moment of inertia, and the continuum is presumed to be the result of motions within the atmosphere, oceans, and core.

With conventional astronomical techniques it is possible to determine the annual and semiannual variations and the continuum for periods between several years and several months. It is barely possible to determine the tidal effects for the monthly and fortnightly periods. For a variety of reasons the higher period fluctuations are of great interest to geophysicists. The long-term slowing down of the earth's rotation due to the tidal friction interaction with the sun and moon depends on the amplitude and phase of the semidiurnal tides. In the past, this rate has been determined by combining astronomical observations over several centuries or observations of eclipses over longer periods. A precise determination of the semidiurnal fluctuation in the length of day will permit a fresh determination of the amplitude and phase of semidiurnal tides and thus will permit examination of whether or not the long-term retardation is constant or fluctuating. This is important in considerations of the origin of the earth and moon, since discussions of the dynamical history of