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Quantitative Growth of the Mathematical Literature

Abstract. *Since 1868 the number of mathematical publications per year (measured by counts of titles abstracted) has grown from about 800 to 13,000 at an average continuous compound rate of about 2.5 percent per year, doubling about four times a century. Deviations from the exponential curve are clearly related to war, depression, and recovery. If the total number of publications prior to 1868 is estimated by extrapolating from the curve of annual output, the cumulative grand total of mathematical titles grows from 41,000 in 1867 to 419,000 by the end of 1965. Deviations from an exponential growth of 2.5 percent per year are negligible except for two "pauses" during world wars, after which the observations continue parallel to the theoretical curve. The well-known hypothesis of exponential growth of the scientific literature is strongly confirmed but at a rate less than half that found by Price and other investigators. The discrepancy appears to be due to the failure of previous studies to take into account the titles published before the beginnings of the time series used.*

There is overwhelming evidence that scientific literature increases exponentially (for examples, see *1–3*). Yet in spite of the existence since 1868 of at least one journal abstracting "all" mathematical titles, published data on the mathematical literature are lacking (*4*). Figure 1 shows the annual output of mathematical titles as measured by complete counts of author indexes (jointly authored papers counted only once) in the *Jahrbuch über die Fortschritte der Mathematik* for 1868 through 1940 and in the *Mathematical Reviews* for 1941 through 1965 (*5*). Because of delays in publication and abstracting, differences between nominal and actual dates of publication, and other random factors, the dating of a title is subject to significant errors. Hence we have plotted a 5-year moving average located at the middle year. The interruptions of steady growth are clearly related to the two world wars and the great depression of the 1930's. It is interesting to note that the wars produced almost equal minima, and that recoveries occurred at the same absolute linear rates, as indicated by the parallel straight lines. The break with linear recovery around 1957–1961 was due to a failure of abstracting to keep up with the literature (*6*).

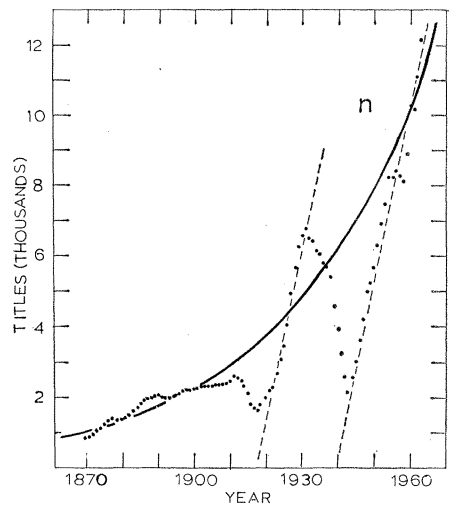

Fig. 1. Five-year centered moving average of the annual number of abstracts 1868–1965. The smooth curve n is given by Eq. 1 in the text. The broken lines suggest parallel linear growth in the postwar periods.

The smooth curve in Fig. 1 is the exponential

$$n = 1400\, e^{0.025(t\,-\,1880)} \qquad (1)$$

chosen to achieve close fit during the "normal" years prior to the first world war and to pass through what appear to be middle points of the war-induced oscillations (*7*).

Because of the violent oscillations in the war periods, Fig. 1 is hardly conclusive evidence for exponential growth, but it is essential to our main purpose of analyzing the growth of the cumulative total of mathematical titles. We could obtain cumulative totals by simply summing the successive yearly counts graphed in Fig. 1, but this would underestimate the total by ignoring the literature prior to 1868. To avoid this pitfall we estimate the cumulative total through 1867 by assuming that the annual output is given approximately by the smooth curve, that is, by Eq. 1. This gives an estimate of 41,000 titles through 1867 (*8*). We then obtain the cumulative totals by adding to 41,000 the successive annual numbers of titles shown in Fig. 1. These points are plotted in Fig. 2 together with the corresponding theoretical curve

$$N = 56{,}000\, e^{0.025(t\,-\,1880)} \qquad (2)$$

The fit in Fig. 2 is extraordinarily close, and deviations are clearly due to the two world wars, which appear to postpone growth rather than to alter its character or rate. These findings strongly support those of Price for physics (*1*, pp. 102–104; *2*, pp. 17–19). On the other hand, the rate of growth found here is only about half that found by Price. On the basis of data from several fields he conjectured "It seems beyond reasonable doubt that the literature in any normal, growing field of science increases exponentially, with a doubling in an interval ranging from about ten to fifteen years" (*1*, footnote, pp. 102). Such a doubling interval corresponds to an annual increase of from about 7 to 5 percent, whereas we have found here for mathematics an annual increase of about 2.5 percent and doubling about every 28 years.

