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1 January 1967; 7 March 1967

Rapid Rotation of the Solar Interior

Abstract. *A model proposed for the sun apparently can account for the maintenance of photospheric rotation against deceleration by the solar wind. The rotational period is about half a day near the top of the radiative mass, and 1.6 times shorter at the center.*

The solar wind produces a net flow in the radial (r) direction of the azimuthal (φ) component of angular momentum, $\tau_\varphi{}^r$. According to independent estimates (*1–3*), this flux of angular momentum is approximately 1×10^8 g sec^{-2} when averaged over the photosphere. The corresponding torque would be sufficient to halve the present angular momentum of the sun in 4×10^9 years, if the sun were in uniform rotation. Below the hydrogen convection zone (HCZ), however, the processes known to be available for the transport of angular momentum would all appear to be of low efficiency (*4*). Because the angular momentum of the HCZ alone has a half-life of less than 1×10^8 years, it is necessary to examine the processes that could support its continued rotation against the decelerating torque of the solar wind.

Several authors (*1, 5, 6*) have suggested that throughout most of the solar interior the rotational period is possibly much shorter than it is at the photosphere. Some evidence is also available for rapid rotation in the interiors of certain other main-sequence stars of solar type (*7*). If the angular velocity actually is large in the solar interior, then the radiation flowing from it could carry enough angular momentum to replace the losses in the solar wind. In a part of a star where the angular velocity is ω, Jeans showed that there is a radiative flux of angular momentum that may be written approximately

$$(\tau_\varphi{}^r)_{\rm rad} = (\omega r^2 \sin^2\theta)\, F/c^2 \qquad (1)$$

where θ is the polar angle, and F is the usual radiative flux of energy (*8, 9*). Outside of the energy-generating region at the center, the Jeans flux of angular momentum is therefore

$$\langle(\tau_\varphi{}^r)_{\rm rad}\rangle = \frac{\omega L}{6\pi c^2} \qquad (2)$$

when averaged over the sphere.

The arguments of Dicke (*1*), Deutsch (*7*), and Plaskett (*6*) led them to characterize the solar interior with a rotational period of the order of half a day. Let us adopt this value for $r/r_0 = 0.8$, at a level near the top of the radiative mass. The corresponding rotational velocity at this level is 100 km per second. We then find that $\langle(\tau_\varphi{}^r)_{\rm rad}\rangle = 3 \times 10^7$ g sec^{-2}. This is to be compared with the solar-wind flux which, when reduced to $r/r_0 = 0.8$, is 2×10^8 g sec^{-2}.

The uncertainties of both calculations admit the conclusion that the HCZ can retain the slow angular velocity now seen at the photosphere, provided that the Jeans flux of angular momentum can be transferred from the radiation field to the matter near the base of the HCZ. This transfer can be effected by the radiative viscosity acting in an angular-velocity gradient in the outer part of the radiative mass. When the viscous transport of angular momentum is averaged over the sphere, it yields (*10*)

$$\langle(\tau_\varphi{}^r)_{\rm vis}\rangle = \tfrac{2}{3}\,\eta\, r^2\,(d\omega/dr) \qquad (3)$$

The coefficient of radiative viscosity is (*8, 11*)

$$\eta_R = 4\sigma T^4/15\,\kappa\rho\, c \qquad (4)$$

At $r/r_0 = 0.8$ in the Schwarzschild-Weymann model of the sun (*12*), this is $\eta_R = 0.46$ g cm^{-1} sec^{-1}, which is an order of magnitude larger than the coefficient of molecular viscosity. If we introduce this value into Eq. 3, and set $\langle(\tau_\varphi{}^r)_{\rm vis}\rangle$ equal to the flux "observed" by Brandt and by Weber and Davis, we find that the requisite gradient is $d\omega/dr = 1 \times 10^{-13}$ sec^{-1} cm^{-1}. The thickness of the transition zone that supports this gradient is then approximately

$$\delta r = \omega / |\, d\omega/dr \,| \qquad (5)$$

evaluated at $r/r_0 = 0.8$. This turns out to be 1×10^4 km. Let us take $r = r_2$ at the inner boundary of the zone, and r_1 at the outer boundary.

We may expect that the outflow of angular momentum in the solar wind would increase in proportion to $\omega_0{}^n$, where $n \geqq 1$, according to Weber and Davis (*3*). On the other hand, the radiative torque on the HCZ evidently depends only on the difference, $\omega_2 - \omega_0$, between the angular velocities at the inner boundary of the transition zone and at the photosphere. With $\omega_0/\omega_2 \ll 1$, the radiative torque is relatively insensitive to changes in ω_0. The actual angular velocity of the HCZ therefore represents a true dynamical equilibrium between the opposing torques associated with the Jeans flux and the solar wind.

In view of the fact that the Jeans flux of angular momentum maintains the requisite flow of angular momentum into the base of the transition zone, there need be no appreciable gradient of angular velocity below this level. A model that seems to satisfy the requirements is that of Roxburgh (*13*), in which the angular velocity is a function $\omega(r)$ that rises gently from its value ω_2 at the top of the radiative mass to a maximum $\omega_c \simeq 1.6\,\omega_2$ at the center of the star. Roxburgh's model is characterized by vanishing meridional circulation, but it incorporates a toroidal magnetic field of the type first described by Biermann (*14*), which can arise from the battery effect of the centrifugal force on the free electrons. Everywhere in Roxburgh's model the gravitational force is large compared to the centrifugal force, and the centrifugal force is large compared to the magnetic body force. Over intervals of order 5×10^9 years, the magnetic field has insufficient time to reach its steady state (*15*). However, Roxburgh has also shown (*16*) that $\omega(r)$ can achieve its steady state within this time scale, provided that the angular momentum of the sun is high enough so that $r_2\omega_2 \gtrsim 60$ km per second in the steady state—a condition probably satisfied.

