power was beyond the spectrometer bandwidth, this must be considered as a lower bound. The curve was obtained by scaling the area under each spectrogram. The $\pm\sigma$ marked on these curves is the result of both calculation and measurement; the agreement between the two is excellent.

All of the runs of the experiment were averaged together to produce the average Mars spectrogram shown in Fig. 9. Because so many runs have been averaged, the $\pm\sigma$ interval has been reduced to a very small value. The asymmetry in this spectrogram may be explained in two ways. (i) The surface may have a slightly preferred slope, as sandy places on Earth have when winds from a preferred direction ruffle the surface. (ii) The spectrogram is the result of the integration of hundreds of *hours* of signal plus noise and the subtraction of an equally long (but interleaved) integration of noise only. The asymmetry of the spectrogram may be the result of some residual instability of that process.

R. M. Goldstein
*Jet Propulsion Laboratory, California Institute of Technology, Pasadena 91103*

### References and Notes

1. R. M. Goldstein and W. F. Gillmore, *Science* **141**, 1171 (1963).
2. R. M. Goldstein and R. L. Carpenter, *ibid.* **139**, 910 (1963).
3. Research supported by NASA contract 7-100.

25 October 1965

## Theory of Rotation for the Planet Mercury

Abstract. *The theory of the rotation of the planet Mercury is developed in terms of the motion of a rigid system in an inverse-square field. It is possible for Mercury to rotate with a period exactly two-thirds of the period of revolution; there is a libration with a period of 25 years.*

By radar, Pettengill and Dyce (*1*) have observed that the rotation of the planet Mercury is direct with a sidereal period of 59 ± 5 days. McGovern *et al.* (*2*) have refined this value to 58.4 ± 0.4. Mercury's period of revolution is 87.97 days; for synchronous rotation the sidereal period of rotation would be the same. The observed value of 58.4 ± 0.4 days, has interesting theoretical implications. The role of tidal torque and tidal friction in bringing Mercury to this period has been calculated by Peale and Gold (*3*), and by Goldreich (*4*).

The torque exerted by the sun on Mercury arises from a term in the potential which varies inversely with the cube of the distance. For an eccentricity of 0.2, the variation between perihelion and aphelion in this term is a factor of 3.4. Hence, as pointed out by Peale and Gold (*3*) and by Goldreich (*4*), the rotation of Mercury tends to be controlled by the situation at perihelion; it tends to rotate so as to match the rotation velocity with the instantaneous orbital angular velocity at perihelion or near it.

But since the period demanded by this condition is nearly two-thirds of the orbital period, it is reasonable to ask whether a resonance lock is possible at exactly two-thirds of the orbital period, or 58.65 days. This seems plausible because the second-harmonic term in the planetary potential will have fore-and-aft symmetry; up to the second degree, the two ends of the axis of minimum moment of inertia behave in the same way in the gravitational field of the sun. Colombo (*5*) has already surmised that the lock is possible; our own work was begun before we were aware of his.

If $A < B < C$ are the principal moments of inertia at time $t$, and if $C$ is taken perpendicular to the orbit plane, then the potential energy of the planet Mercury is, by MacCullagh's formula

$$V = \frac{-KM}{r} - \frac{K(A+B+C-3I)}{2r^3} \qquad (1)$$

where $K$ is the gravitational parameter, $M$ is the mass of Mercury, and $I$ is the moment of inertia around the radius vector $r$

$$I = A\cos^2\phi + B\sin\phi \qquad (2)$$

where $\phi$ is the angular displacement of the principal axis, $A$, in the counterclockwise direction as seen from north, from the position vector, $r$.

The Lagrangian of Mercury's motion is

$$L = \tfrac{1}{2}M\left[\left(\frac{dr}{dt}\right)^2 + r^2\left(\frac{df}{dt}\right)^2\right] + \tfrac{1}{2}C\left(\frac{d\phi}{dt}+\frac{df}{dt}\right)^2 + \frac{KM}{r} + \frac{K[A+B+C-3I]}{2r^3} \qquad (3)$$

where $f$ is the true anomaly.

To the second order in $\phi$, rotation of the planet Mercury is governed by

$$\frac{d}{dt}\left[C\frac{d(\phi+f)}{dt}\right] + 3\frac{K}{r^3}(B-A)\cos\phi\sin\phi = 0 \qquad (4)$$

The instantaneous motion of the planet Mercury is described by

$$r = a(1-e^2)/(1+e\cos f) \qquad (5)$$

where $a$ is the semimajor axis and $e$ the eccentricity of the orbit. From the law of invariant areal velocity, the orbital angular momentum, $h$, is

$$h = r^2\,(df/dt) \qquad (6)$$

If the independent variable is changed from the time $t$ to the true anomaly $f$, then Eq. 4 becomes, if $A$, $B$, and $C$ are constant,

$$\frac{d^2\phi}{df^2} - \frac{2e\sin f}{1+e\cos f}\left(\frac{d\phi}{df}+1\right) + \frac{3\lambda}{1+e\cos f}\cos\phi\sin\phi = 0 \qquad (7)$$

where $\lambda = (B-A)/C$.

In a circular orbit ($e = 0$) and in the case of a body with dynamic symmetry and the axis of symmetry perpendicular to the plane of the orbit ($\lambda = 0$), Eq. 7 can be integrated by quadratures. Hence for small $e$ and $\lambda$, one can find an approximate expression for the solution of this equation, working from the Poincaré small-parameter method or the Krylov-Bogolyubov averaging method. Since $e \neq 0$ and $\lambda \neq 0$ represents a nonintegrable case, only qualitative investigation and numerical analysis of Eq. 7 appear to be readily obtainable.

By repeated numerical integration of Eq. 7 over a period of 100 years we find that for $\lambda = 0.00005$ (that is, somewhat less distortion than the moon) Mercury will lock at an average period of 58.65. The instantaneous period oscillates with an amplitude of the order of 0.008 days and a period of 25 years.

Han-Shou Liu
John A. O'Keefe
*Goddard Space Flight Center, Greenbelt, Maryland*

### References and Notes

1. G. H. Pettengill and R. B. Dyce, *Nature* **206**, 1240 (1965).
2. W. E. McGovern, S. H. Gross, S. I. Rasool, *ibid.* **208**, 375 (1965).
3. S. F. Peale and T. Gold, *ibid.* **206**, 1240 (1965).
4. P. Goldreich, *ibid.* **208**, 375 (1965).
5. G. Colombo, *ibid.*, p. 575.
6. We thank R. K. Squires and E. R. Lancaster for assistance. The numerical analysis was performed by W. R. Trebilcock.

10 December 1965