Before jumping to the conclusion that mathematics has a different growth rate than other sciences, note that although Price speaks of "the literature" as though he were referring to the total literature, his data are actually for the literature in each field after a certain time, in each case the beginning of an abstracting service: 1900 for physics, 1908 for chemistry, 1927 for biology, and 1940 for mathematics (*2*, p. 10). Figure 3 shows the effect of similarly ignoring the mathematical literature prior to the various dates. The straight line is the semilogarithmic graph of the exponential curve of Fig. 2 and Eq. 2.

The curves P, B, and M are obtained by ignoring the literature prior to 1900, 1920, and 1940. Of course, all these curves will eventually approach parallelism with N, but up to 1950 they seem to be straightening out at much smaller doubling periods and much higher rates of growth. By ignoring the prior literature we have obtained growth rates comparable to those of Price. Indeed, curves P, B, and M look very much like his curves

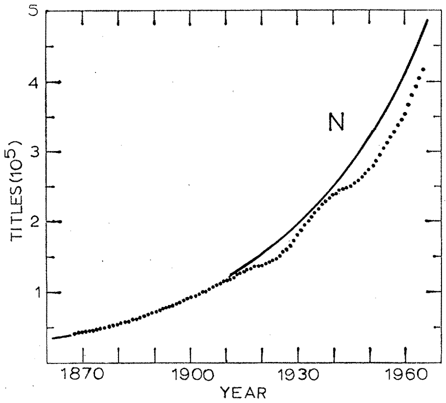

Fig. 2. The cumulative total of titles abstracted plus 41,000 titles estimated to be the cumulative total through 1867. The smooth curve N is given by Eq. 2 in the text.

for physics, biology, and mathematics respectively (*2*, fig. 2, p. 10). It appears likely that if Price and others took into account the literature prior to their statistical series, they would obtain substantially lower growth rates. This analysis supports the conjecture that the overall total scientific literature has been accumulating at a rate of about 2.5 percent per year, doubling about four times a century. When the literature prior to any year is ignored, the rate of growth is overestimated, and we may expect the future observation of spurious declines in growth, if growth rates actually remain constant, or failure to observe accelerations if they actually occur.

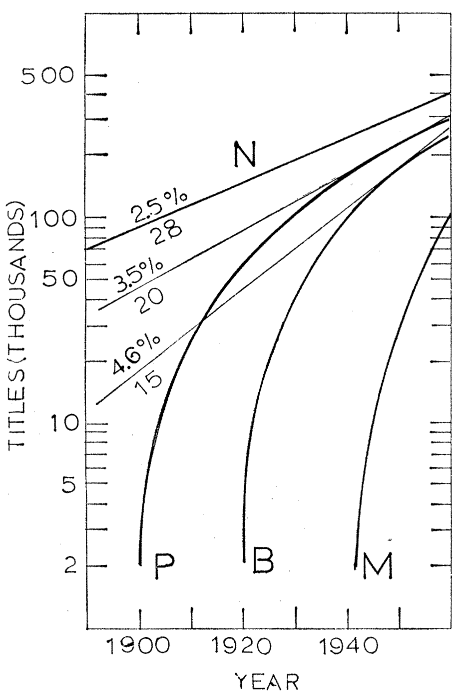

Fig. 3. Graphs on semilogarithmic paper of the theoretical curve N of Fig. 2 and of $P = N - 90{,}000$, $B = N - 150{,}000$, and $M = N - 250{,}000$ obtained by ignoring the literature prior to 1900, 1920, and 1940, respectively. The tangent lines show estimated growth rates increasing from 2.5 to 4.6 percent and doubling periods decreasing from 28 to 15 years. Note that relatively small changes in inclination of a straight line produce significant changes in estimated growth rates and doubling periods.

