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Three-Dimensional Map Construction

Use of the crossed-slit anamorphoser simplifies the technique and shortens construction time.

George F. Jenks and Dwight A. Brown

Three-dimensional maps are perspective representations of obliquely viewed statistical or topographic surfaces. Many readers are familiar with isometric topographic or geologic block diagrams, but these comprise only a fraction of the three-dimensional maps which have been, or could be, made. Maps showing some of the potential of this type of illustrative device are shown in Fig. 1. Why not, however, use maps of this kind to illustrate submarine or fish environments; airport facilities and landing obstacles; the development of hurricanes; intersecting statistical surfaces; the movement of fog, smog, and other air pollutants; and any other areal phenomenon which can be thought of as a three-dimensional surface?

At the turn of the century, block diagrams were common in geologic and geographic literature, but they are not often used today. Why have such useful illustrations been, in large part, discarded by modern authors? The answer seems to lie in both the economic fact and the mistaken belief that preparation of such maps requires special training. True, conventional three-dimensional map construction is so time-consuming that the cost of individual illustrations has become prohibitive, and it requires considerable skill on the part of the draftsman. However, the construction of three-dimensional maps need be neither costly nor overly demanding of artistic skill, as we hope to show. More specifically, this article contains a brief statement on the basics of map perspective, a section in which a unique construction technique is introduced, and discussion of a new method of optical transformation of map coordinates. Furthermore, the possibility of using the transformation technique in fields other than cartography is discussed.

The techniques introduced here are not based upon computer-driven plotout devices, but mention should be made of experimental work in this area. Computerized perspective illustrations have been developed to a high degree by the computer and aerospace industries, which have produced three-dimensional views of aircraft and aircraft parts, rendezvous, cosmic radiation belts, and so on, to bridge the information gap between the scientist and engineer and the nonscientist (*1*). It is quite apparent that computer technology will be refined to the point where many kinds of maps will be produced by the computer in final drafted form. Unfortunately, however, the time lag between what is to be and what is now technologically or economically feasible may be considerable. During this interim period the need for three-dimensional maps will have to be met by other means, such as those presented here.

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Basics of Three-Dimensional Map Perspective

Three-dimensional maps present earth surfaces as viewed from a real or assumed position in space. The viewing point can be any compass position, at any angle above or below the horizon (*2*). The map image that is presented for any viewing point is theoretically formed on a transparent perspective plane. This plane is perpendicular to the central line of vision and lies at a point between the eye of the viewer and the earth surface being mapped. In cartographic presentation the perspective map image, or transformed map, is constructed geometrically, but it can be visualized as being formed by converging projectors (in angular-perspective presentation) or by parallel projectors (in parallel-perspective presentation) (see Fig. 2).

Three-dimensional maps are conceptualized views of what the distributive surface *might look like,* and they cannot be regarded as oblique photographs of earth models. As a result, it is best to think of the process of construction as that of proceeding from a planimetric map to a transformed or foreshortened view of this flat map. The steps in construction are illustrated in Fig. 3. The planimetric map (Fig. 3A) is transformed by angular perspective to Fig. 3B, and then the various features are elevated on the vertical axis (Fig. 3D). In Fig. 3C the same map (Fig. 3A) is transformed by parallel perspective (*3*), and in Fig. 3E it is elevated, with a vertical scale equal to that used in the foreground of Fig. 3D.

The three-dimensional maps of Fig. 3 illustrate some of the differences between angular- and parallel-perspective presentations of earth surfaces. In terms of visual characteristics the angular-perspective map (Fig. 3D) is superior because the convergence cues contribute to the illusion of depth without detracting from the overall configuration (*4*). The parallel-perspective map (Fig. 3E) appears to be a contrived version of the surface because the parallel sides of the map give an illusion of divergence and the background appears to be tilted forward. There is little question that the angular-perspective map more closely simulates the "real" surface than the parallel-perspective map does.

At this point one might ask, Why bother with parallel perspective in three-dimensional mapping? The question must be answered in two parts: in terms of scaling and in terms of ease of construction. Three-dimensional maps are tools for transferring information, and measurements on the map surface are often a means to this end. On maps constructed in angular perspective, the scale changes continuously over the entire surface of the map in both the vertical and the horizontal planes. Thus, measurements on this changing surface can be made only with a perspective scale constructed to fit a given map. On a parallel-perspective map the scale is constant along any series of parallel lines, and thus measurements can be made along these orientations in both the horizontal and the vertical planes. Because of this, the height or length of a feature can be accurately compared with the height or length of another feature anywhere on the map.

Readers of three-dimensional maps are often more aware of the convergence of lines in the horizontal plane than in the vertical plane. In construction, however, it is the convergence in the vertical plane that adds to the cost of constructing angular-perspective maps. For example, the top circle of Fig. 3D is elevated more in the foreground than in the background, because the vertical scale decreases from front to back on the map. Compare this elevation with that of the same feature in map 3E; in the latter map there is no convergence and the whole feature is elevated in one operation.

The cartographer selecting the perspective basis for constructing a three-dimensional map must choose between realism and practicality. Practicality compels him to select parallel perspective; realism, to select angular perspective. When three-dimensional maps become readily available from computer output, construction costs of the two types will be comparable and the choice will depend on the user's requirement. Until that time comes, production of angular-perspective maps will not be economically feasible (*5*). The acceptability of parallel-construction presentations varies from map to map, as may be seen from a comparison of Figs. 3E and 4: Fig. 3E is rather unsatisfactory; Fig. 4 is quite satisfactory. In general, parallel-perspective maps with low-to-moderate vertical-scale exaggeration and low angles of view are acceptable, while those with great vertical scale exaggeration, or high viewing points, are less desirable.

