Reports

Drumlin Formation: A Rheological Model

Abstract. *Drumlins are formed in a layer of boulder clay separating a glacier from certain types of terrain. Under certain conditions the large particles in the clay form a dilatant system. When flow in the separating layer is interrupted, part of the layer packs into an obstruction and the rest of the material flows around this obstruction, giving it a streamlined form.*

Certain previously glaciated areas are characterized by groups of low, smooth hills called drumlins. That the drumlin has been shaped by something flowing past it is apparent from the external form and from the fact that the planform shows a shape that can be described by an equation for a streamlined body (*1*). The basic problems concerning drumlin formation are the origin of the material and the nature of the shaping mechanism. Existing theories, which have been reviewed at some length (*2*), tend to fall into two groups maintaining that: (i) drumlin material was deposited from debris carried by the glacier and then shaped by the flowing glacier, or (ii) the material is that of the terrain, shaped by glacial action. The mechanism that I suggest incorporates a little of each theory.

The relative scarcity of drumlin forms suggests that the conditions necessary for their formation were rigorous and infrequent. I propose that the conditions were these:

1) The glacier-terrain relation was such that a relatively thin mobile layer of material separated the moving glacier from the terrain. The thickness of this layer, which acted like a film of lubricant, was about 1 to 5 percent of the thickness of the glacier.

2) The lubricating layer was composed of a concentrated dispersion of boulders in a dense clay-water system —the material usually called boulder clay or till.

The boulder-clay layer flowed with the glacier, and if the proportion of boulders was right the material was dilatant. A dilatant material expands when deformed, and when fully expanded flows fairly easily; but it is resistant until fully expanded. When a natural dilatant material is at rest the particles are in a tight random packing and this packing must be disturbed before flow can occur. Since the static packing is a tight packing, any adjustment must result in expansion. The mechanism of dilatancy has been investigated by Andrade and Fox (*3*), and the relevance of dilatancy to geology has been considered by Mead (*4*) and Boswell (*5*).

The flowing layer of boulder clay goes on flowing unless disturbed in some way. At points at which the flow is interrupted the expanded dilatant system tends to form a local packing; once it has packed it is stable, and the flow adjusts by flowing around the resultant obstruction. A bedrock knob would cause such an interruption in flow, and the boulder clay would tend to pack at the lee side of the obstruction; thus the eventual drumlin has a rock embedded near its upstream face. The clay layer is stable in either the static, tightly packed form or the flowing, expanded form. The great pressure exerted by the glacier causes the system to favor the condensed form, so that packing tends to occur whenever it can, whenever the flow stress drops below a critical value.

Shaping of the drumlin is aided by the rheological complexity of the boulder clay. In addition to the clay being dilatant because of the dispersion of large particles, the peculiar properties of clay cause the medium in which the large particles are dispersed to be thixotropic: an increase in stress causes reduction in viscosity (*5*). The clay material tends to flow more easily in the vicinity of a local packing than elsewhere in the flow, because of high local stresses caused in the flow as it adjusts its movement around the obstruction. The clay part of the system, flowing more easily under these circumstances, facilitates formation of a smooth shape that causes minimum disturbance in the flowing layer. Certain shapes permit streamline flow and the drumlin is shaped accordingly.

A polar equation of the form $r = \cos n\theta$ has been successfully used (*1*) to describe planforms; r and θ are variables and n is a dimensionless number expressing the elongation of the lemniscate loop that the equation generates. Reed, Galvin, and Miller (*7*) described the shape of drumlins by using the equation for an ellipsoid; this gives a three-dimensional description, but Chorley's two-dimensional description is preferable because it not only describes the shape of the drumlin but also indicates its mode of formation. The Chorley method also allows drumlins to be classified by use of the single value n in the equation $r = \cos n\theta$.

Drumlins may be finally shaped by the clay flow or the glacier; the shaping agency is probably the clay flow rather than the ice itself. Ice of the glacier appears to be too potent an abrasive agency to have left such precisely shaped forms in a plastic material such as boulder clay. The glacier is an efficient eroder of hard rocks, and it seems unlikely that a heap of boulder clay could exert enough resistance to require formation of a streamlined shape. In fact the shear strengths of ice and boulder clay are very similar (*5*), and it may be that subsequent ice cover eroded the deposit. This situation could arise if the clay layer were not continuous.

If drumlins are formed beneath a layer of boulder clay, a mechanism must be devised by which they can become exposed and yet retain their characteristic shape. This could occur because of the wide difference in properties between flowing and static boulder clay. The packed and shaped forms are tightly packed and very stable, and their surfaces represent the interface between the flowing and static parts of the system. When the glacier melts, the relatively plastic mobile layer is dispersed completely, but the streamlined molded forms remain.

Alternatively, if the boulder clay layer is discontinuous and if the shaping is done by subsequent direct glacial action, the glacier will expose the drumlins when it melts.

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References and Notes

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Mechanism of Lunar Polarization

Abstract. *A theoretical model to explain the negative polarization of moonlight at small lunar phase angles is developed. The model is based on the polarization of light in the diffraction region bordering the geometric shadow of an opaque dielectric obstacle.*

The polarization of moonlight as a function of lunar phase angle θ (earth-moon-observer angle) has been studied by many workers (*1, 2*). Surfaces have been made of various powdery substances which simulate the polarization behavior of the lunar surface (*1–3*). A typical polarization curve for the lunar disk is shown in Fig. 1. The sign convention for polarization by reflection assigns a positive polarization to light whose stronger component is polarized with its *E*-vector perpendicular to the plane of incidence. While the positive polarization at large phase angles is characteristic of reflection from any dielectric (as in Rayleigh scattering, for example), the mechanism which generates the negative polarization has not been understood. I now suggest a mechanism which explains the origin and magnitude of this negative polarization.

