Texts in Abstract Algebra

Undergraduate programs designed to provide preparation for serious graduate work in mathematics usually include a one-year course in abstract algebra beyond the elementary linear algebra. Too often such courses consist of little more than a cataloging of definitions and superficial theorems, possibly spiced with uninformative trick exercises. On the other hand they are frequently very sound introductions to abstract algebra and include careful treatments of such topics as finitely generated abelian groups, elementary ideal theory in commutative algebra, and Galois theory. Among the new texts intended for courses of the latter variety are the three reviewed here: **Elements of Abstract Algebra** (Wiley, New York, 1966. 340 pp., illus. $7.95) by Richard A. Dean; **University Algebra** (Prentice-Hall, Englewood Cliffs, N.J., 1966. 285 pp., illus. $10.60) by Richard E. Johnson; and **A First Course in Abstract Algebra** (Holt, Rinehart, and Winston, New York, 1966. 350 pp., illus. $8.50) by Hiram Paley and Paul M. Weichsel.

There are differences among these books in organization and emphasis. In scope there is much in common. Thus in each we find a treatment of the integers, including a characterization and some elementary arithmetic; group theory, including elementary facts about permutation groups, structure of finitely generated abelian groups, and some arithmetic properties of finite groups (for example, Lagrange and Sylow theorems); basic commutative algebra, including factorization, polynomial domains, fields of quotients, and field extensions; a minimal treatment of finite dimensional vector spaces; and some general ring theory.

Except for some additional linear algebra and a chapter on lattices, this essentially describes the scope of Johnson's book. The other two both include rather complete treatments of elementary group theory. The final chapter and high point of Dean's book is Galois theory for fields of characteristic zero with applications to the theory of equations. Paley and Weichsel's final chapter, "Topics in the theory of rings," includes the Wedderburn and the Jacobson commutativity theorems, the elements of semi-simple rings, and Maschke's theorem. Thus, although some topics (for example, group extensions, which provide the proper context for the notion of a direct product) are omitted, each book contains the material for a good introduction to abstract algebra, and the Dean and the Paley-Weichsel books provide material for a considerably enriched course.

The exercises in these books are uniformly good and include many capable of challenging the better students. For instance, Johnson in one of his "theoretical projects," a neat pedagogical device, calls for the student to develop Giffen's slick proof that $R[X]$ is a unique factorization domain whenever R is.

An introduction to abstract algebra is probably most effective if it begins with a brief treatment of the elementary facts about binary operations, closure, identities, inverses, homomorphisms, and isomorphisms; that is, the really basic concepts of algebra. None of these books follows such a program, and each suffers from pedagogical and organizational difficulties as a result.

There are essentially two schools of thought on the treatment of the integers in abstract algebra, and both are represented here. Johnson and Paley-Weichsel, representing one school, start right off listing all dozen or so axioms for a naturally ordered integral domain. Dean, representing the other, treats induction informally in Chapter 0 and defers formal discussion of the integers until after the introductory chapters on groups and rings.

Pedagogically, the former approach is nearly indefensible, whereas the latter is quite sound. Initially, the integers are important only as a source of examples, and surely students can be trusted to use them that far. Having gained some mastery of the notions of groups and rings, the student will find a formal study of the integers understandable and pertinent.

Of the three, Johnson's book is the most elementary, and the material is, with few exceptions, within the grasp of most beginners. However, it appears to have been hastily written and is an inferior product of its author. The teacher who uses this book will be kept busy providing motivation, adjusting the emphasis and pace, and explaining the many careless statements. The pointless generality and blazing speed of his treatment of finitely generated modules over a principal ideal domain are not consistent with the progression of the rest of the book. That a linear transformation is characterized by its action on a basis is slighted but that $L(V)$ is von Neumann regular is emphasized. The term "permutation" is used (p. 32) long before it is defined (p. 168). It is not clear (p. 145) whether if $x \neq 0$ is a vector, $\{x\} = \{x, x\}$ is dependent or independent. The statement of Lagrange's theorem given on page 167 does not, as claimed (p. 180), give the order of S_n/A_n.

Paley and Weichsel's text is both the most versatile and the most standard. It could be used for a short elementary course, but it is better suited for a year course for reasonably mature undergraduates. It contains much challenging material and should provide good preparation for serious graduate work in algebra. The book is basically sound, but occasionally there is weak motivation and careless writing. Proper emphasis is lost by calling too many minor facts theorems. Important theorems are sometimes poorly stated. A particularly obnoxious example: given V an n-dimensional D-vector space, Paley and Weichsel state only that $L(V)$ and D_n are cardinally equivalent. What errors exist are mostly minor (although they do bungle recursive definition) and do not seriously mar the text.

The text by Dean is in many ways the most interesting, but will probably prove too much for the average undergraduate. In the preface he states, "If this book has a theme, it is the group concept. . . . If this book has a goal, it is Galois theory for fields of characteristic zero. . . ." The treatment is detailed almost to a fault, often tedious, and somewhat out of style. Thus, for instance, the subgroup generated by $A \subseteq G$ is described as words on A rather than as the meet of overgroups of A. There are some lapses and a large number of misprints. One annoying feature is that the numbering system for results is virtually useless. In spite of these objections this is a good tough book, and the highly motivated student should find it more profitable than the other two. It would certainly be a pleasant challenge to use it as a text.

These are generally good texts that should have been outstanding. Each has the characteristics of a preliminary draft rather than of a finished product, and I hope that future editions of these books will achieve their great potential.

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