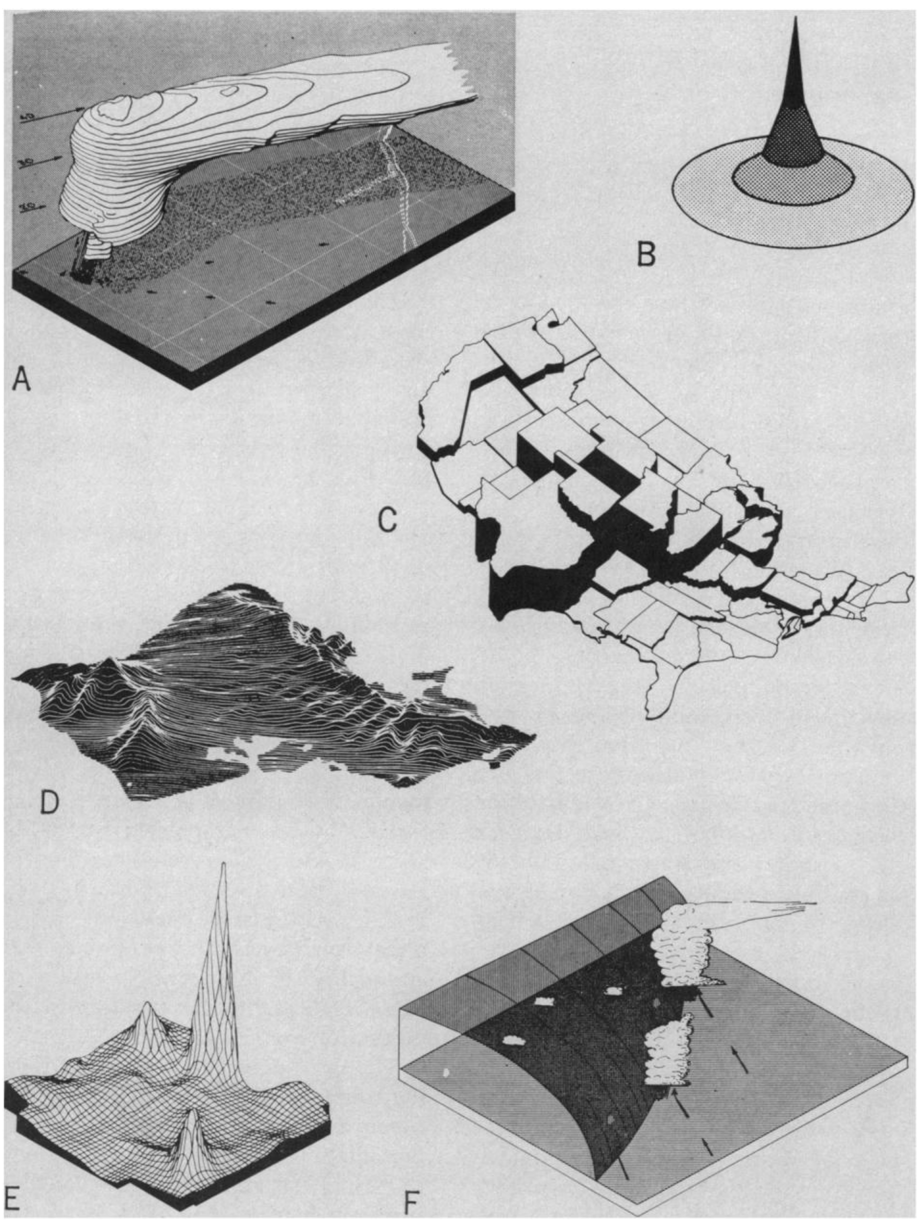


Fig. 1. Three-dimensional maps vividly portray physical and statistical surfaces that are difficult to visualize in conventional two-dimensional format. The examples illustrated here are (A) a thunderstorm with hail; (B) land values in a theoretical city; (C) value of goods shipped from various states to Kansas; (D) the topography of Oahu, Hawaii, as viewed from the south; (E) population density in central Kansas; (F) passage of a cold front across the Great Plains.

Construction of Three-Dimensional Maps in Parallel Perspective

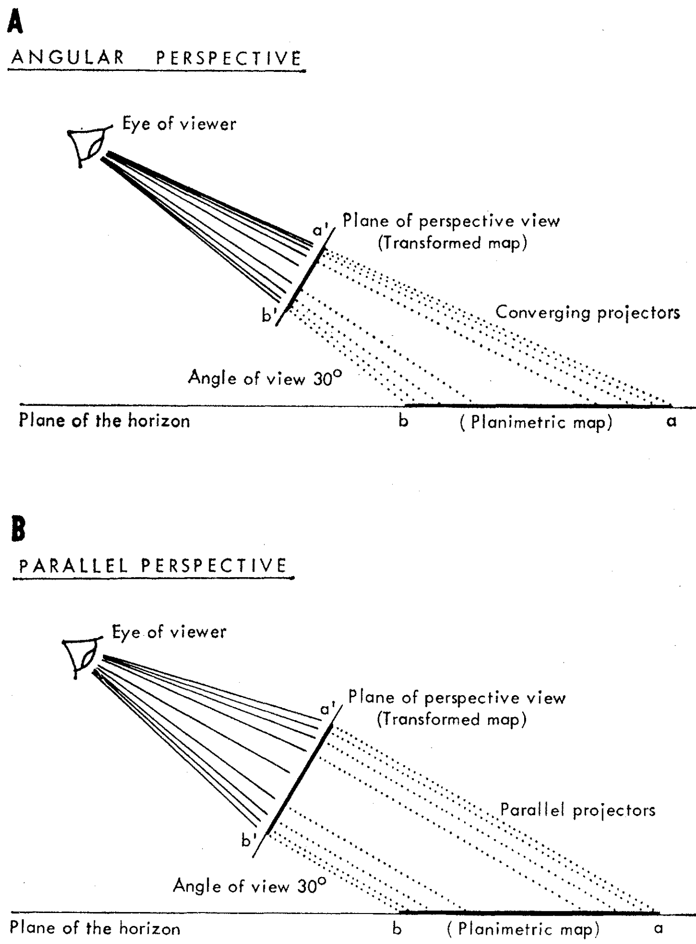
The labor expended in constructing a three-dimensional map can be divided into two major efforts. In the first, two-thirds to three-fourths of the cartographer's work is spent in transforming the planimetric map to a foreshortened map. In the second, the map-maker develops an undulating surface above the transformation. This latter process requires much less effort than the first, but the untrained map-maker is often dismayed at the prospect of the drafting. Actually, the complete development of the map can be accomplished with a pencil, a straight edge, and a drafting fountain pen. The procedure is as follows.

