Meetings

Mathematics: International Congress

The International Congress of Mathematicians (ICM) held its quadrennial session 16–26 August 1966, this time at Moscow University. This marked the first time the Congress had assembled in the U.S.S.R. Approximately 4300 persons attended. They came from 54 countries: 1500 from the U.S.S.R., 725 from the United States, 300 from Great Britain, 300 from France, and 230 from the German Democratic Republic. Smaller groups represented Cuba, North Korea, and North Vietnam. Approximately a dozen countries (African and Asian) made their first ICM appearances. From nearly every point of view this was the largest ICM ever held.

Invitations had been sent to the mathematical organizations of all countries and regions, whether or not they are on diplomatic speaking terms with the U.S.S.R. Arrangements were made and publicized for the issuance of visas en route for those coming from places without diplomatic relations.

Neither Mainland China nor Taiwan was represented. Taiwan had sent representatives to the 1962 Stockholm Congress. Mainland China has never sent anyone to an ICM, presumably in keeping with its policy of not participating in organizations which give any sort of recognition to Taiwan. The ICM operates under the aegis of the International Mathematical Union, an organization of 41 members, one of them the Taiwanese mathematical association. The ICM secretariat told me that it received no communication from either part of China, although it did receive about 80,000 letters, including some 200 containing purported proofs of Fermat's Last Theorem.

The enormous growth of the science was reflected in the increase from two to four of the number of Fields Medals awarded. Two of these medals, which may be described as "Nobel prizes" for younger mathematicians, were awarded to Americans—Paul J. Cohen (Stanford) for his work in the foundations of mathematics and Stephen Smale (Berkeley) for his contributions to differential topology. M. F. Atiyah (Great Britain) received one for his contributions to topology and partial differential equations, as did A. Grothendieck (France) for his contributions to algebraic topology.

Grothendieck did not attend the Congress. The Organizing Committee, which had included him (and the other medalists) among the 83 distinguished scholars invited to give special addresses, informs me that it received no explanation from him. The other three medalists received their awards at the opening ceremonies held in the Palace of Congresses at the Kremlin.

There appeared to be universal agreement that all four eminently deserved their awards. But there was also substantial feeling that the awarding committee might well have recognized the achievements of at least one of what the president of the International Mathematical Union, G. de Rham (Switzerland), characterized in his closing address as "the abundance of brilliant young Soviet mathematicians," especially since the number of such awards is not fixed. (No Soviet mathematician has ever received a Fields Medal.)

Of the work of Smale some mention has already been made in *Science* (7 October 1966) in an article devoted mainly to his difficulties with the House Committee on Un-American Activities, other Congressmen, and agencies. For his scientific work he was awarded in January 1966 a Veblen Prize by the American Mathematical Society.

Given present-day specialization in mathematics, it is likely that only Cohen's work, being in the foundations of mathematics, is in an area with which all mathematicians feel they should have a nodding acquaintance. This work, published in summary form [*Proc. Natl. Acad. Sci. U.S.* **50**, 1143–1148 (1963); **51**, 105–111 (1964)], is in the theory of sets, and is concerned with the problem of counting infinite collections. Two sets are regarded as having an equal "number" of elements if the elements of one can be put into one-to-one correspondence with those of the other.

The set P of all positive integers can clearly be put into such correspondence with the set Q of all positive even integers by associating with each element of P its double in Q, and conversely. This common transfinite cardinality was denoted as aleph-null by the founder of the theory of sets, G. Cantor (Germany). He showed that no infinite set has fewer than aleph-null elements, but that many have more, for example, the set of all real numbers. He also showed that there is a next larger transfinite cardinal, aleph-one, and conjectured that there are precisely aleph-one real numbers, that is, sets of integers. This became known as the continuum hypothesis.

What Cohen has established is that the continuum hypothesis can be neither proved nor disproved on the basis of the standard structure (axioms) of the theory of sets. Moreover his work showed that none of the additional axioms that have been proposed can be of any assistance in resolving this question.

Perhaps subsequent investigations will reveal new principles on the basis of which Cantor's continuum hypothesis can be settled, perhaps not. On this point intense controversy now centers.

Cantor was led to his studies in the theory of sets by his earlier work on trigonometrical series. In this subject too there was presented a solution of its most celebrated problem. On the eve of the Congress, L. Carleson (Sweden) published [*Acta Math.* **116**, 135–157 (1966)] a proof of the famous conjecture of N. Lusin (U.S.S.R.) that the Fourier series of a periodic continuous function (more generally, even only an L_2 function) converges, except possibly on a set of measure zero. This proof, now undergoing intense study by specialists everywhere, was contrary to the expectations of many leading authorities who had come to believe that Lusin's conjecture was wrong. About 40 years ago, A. Kolmogorov (U.S.S.R.) had constructed a Lebesgue integrable function whose Fourier series diverges everywhere. This famous example does not, of course, conflict with Carleson's result, since Kolmogorov's function is not continuous, nor even L_2.

In a paper that appeared immediately after the Congress, J.-P. Kahane

(France) and Y. Katznelson (Israel) showed that Carleson's result is "best possible" [*Studia Math.* **21,** 305–306 (1966)]. They proved that, given an arbitrary set of measure zero, there exists a continuous periodic function whose Fourier series diverges on the given set.