Turning back to the transition zone near the top of the radiative mass, we may approximate its behavior by neglecting effects of compressibility and viscosity gradients, and idealizing the problem as follows: to find the steady motion of a uniform viscous liquid that is confined to the shell between two concentric spherical surfaces, the inner surface in rotation with constant angular velocity ω_2, and the outer surface in rotation on the same axis with constant angular velocity $\omega_1 < \omega_2$. If Φ is the gravitational potential, the equation of motion for this problem is

$$\rho\mathbf{v}\cdot\nabla\mathbf{v} = -\rho\nabla\Phi - \nabla p + \eta\nabla^2\mathbf{v} \qquad (6)$$

with $\rho\nabla\Phi \simeq -\nabla p$, due to the fact that the centrifugal force is small. This problem appears not to have been solved in any generality (*17, 18*). Nevertheless, it is possible to anticipate some of the properties that the solution will have. In particular, as r increases, the rotational velocity v_φ obviously falls much more steeply at low latitudes than at high (its gradient vanishes on the axis). The viscous stresses will therefore be largest at low latitudes, and, on a given sphere in the fluid, these stresses will produce a larger angular velocity at low latitudes. But this is just the kind of phenomenon that we observe in the equatorial acceleration of the photosphere.

That the equatorial acceleration may be taken as evidence for a more rapid rotation of the solar interior than that at the photosphere is an old idea, long ago investigated by Belopolsky (*19*), Sampson (*20*), and Wilsing (*21*). Because none of these authors knew the physical conditions that prevail in the solar interior or were aware of the existence of the HCZ and the solar wind, their discussions are inapplicable in any detail. Nevertheless, the central idea still appears to be of value.

Although an equatorial acceleration is to be expected in the transition region, where the angular velocity has a steep gradient, an explanation is required for its persistence through the HCZ to the photosphere, where it is observed. The cause probably lies in the anisotropy of the motions in the HCZ. Being predominantly radial, these motions should efficiently exchange angular momentum in this coordinate. However, they may be far less effective in transverse directions, and they would then tend to preserve the equatorial acceleration that characterizes the transition region. At the photosphere there may be additional effects arising from the unknown latitude dependence of the solar wind.

A weak toroidal magnetic field spontaneously arises in the outermost radiative parts of Roxburgh's model. Moreover, the gas in these parts must slowly flow into the base of the HCZ to supply the mass lost in the solar wind, and also in connection with the ordinary evolutionary brightening of the sun. Perhaps this flow can provide the seed field for amplification and then diffusion in the HCZ, in accordance with the general ideas of Babcock (*22*) and Leighton (*23*). Although no specific mechanism can be foreseen, other manifestations of the solar cycle may also be shown to depend on instabilities originating in the transition zone (*24*). Indeed, a connection between the rapidly rotating interior and some features of the solar cycle was sought 80 years ago (*19–21*).

If the hypothesis is true that most solar-type stars are in rapid rotation, this could relieve the difficulties associated with the apparently very high concentration of angular momentum in early-type stars relative to late-type ones and—within the solar system—in the planets relative to the sun (*25*). Other consequences for the structure and evolution of solar-type stars have been surveyed (*25, 26*).

For main-sequence stars of sufficiently low mass, the HCZ extends all the way to the center (*27*). Since objects of this kind generally show no evidence for appreciable rotation at the surface, we must assume that they are really objects of low net angular momentum. Such a star may have condensed from a protostar that was poor in angular momentum, or its stellar wind may have been sufficiently vigorous and long-lived to decelerate the whole mass. The possibility should be explored that in these stars massive stellar winds are capable of accelerating the Hayashi contraction phase, as evidenced in the luminosity functions of some young star clusters and associations.

Main-sequence stars with types earlier than F5 generally rotate with periods of the order of a day at the surface. Schatzman (*28*) has shown that, since these objects lack deep hydrogen-convection zones, they do not support stellar winds, and they experience no decelerating torques once they have passed through the Hayashi phase. In contrast to solar-type objects, upper main-sequence stars appear to find steady states in which the angular velocity rises from the interior toward the surface (*29*). Their spectroscopic characteristics appear to differ depending on whether in their photospheric layers the initial magnetic energy density was greater than, or less than, the kinetic energy density associated with rotation (*7, 30*).

Studies should be made of the rearrangement of angular momentum in stellar models as the result of their evolution off the main sequence. For stars in which the gravitational forces appreciably exceed centrifugal forces, most evolutionary changes probably occur relatively unchanged by the stellar rotation. However, as mass shells contract, or as they pass from radiative zones to convective zones, they will redistribute the angular momentum they carry. The appropriate assumptions for describing this redistribution are as follows. In radiative zones, angular momentum is conserved in shells, and in convective zones it is distributed to maintain a constant angular velocity. Kraft (*25*) reviewed the existing calculations and noted some implications of nonuniformly rotating main-sequence models like those of Roxburgh and Strittmatter. Strittmatter and Ezer are systematically examining angular-momentum transport in stars originating at various points along the main sequence.

ARMIN J. DEUTSCH
Mount Wilson and Palomar Observatories, Carnegie Institution of Washington, California Institute of Technology, Pasadena 91106

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14 February 1967