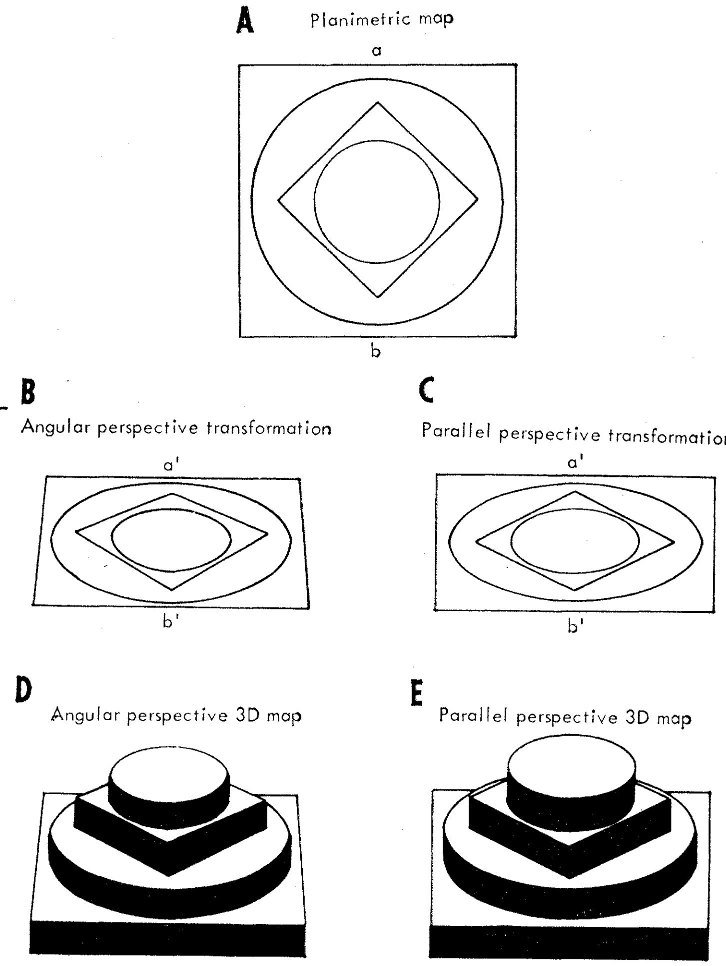
1) Selecting a viewing point. Selection of the viewing point should be based on the orientation of the more significant features on the surface being

mapped. Since low-lying areas are usually blocked from view by higher features, a viewing point is often selected such that low areas occupy the foreground of the final map (Fig. 5A).

2) Preparing a transformation and selecting a vertical scale. The dimensions and shape of the transformation are determined by the elevation and orientation of the viewing point. Foreshortening is a function of the sine of the elevation angle, and the shape of the transformation depends on the orientation of the view. A square map transforms to a rectangle if the viewing point is located on a sight line perpendicular to a side of the map; in any other orientation the transformation takes the form of a nonrectangular parallelogram. Traditionally, transformations are made by hand with point-by-point transferal of detail from a square to a foreshortened grid. Optical transformation (discussed in the next section) shortens this process. A vertical scale which exaggerates reality is selected, because most earth surfaces appear relatively flat when viewed from above. At present there are few guidelines in the selection of vertical scales, but work on this problem is now being done. Note (Fig. 5B) that the vertical scale is plotted along the viewing axis and in inverted form.

3) Elevating the data surface. A sheet of drafting medium is placed over the transformation, and an alignment axis and tick are drawn on it. These two sheets can now be registered in position





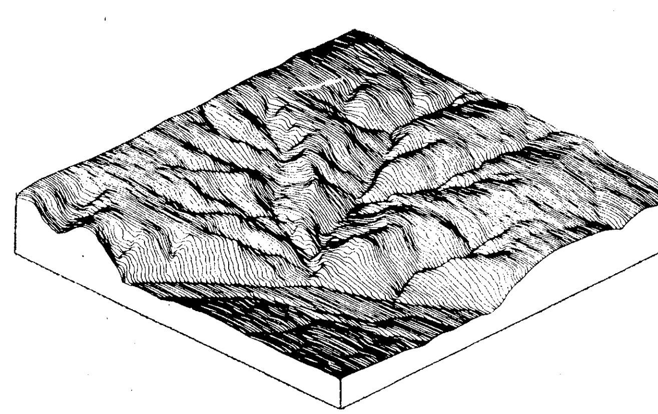


Fig. 2 (above). In two-dimensional maps, perspective illustrations are constructed with angular or parallel geometric projection. The perspective images are theoretically formed on transparent planes which are perpendicular to the central line of sight. In angular perspective (A) an image is formed upon this plane by converging projectors; in parallel perspective (B) the image is formed by parallel projectors. Fig. 3 (top right). Both angular and parallel perspective can be used in the construction of three-dimensional maps. Both types of three-dimensional maps are constructed from planimetric maps (A); the planimetric maps are transformed to perspective views (B and C), and then the surface is elevated on the vertical axis (D and E). The final product is a cartographic interpretation developed from a two-dimensional map; this interpretation varies with the type of perspective used, the viewing axis and the elevation, and the amount of vertical exaggeration. Fig. 4 (bottom right). A parallel-perspective three-dimensional map of a small drainage basin in southern Kansas. The viewing point is 30 degrees above the horizon, and the viewing axis is aligned with the major trend of the basin. While the elevated background of the map is unrealistic, this disadvantage is slight as compared with that of the greater expense of constructing an angular-perspective map of the same feature.

for any elevation on the vertical scale, and data are transferred by tracing directly from the transformed surface. The draftsman starts at the top of the surface and works down, so that areas blocked from view by higher features can be omitted (Fig. 5, C and D).

4) Overlaying the elevated surface with parallel lines and drawing profiles. The parallel lines are oriented in any position that the cartographer chooses, except in the case of profiles along the alignment axis. Profiles along that axis are all straight lines, and no illusion of depth results when this view is taken. For vertical profiles, the spacing of the parallel lines should correspond with the vertical scale; for slanted profiles the spacing should be wider. In either case, profiles are drawn through intersections of the parallel lines with the isometric lines on the elevated data surface. The cartographer can begin at any intersection of a parallel line and an isometric line. He locates the next isometric line and determines whether it is up-slope or down-slope. If up-slope, he draws the profile from the first intersection to the intersection of the up-slope isoline and the next higher parallel line (Fig. 5E).