A set of measure zero (equivalent to the concept of zero probability) is a set on which one can change arbitrarily the values assumed by a (Lebesgue) integrable function without altering the value of the integral.

The failure of Fourier series to reproduce for all values its generating function, even when that generating function is continuous (a fact known since 1876) naturally has led mathematicians to consider the problem of constructing, if possible, systems analogous to the Fourier trigonometric system $\{1, \sin x, \cos x, \ldots, \sin nx, \cos nx, \ldots\}$ which have the property that the Fourier series constructed from them will reproduce continuous generating functions. Systems of great importance having this property were brought to light, but none of them possessed all the fundamental properties of the trigonometric Fourier sequence.

At the Congress, a young Soviet mathematician, A. M. Olevskii, showed that nothing better can be done. More precisely, he proved that there exists no uniformly bounded, orthonormal system such that the Fourier series (with respect to that system) of an arbitrary continuous function must always reproduce that function everywhere. Together with related interesting results, he has published this in the *Izvestiya* of the Academy of Sciences of the U.S.S.R. [Math. Series, **30**, 387–432 (1966)].

From the work which I have described, the Moscow conference would seem to be characterized more by the solution of famous problems than by the indication of new directions. Those able to evaluate other work presented may provide a different impression.

The most important new paths will probably result from the informal discussions among the 4300 mathematicians who gathered from 54 countries. This represented the first large-scale contact between the mathematical communities of the U.S.S.R. and non-socialist countries, undoubtedly the most valuable contribution of the Congress.

Another value of the Congress, simply by virtue of its existence, is that it assembled enough mathematicians in one place so that nearby areas could schedule highly specialized conferences for much smaller groups (about 300 each) to present and discuss research on tightly knit topics. Czechoslovakia, Finland, Hungary, Italy, and Poland were sites of such gatherings, either just before or just after the Congress.

The holding of a scientific congress is clearly regarded as a great event in the U.S.S.R. A special stamp was issued by the postal authorities; the Soviet press carried extensive accounts both of the ICM and on the subject of mathematics itself, before, during, and after the Congress. For example, both academician I. G. Petrovskii (rector of Moscow University and president of the ICM) and V. G. Karmanov (Secretary of the ICM Organizing Committee) published feature-length articles on mathematics.

There were interviews with both Soviet and foreign mathematicians. In one such interview, Fields Medalist Cohen expressed high praise for Moscow University, for Soviet mathematical life generally, and characterized the organization of the Congress as "perfect." He added that the participants had "every opportunity for fruitful work, to see Moscow, and the life of Soviet people."

In closing this report, it may be particularly appropriate to recall the words of the late O. Veblen, after whom the American Mathematical Society named its research prize in geometry. As president of the ICM in 1950, when it met in the United States, he concluded his address with these words: "To our non-mathematical friends we can say that this sort of a meeting, which cuts across all sorts of political, racial, and social differences and focuses on a universal human interest, will be an influence for conciliation and peace."

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Chromosomes and Leukocytes

At a conference "Leukocyte Chemistry and Morphology Correlated with Chromosome Anomalies" (New York, 3–5 November 1966), several papers on the etiology of Down's syndrome (trisomy-21 or mongolism) were presented. These included data that the familial form of trisomy-21 represents a hereditary tendency for ovogonial nondisjunction, which increases with maternal age. Significantly more mothers of patients with Down's syndrome and gonadal dysgenesis had thyroid autoantibodies.

The status of information on meiotic chromosomes, the incidence of sex-linked and autosomal chromosomes, and the phenotypic expressions in patients with sex chromosome anomalies was discussed.

Viruses and radiation were subjects of papers on the absence of metaphase figures and other chromosomal aberrations in cultures of leukocytes from patients infected with measles virus, and on polyploidy and endoreduplications in cultures of leukocytes taken from patients after therapy with cobalt-30, iodine-131, and x-irradiation.

Morphological changes in the nucleus and the limiting membranes of polymorphonuclear leukocytes were also described. Characteristic nuclear projections were present in neutrophils of patients with trisomy D(13/15), but did not occur in neutrophils of patients with trisomies E or 21. A direct correlation was reported between the size of the X chromosomes and that of the drumsticks in neutrophils. The XXY karyotype lowered the incidence of drumsticks, but did not affect that of Barr bodies. Limiting membranes in the polymorphonuclear leukocytes are abnormal in patients with the Chediak-Higashi and Batten's syndromes, both of which represent the homozygous expression of autosomal recessive genes. A high incidence of abnormal granules in leukocytes is associated with the carrier state of Batten's disease.

Leukocyte alkaline phosphatase activity (LAPA) is absent or very low in most patients having chronic myeloid leukemia (CML) and the Ph^1 chromosome. However, reports in the literature that the LAPA index increases during remissions after busulfan therapy were not confirmed. Elevations of the LAPA index are considered of diagnostic value in indicating the possibilities of infection and ulcerative colitis in patients with CML. In addition, this LAPA index is useful in differentiating polycythemia vera from secondary erythrocytosis, and chronic granulocytic leukemia from leukemoid reactions. The absence, or a very low level, of LAPA is not characteristic of atypical cases of CML, and this index is not useful in differentiating other myeloproliferative syndromes.

Cytochemical assessment of LAPA by Kaplow's method is an inexpensive,