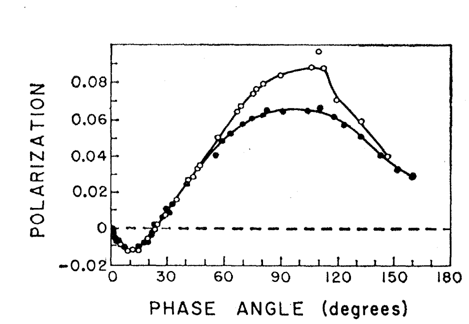

Fig. 1. The polarization $P \equiv (I_\perp - I_\parallel)/(I_\perp + I_\parallel)$ of moonlight as a function of lunar phase angle. The solid circles are results for positive phase angles, and the open circles for negative phase angles (after Lyot, *9*). $I_\perp$ and $I_\parallel$ are the intensities of light polarized with their *E*-vectors respectively perpendicular and parallel to the plane of incidence.

The strong increase in the brightness of the lunar surface for small phase angles is believed due to the effect of shadow (*4*). When a distant object illuminated by a point source is viewed from the direction of the source, all visible regions of the object are also visible to the point source, hence are illuminated. No shadow is visible. Viewed from some other direction, some observable regions are in shadow, and the mean brightness of the surface will be correspondingly reduced. Recent measurements by van Diggelen (*5*) have shown that the lunar surface brightness increases very rapidly with decreasing phase angles even for phase angles as small as 1° to 2°. It seems natural to search for an explanation of the negative polarization, which also increases rapidly at very small phase angles, by the same shadow mechanism.

Let us examine the shadow problem in more detail. Consider the problem of a square opaque object mounted above a diffusing screen, illuminated at normal incidence from above by an unpolarized source (Fig. 2). Observer N, observing the screen at normal incidence, sees no shadow. Observer O, observing at oblique incidence, sees into the shadow region and sees part of the brightly illuminated region obscured. Well outside the geometric shadow region, the illumination falling on the screen is unpolarized. Well inside the region of geometric shadow, the light intensity on the screen is negligible. In the diffraction region, however, the light can be polarized by diffraction. If so, the integrated light from the screen arriving at O might be partially polarized. Observer N, from the symmetry of his location, must observe no net polarization.

The light seen by O may have been polarized by diffraction either in traveling to the diffusing screen or in returning to O. Polarization by diffraction simultaneously in both paths is a higher order effect. Thus, the polarization seen by O is the same as the polarization of the light falling on the screen in the area of the screen directly observable by geometric optics to the observer at O, multiplied by a factor of 2 to account for the light polarized by diffraction in returning to O. It is necessary then only to calculate the polarization of the flux arriving at the diffusing screen.

To calculate approximately the polarization pattern in the vicinity of the shadow line, the opaque obstacle will

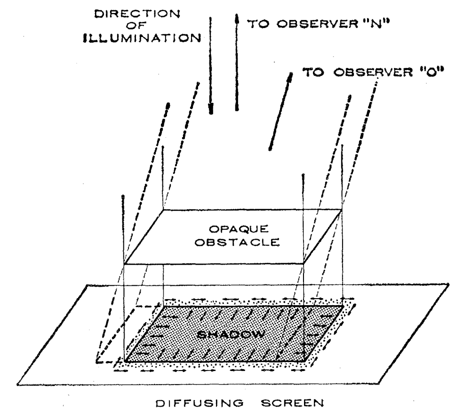

Fig. 2. The geometry of the problem of ideal shadow polarization. The shadow region and the geometric field of view of the two observers are shown, and the dominant polarizations in the diffraction regions indicated by small arrows.

be replaced by a perfectly conducting halfplane. The solution to this problem, first given by Sommerfeld, can be readily calculated in terms of equations given in Born and Wolf (*6*) and Fresnel integral tables. Let I_0 be original unpolarized incident flux, and $I_\parallel$ and $I_\perp$ be the intensity of the light incident on the diffraction, screened polarized parallel and perpendicular respectively, to the plane of incidence (but respectively perpendicular and parallel to the edge of the half-plane). The polarization $P(x)$ at the screen is given by

$$P(x) \equiv \frac{I_\perp - I_\parallel}{I_0} = \left(\frac{\lambda}{8\pi^2 r}\right)^{\frac{1}{2}} A(x) \;\ldots . (1)$$

where x is the distance from the geometric shadow line, r is the obstacle-screen distance, and λ is the wavelength of the light.

$A(x)$ is plotted in Fig. 3, with x

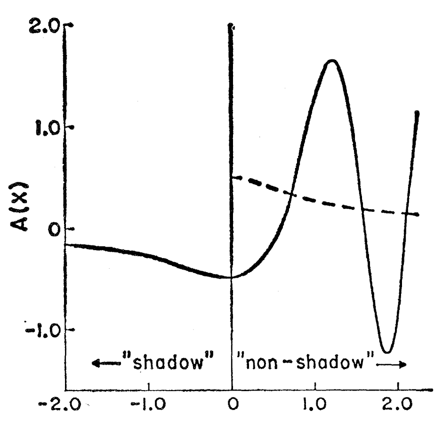

Fig. 3. The function $A(x)$ in the vicinity of the geometric shadow edge. The dotted line shows the monatonic part of $A(x)$ in the region of full illumination.