5) Inking the profiles for final copy. Figure 5F shows a completed map, after inking. Figure 5F is a section (upper right background) of Fig. 4, a smaller-scale map than Fig. 5, A–E.

The construction procedures outlined above are essentially adaptations of techniques previously reported by Lobeck, by Schuster, by Stacy, and by Tanaka (*6*). The method of drawing vertical profiles without constructing numerous verticals is unique, quick, and reliable. Even though each vertical profile is a simple uniform line, the combination of profiles gives the final map an appearance of a continuous surface. Furthermore, the technique makes it possible to render minor nuances in surface configuration which are frequently lost in other forms of symbolization.

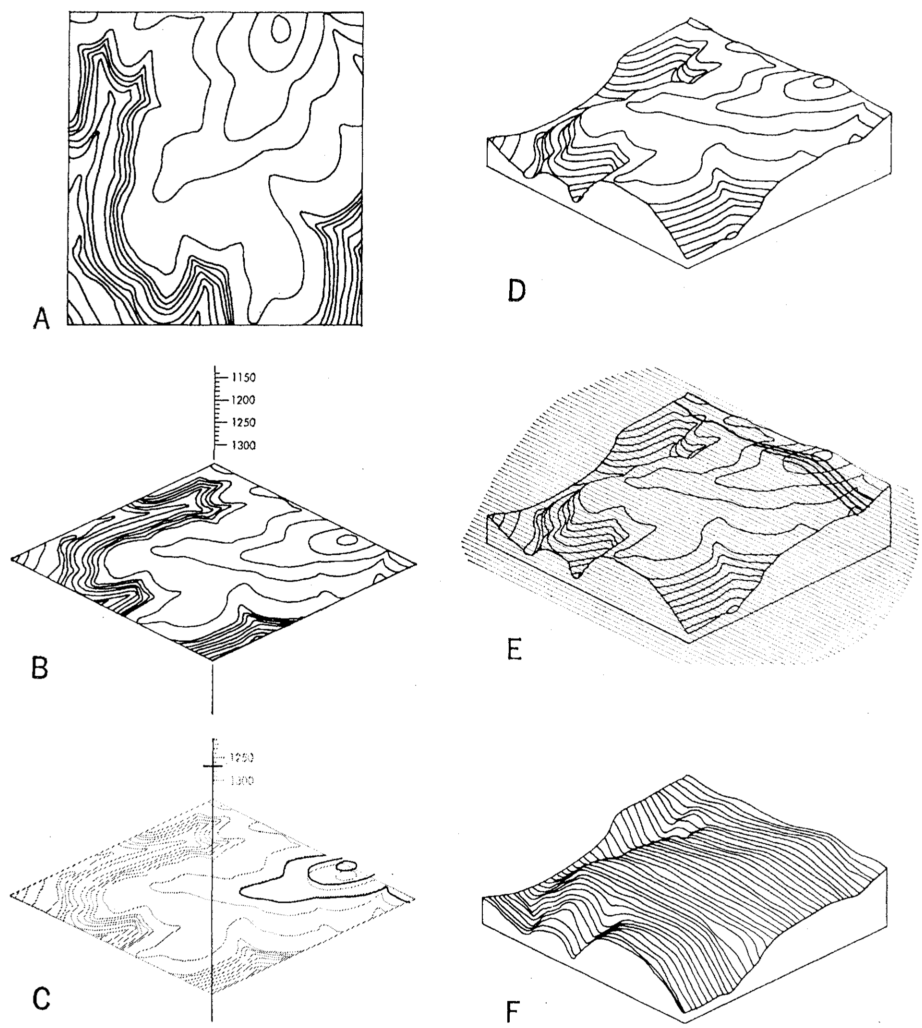


Fig. 5. Steps in the construction of a three-dimensional map with vertical profile symbolization. After the planimetric map has been analyzed, a viewing axis and elevation are selected; in this case (A) the axis is SE and NW and the elevation is 40 degrees above the horizon. (B) A perspective view is then constructed and a vertical scale selected (note that the vertical scale is plotted inversely in preparation for the next construction step). (C and D) A sheet of tracing medium is placed over the transformation and raised along the vertical axis until all the surface data are transferred to elevated positions. (E) Vertical profiles are constructed by overlaying the elevated surface with a series of parallel lines spaced to correspond with the vertical scale, and then drawing profile lines through the intersections of the parallel and the isometric lines. The final map (F) is a clean ink tracing of the profiles.

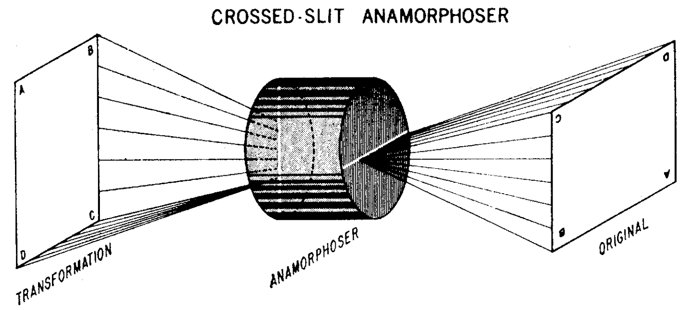
Fig. 6 (opposite page, top left). When light coming from an object passes through the slits of an anamorphoser, the positions of coordinates on the original are transformed to a new set of *x*-*y* coordinates. Unlike the camera lens, which essentially enlarges or reduces coordinate patterns, the anamorphoser changes the shape of the image. For example, in this diagram the square original is changed into a rectangular transformation.