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References and Notes

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2. ———, *Little Science, Big Science* (Columbia Univ. Press, New York, 1963), chap. 1.
3. L. N. Ridenour, *Bibliography in an Age of Science* (Univ. of Illinois Press, Urbana, 1951).
4. The only published study of which I know is H. S. White, "Forty years' fluctuations in mathematical research," *Science* **42**, 105 (1915). Since the author gives no numerical data and omits vertical scales on his graphs, this work can be no more than suggestive of interesting lines of inquiry.
5. In some volumes of the *Mathematical Reviews* the total was estimated by sampling. After 1958 abstracts are serially numbered. In 1870 the *Jahrbuch über die Fortschritte der Mathematik* commenced the publication of an annual volume abstracting the entire world literature in mathematics for a single preceding year beginning with 1868. The plan was followed with fair success, but the substantial delays were unsatisfactory, and in 1931 the *Zentralblatt für Mathematik und ihre Grenzgebiete* began to abstract the mathematical literature more rapidly. Dissatisfaction with the Nazi control of these journals led to the founding of the *Mathematical Reviews* in 1940. The *Jahrbuch* stopped publication in 1942, but the *Zentralblatt* continues. Counts in the *Zentralblatt* from 1930 to 1965 are not inconsistent with those obtained from the *Jahrbuch* and the *Mathematical Reviews*. I have not made use of the more recently founded Russian *Referativni Zhurnal* and French *Bulletin Signalétique*, nor of the older Dutch *Revue Semestrielle*, since there seemed little likelihood that they would add information. The tabulated counts and derived data may be obtained from the author.
6. The *Jahrbuch über die Fortschritte der Mathematik* also fell behind, but it tried to publish the literature for a given year together, except for five occasions when two or more years were combined in a single volume. After 1940, the count is no longer of titles published in a year but of titles abstracted. If the tedious task of sorting out titles by year of publication were done, the points after 1940 would be shifted slightly to the left and up. The break in 1960 would disappear.
7. One should not expect too much from extrapolation beyond the range in which a curve is fitted, but Eq. 1 gives $n = 6$ for the year 1665, and this is not unreasonable for the period of the founding of the first scientific journals.
8. The estimate was obtained by integrating Eq. 1 from minus infinity to 1867. This amounts to adding together the theoretical annual frequencies given by Eq. 1 for all years prior to 1868. Equation 2 is just the integral of Eq. 1 that is asymptotic to the t-axis as time recedes. Any other integral of n is given by $N + C$, where C is a constant, but only $C = 0$ yields a curve of exponential growth. Taking $C < 0$ corresponds to ignoring some of the previously accumulated literature.

9. This work was done while I was on leave from Carleton College as visiting research mathematician at the University of California at Berkeley. It was partially supported by a grant from the Society of Sigma Xi. Counts and calculations were done by Robert H. Holmes.

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Hormonal Termination of Diapause in the Alfalfa Weevil

Abstract. *Topical treatment of the alfalfa weevil,* Hypera postica *(Gyllenhal), with the synthetic juvenile gonadotropic hormone 10,11-epoxyfarnesenic acid methyl ester effectively terminated summer diapause.*

Diapause is a condition of physiologic arrest that allows many insects to survive extended periods of cold, heat, and drought. In insects that diapause as adults, there is accumulation of lipid, substantial decrease in respiration, characteristically reduced activity and feeding, and no reproduction (*1–3*). The endocrine-mediated regulation of adult diapause in *Leptinotarsa decemlineata* (Say) has been confirmed in several studies since extirpation of the corpora allata (source of the juvenile gonadotropic hormone) was found to reproduce all the behavioral and physiologic effects associated with normal diapause (*1, 2*). However, these investigations have also shown that, although the surgically induced diapause is completely reversed by implantation of corpora allata, normally diapausing beetles are unaffected by implantation of the glands.

Thus, normally diapausing beetles apparently have some humoral inhibition that represses development of and secretion by the host's own corpora allata as well as implanted glands. Such inhibition is apparently not operative in allatectomy-induced diapause. Therefore, if diapause results from some humoral inhibition of corpora-allata development and secretion, it should be possible to circumvent the effects of the inhibitor and break diapause if the corpus allatum hormone itself or a sufficiently active synthetic hormone is supplied exogenously. We tested this hypothesis by treating diapausing adult alfalfa weevils, *Hypera postica* (Gyllenhal), with the synthetic juvenile gonadotropic hormone *trans-trans*-10, 11-epoxyfarnesenic acid methyl ester (*4*).

Adult weevils, reared in a manner described (*5*), already 2 weeks in diapause that would have normally continued for 12 to 14 weeks longer, were treated topically on the venter of the abdomen with 0.1, 1.0, 10, 50, or 100 μg of 10,11-epoxyfarnesenic acid methyl ester in 0.5 μl acetone. Control groups were either treated with acetone or left untreated. Each group contained 12