Crossed-Slit Anamorphoser

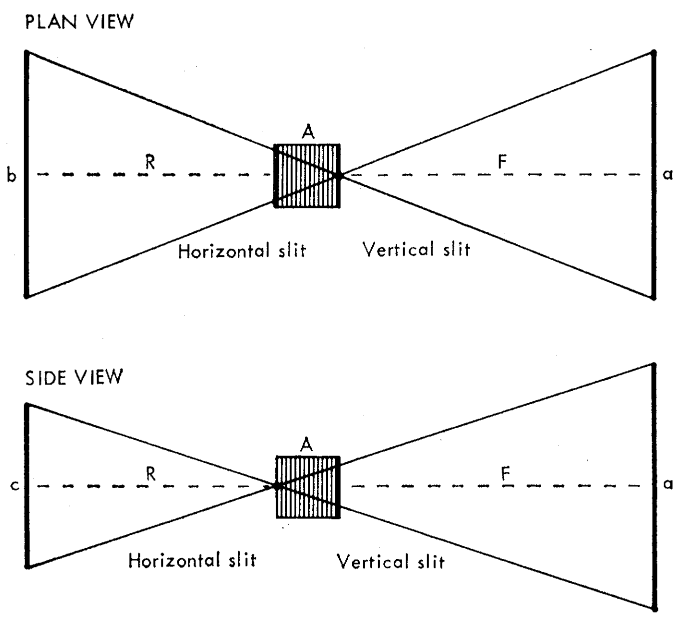
The crossed-slit anamorphoser (Fig. 6) is a simple optical device which transforms photographic images passing through two slits located on parallel planes. The device is not new, having been patented in France in 1895 and in this country in 1927 (*7*). Apparently its inventors were primarily interested in the humorous applications of the device, and as far as can be determined it has not previously been utilized for map-coordinate transformation.

Geometry of the anamorphoser. The transformation of map coordinates takes place when light rays reflected from the original map are passed through the two slits and registered on a photographic medium. Rays entering the first slit converge along its linear axis and then pass through the rear slit, effecting convergence along its line of orientation. The effect of this configuration is much the same as the effect of arranging, if it were possible, an infinite number of pinhole cameras side by side. The quality of the image formed by the anamorphoser is also similar to that of the pinhole camera.

When the anamorphoser is used for parallel transformation the slits should be straight and oriented at right angles. In addition, the original map, the anamorphoser, and the photographic negative should all be maintained in parallel planes perpendicular to a cen-

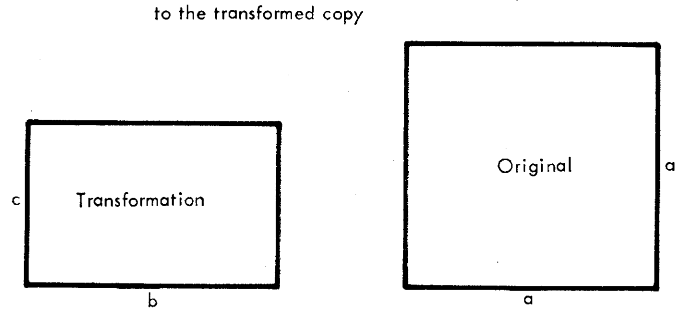


GEOMETRY OF ANAMORPHOSER TRANSFORMATION



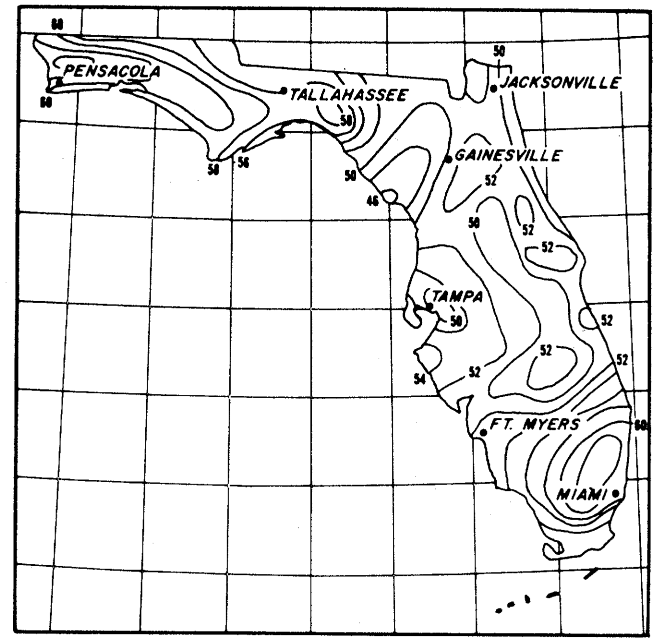
a = a known dimension on the original map
b = the long dimension of the transformed map; dimension "a" multiplied by the reduction or enlargement ratio
c = short dimension of the transformation; dimension "b" multiplied by the foreshortening ratio which equals the sine of the angle of view

A = distance between slits of the anamorphoser
F = distance from the front slit of the anamorphoser to the original map
R = distance from the rear slit of the anamorphoser to the transformed copy

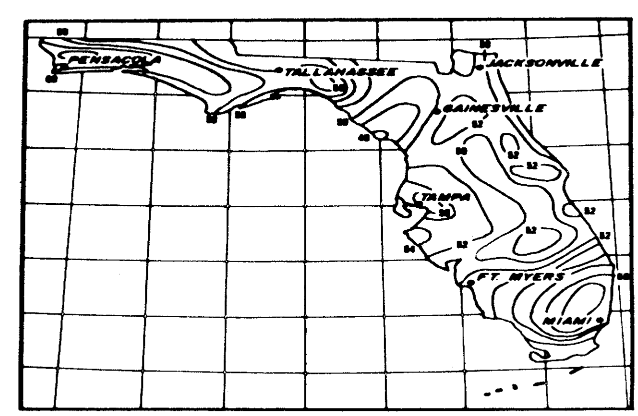


Transformation Formulae

$F = \frac{(a+c)A}{b-c}$ $R = \frac{c(F+A)}{a}$ $b = \frac{(a+c)A + cF}{F}$ $c = \frac{aR}{A+F}$



A



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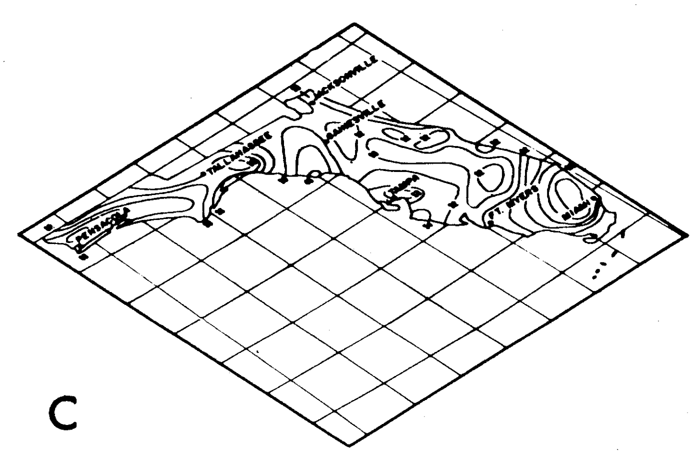
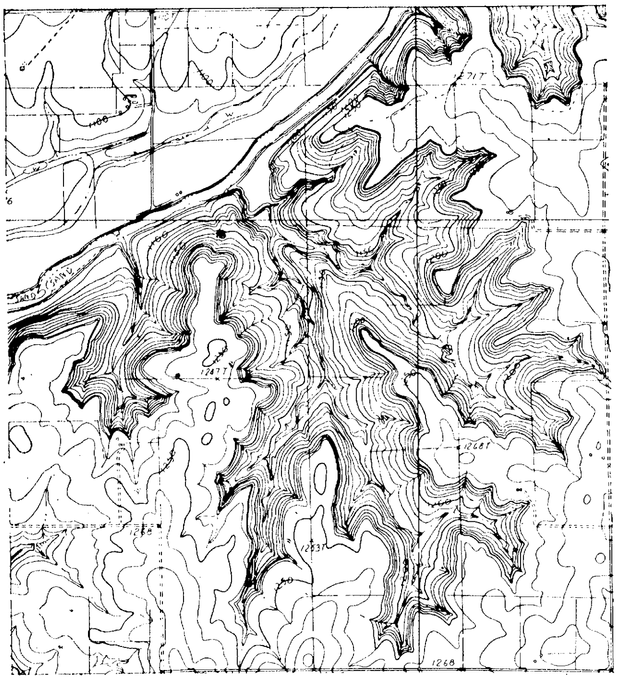
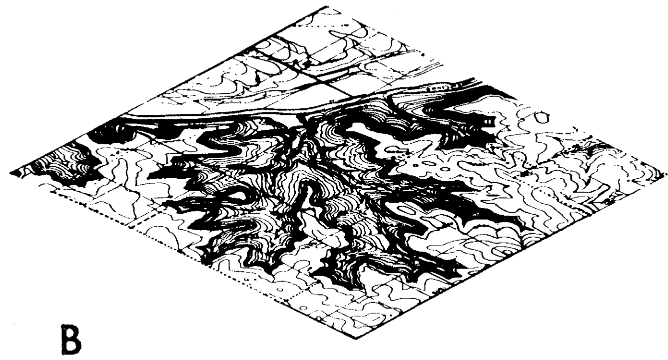


C

Fig. 7 (bottom left). The geometry of anamorphoser transformation is controlled by the relative dimensions of A, R, and F. When any one of these dimensions is altered, the scale or the shape of the transformation will change. For example, the 1:1 ratio of a to b could be altered to 1 : 2 by changing the dimension R. Similarly, the ratio of b to c will change when either A or F is altered. Fig. 8 (right). Maps B and C are optical transformations of planimetric map A. The dimensions of the anamorphoser spacings were calculated from the formulas given in Fig. 7 and are based upon the selection of a viewing elevation of 40 degrees above the horizon. The orientation of map B is S-N; that of map C, SW-NE. Since these are both parallel transformations, map B could be used as a base for both a north and a south view of the area.



A



B

Fig. 9. The area shown is part of the *Silverdale* [Kansas] *SW Quadrangle* published by the U.S. Geological Survey. The planimetric map (A) is here reduced from 1 : 24,000 to approximately 1 : 41,500, and the scale is too small for the contours of the original to be seen in detail. Similarly the details of the transformation (B) cannot be seen in the reproduction, but B does illustrate the capabilities of the anamorphoser. In fact, the value of the anamorphoser increases as map detail increases because it is no more expensive to photograph a complex pattern than a simple one. Conversely, a hand transformation of A to B would require many hours of laborious plotting, and thus would be expensive.

tral axis. If these conditions are met, the amount of transformation and reduction or enlargement are controlled by three critical dimensions. These are (Fig. 7) the distance, A, between slits in the anamorphoser, the distance, F, from the front slit to the original copy, and the distance, R, from the rear slit to the photographic negative. The position of these elements in relation to each other and the formulas for calculating distances for given transformations are given in Fig. 7.

Anamorphoser construction. Crossed-slit anamorphosers can easily be constructed from a variety of materials. They have four parts: (i) a stiff plate cut to fit the lens mount of the camera or enlarger to be used in the transforming process; (ii) a cylindrical barrel (*8*) affixed to the lens mount to space the slits; (iii) a front plate for the barrel, parallel to the lens mount plate; and (iv) a pair of slits mounted on the front and rear plates. The maximum length of the slits is determined by the diameter of the barrel, which in turn is limited by the size of the lens mount. The distance between slits closely corresponds to the length of the barrel, and different

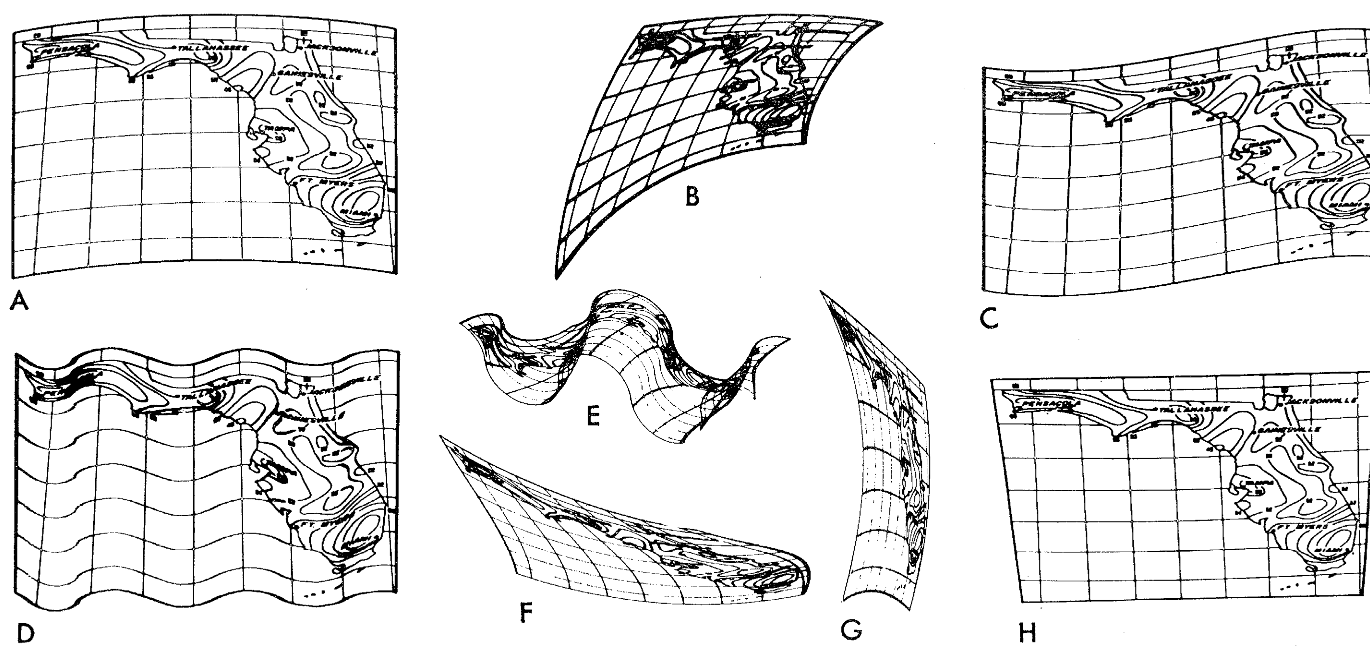


Fig. 10. A series of experimental transformations of the planimetric map of Fig. 8A. Such nonparallel transformations may offer possibilities for changing one projection to another, or for use of the anamorphoser in fields other than cartography. For example, transformations of these kinds might be used in planning printed circuit diagrams which must fit around other components, or in preparing teaching aids.

values of this dimension can be achieved by varying the length of the barrel, or by using a telescoping barrel (Fig. 7).

The anamorphoser is similar to the pinhole camera in many ways. Wide slits, like large pinhole apertures, result in fuzzy images with poor definition of detail. Conversely, narrow slits, while greatly increasing definition, increase exposure time because very little light passes through the apertures. Both devices also possess the characteristic of infinite focus and can therefore be used for close-up photography requiring great depth of field.

Empirical testing of slit widths reveals that poor detail results from use of slits wider than 0.030 inch (0.76 millimeter) and that exposure times become impractical when slit widths of less than 0.005 inch are used. Best results for 1:1 and 1:2 reproductions were obtained with slit widths of 0.010 to 0.020 inch. These observations were based on use of a blue-sensitive continuous-tone film to record images transmitted through a back-lighted negative.

Slits for the anamorphoser can be of open or closed construction. In open construction the slits are cut into an opaque material, or created by bringing two straight edges of an opaque material close together. This type of slit proved to be highly unsatisfactory under normal working conditions because the edge of the material attracts dust particles, causing striping of the photographic copies. Closed slits are prepared by making photographic negatives of lines drawn at a larger scale and then reduced. This construction offers advantages in that dust does not block the slits, construction is simple, and the slit width can be accurately controlled. In addition, slits can be prepared by this method in a wide variety of configurations. Closed slits have given very satisfactory results, but they must be handled with care because film emulsion and the film base scratch easily.

Photographic materials and techniques. Ordinary photographic materials and common darkroom procedures are used in making transformations. Exposure times are much longer than they usually are with conventional lens systems, and because of this the problems of lighting and film speed become very important. Exposure time can be shortened by transmitting light through a negative rather than reflecting light from a positive. Exposure times also vary with the dimensions of A, F, and R (Fig. 7).

As yet no experiments with continuous-tone negatives have been carried out, and the discussion that follows concerns only materials found to be satisfactory in line-copy transformation. The transformation is first recorded in positive form on commercial blue-sensitive continuous-tone orthographic film, with exposures varying from 2 to 5 minutes. These exposures relate to copy in which the b dimension of the transformation corresponds to the a dimension of the original, and the relationship could be compared to a 1:1 ratio except that there is a foreshortening of the c axis. For enlargements, exposures are lengthened; for reductions they are reduced.

The line images made with the crossed-slit anamorphoser on blue-sensitive continuous-tone commercial film are comparable in quality to sharp pencil lines on drafting film. The quality of an image enlarged to drafting size in the transforming process is more than satisfactory for work under any normal lighting condition.

The transformation capability of the crossed-slit anamorphoser in parallel perspective is illustrated in Fig. 8. The planimetric map (Fig. 8A), when transformed as though viewed from the south and 40 degrees above the horizon, is shown in Fig. 8B, and, when transformed as though viewed from the southwest at 40 degrees above the horizon, in Fig. 8C. The base map for Fig. 4 and the transformed copy of it are shown in Fig. 9. These examples make quite clear the great usefulness of the crossed-slit anamorphoser in parallel transformation, because each of the transformations, if made by hand, would require many hours of work. With this device each one can be prepared in a matter of 30 minutes or so, and since this is the most time-consuming step in the preparation of a three-dimensional map, the saving in time is significant. In fact, when complex planimetric maps are used, the time saved may be as much as 30 hours out of a total of 40 hours.

Summary and Speculation

Three-dimensional maps are useful tools which have been neglected for some time. They should be more commonly used, and familiarity with the techniques discussed in this article should dispel any qualms anyone might have about needing artistic talent to construct them. The saving in time resulting from the use of an anamorphoser provides a further incentive.

The anamorphoser transformations discussed above were all prepared by using straight slits, oriented at right angles to each other and placed so that all planes of the elements were parallel to each other. It is possible to vary these conditions in an infinite number of ways and thereby produce nonparallel transformations. Some of these variations are illustrated in Fig. 10. All the illustrations in Fig. 10 are transformations of the planimetric weather map shown in Fig. 8A. The variations used for the maps of Fig. 10 are as follows. (A) All planes parallel, with a curved rear slit; (B) all planes parallel, with curved slits front and rear; (C) all planes parallel, with S-shaped rear slit; (D) all planes parallel, with an undulating rear slit; (E) all planes parallel, with curved front and undulating rear slit; (F) plane of the original rotated on the horizontal axis—both slits curved; (G) plane of the original rotated on the vertical axis—both slits curved; (H) plane of the original rotated on the horizontal axis—both slits straight.

These are only a few of the many transformations which can be made with an anamorphoser, but they do point toward some interesting possibilities. For example, it appears that maps based on one projection might be altered to satisfy the coordinates of a completely different projection. Note, for example, the change of parallels from concave to convex curves (Figs. 8A and 10A) and the change from converging meridians to diverging meridians (Figs. 8A and 10G). Similarly, the grids of maps B, F, and H of Fig. 10 approximate projections which are quite different from the original.

Other possible, and noncartographic, uses also come to mind. For example, transformations of front and side views of architectural and engineering drawings could be made to show views from different elevations and orientations. Or a geologist might be able to illustrate folding of sedimentary strata by preparing a drawing of a series of horizontal beds and then using an anamorphoser with an undulating slit to alter the beds to any degree of folding desired. With a movie camera and a moving lens mount this "folding" process could be photographed with different settings, so that the parallel beds would gradually and continuously change to irregularly folded beds.

The transformations shown in Fig. 10 resulted from uncontrolled experimentation with the anamorphoser, and as yet no geometric solutions for setting up the transformation elements are available. Similarly, the suggested uses are mere flights of fancy, and no claim is made for their practicality. It is hoped, however, that they will stimulate the reader's imagination and lead him to look for uses of the anamorphoser in fields other than cartography (*9*).

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5. We find it difficult to compare the techniques presented here with those now, or soon to be, practical with computers. As far as can be determined, few computation centers now have the equipment needed to produce three-dimensional maps economically, and we are unaware of any time-cost analyses which would provide meaningful comparisons. At present we assume that the major deterrent to computer mapping is the nonavailability of scanners which can digitize the *x, y, z* values along profiles of a complex topographic surface. Further, we do not know of any plotters able to work with sufficient speed or able to drop out "hidden profiles," although it may well be feasible to develop them.
6. A. K. Lobeck, *Block Diagrams* (Emerson-Trussell, Amherst, Mass., 1958); M. Schuster, *Das Geographische und Geologische Blockbild* (Akademie-Verlag, Berlin, 195→ J. R. Stacy, *Ann. Ass. Amer. Geographers* **48**, 232 (1958); Tanaka Kitiro, *Geograph. J.* **79**, 213 (1932).
7. For details of the history of this device see G. Potonniee (E. Epstean, trans.), "Louis Ducos du Hauron: His Life and Work," *Photo-Engravers Bull.* **1939**, 18 (Feb. 1939); U.S. patent 1,652,271, assigned to Cooke Patents, Inc., 13 Dec. 1927; J. E. Mack and M. J. Martin, *The Photographic Process* (McGraw-Hill, New York, 1939), p. 51.
8. Tin cans with both top and bottoms removed make satisfactory barrels.
9. Experiments with these noncartographic transformations have been limited by nonavailability of time and by the fact that we have no measure of their practicality. We do know, however, that it would be possible to obtain some, if not all, of the inverse transformations if sufficient time were devoted to the problem.
10. This article is a report on part of the research on three-dimensional mapping carried on under Office of Naval Research contract Nonr 583 (15).

Development of Quantum Electrodynamics

Personal recollections

Sin-itiro Tomonaga

In 1932, when I started my research career as an assistant to Nishina, Dirac published a paper in the *Proceedings of the Royal Society, London* (*1*). In this paper he discussed the formulation of relativistic quantum mechanics, especially that of electrons interacting with the electromagnetic field. At that time a comprehensive theory of this interaction had been formally completed by Heisenberg and Pauli (*2*), but Dirac was not satisfied with this theory and tried to construct a new theory from a different point of view. Heisenberg and Pauli regarded the (electromagnetic) field itself as a dynamical system amenable to the Hamiltonian treatment; its interaction with particles could be described by an interaction energy, so that the usual method of Hamiltonian quantum mechanics could be applied. On the other hand, Dirac thought that the field and the particles should play essentially different roles. That is to say, according to him, "the role of the field is to provide a means for making observations of a system of particles" and therefore "we cannot suppose the field to be a dynamical system on the same footing as the particles and thus be something to be observed in the same way as the particles."

Based on such a philosophy, Dirac proposed a new theory, the so-called many-time theory, which, besides being a concrete example of his philosophy, was of much more satisfactory and beautiful form than other theories presented up to then. In fact, from the relativistic point of view, these other theories had a common defect which was inherent in their Hamiltonian formalism. The Hamiltonian dynamics was developed on the basis of non-relativistic concepts which make a sharp distinction between time and space. It formulates a physical law by describing how the state of a dynamical system changes with time. Speaking quantum-mechanically, it is a formalism to describe how the probability amplitude changes with time t. Now, as an example, let us consider a system composed of N particles, and let the coordinates of each particle be r_1, r_2, . . ., r_N. Then the probability amplitude of the system is a function of the N variables r_1, r_2, . . ., r_N, and in addition, of the time t to which the amplitude is referred. Thus this function contains only one time variable in contrast to N space variables. In the theory of relativity, however, time and space must be treated on an entirely equal footing so that the above unbalance is not satisfactory. On the other hand, in Dirac's theory, which does not use the Hamiltonian formalism, it becomes possible to consider different time variables for each particle, so that the probability amplitude can be expressed as a function of $r_1\ t_1$, $r_2\ t_2$, . . ., $r_N\ t_N$. Accordingly, the theory satisfies the requirement of the principle of relativity that time and space be treated with complete equality. The reason why the theory is called the many-time theory is because N distinct time variables are used in this way.

This paper of Dirac's attracted my interest because of the novelty of its philosophy and the beauty of its form. Nishina also showed a great interest in this paper and suggested that I investigate the possibility of predicting some new phenomena by this theory. Then I started computations to see whether the Klein-Nishina formula could be derived from this theory or whether any modification of the formula might result. I found out immediately however, without performing the calculation through to the end, that it would yield the same answer as the previous theory.

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This article is the lecture he delivered in Stockholm, Sweden, 6 May 1966, when he received the Nobel Prize in physics, which he shared with Richard Feynman and Julian Schwinger. It is published here with the permission of the Nobel Foundation and will also be included in the complete volumes of Nobel lectures in English, published by the Elsevier Publishing Company, Amsterdam and New York.