The Cosmical Constant

I should like to discuss a little further one of the questions raised by R. H. Dicke (*1*) in his very generous review of my book *General Relativity and Cosmology*. The point raised refers to the cosmical constant. Dicke made a comment, which I read with great pleasure, on the book as a whole, namely, that I was engaged in "drawing from the observational data, meager as they are, the vitality needed to convert formal mathematics into theory." I believe this statement also gives the reason why I included the cosmical constant in Einstein's equations. If the cosmical constant is omitted from the equations, the predicted average density of matter in the universe comes out to be some 100 times greater than the observed. We may, with Sandage (*2*), simply notice the problem with the words "the reason for this discrepancy is not understood at present"; or say, with Dicke, that the observations are not yet reliable enough. Neither of these ways of escape is open to me if Dicke's description of my attitude to general relativity is accurate. An effort must be made, using the theory in its entirety, to find out if the discrepancy can be avoided. I have, I believe, shown that a negative cosmical constant does resolve the difficulty on the basis of the data we possess, even if they are somewhat inaccurate.

The objection raised by Dicke to the presence of the cosmical constant in Einstein's equations is that, when they are derived from a variational principle, the integral to be varied consists of two parts arbitrarily added together, of which one produces the cosmical constant term. Let us call these integrals A and B, of which A produces the Einstein equations without the cosmical constant. Now A has itself to be carefully chosen by the mathematician; an arbitrarily selected integral will not do the trick. And the only way of choosing A is to know the required answer beforehand. In other words, Einstein's equations must have already been obtained by some alternative method. Since A is thus artificially constructed, it does not seem to me that the combination $A + B$ is any more, or any less, artificial than was A itself. In any case, cosmology requires the Einstein equations valid in a region containing matter or energy. To deduce them from a variational principle, a third integral, also carefully selected to give the predetermined answer, must be added *ad hoc* to either A or $A + B$ (*3*). I conclude that the deduction of Einstein's equations from a variational principle is an ingenious exercise in formal mathematics which does nothing to increase or decrease the validity of the equations.

The point at which readers may be more seriously misled is Dicke's statement that the presence of the cosmical constant introduces into the theory "a large characteristic constant length." If this were a necessary interpretation, the situation would be odd indeed, especially as the standard length, compared with which the cosmical constant "length" is "large," is not indicated. However, general relativity can be developed in terms of ordinary units of mass, length, and time, cgs units for example. When this is done it becomes obvious that proper-times should be expressed in time-units and not as lengths, and the method also enables the reader to keep the physical dimensions of all variables clearly before him. It then turns out that the cosmical constant is the inverse square of a time-interval (*4*). In the static Einstein universe this time-interval is the radius of this universe divided by the local velocity of light. In the expanding universe models it is the reciprocal of the Hubble constant. In this second class of models the magnitude of the cosmical constant is of the order of 10^{-35} sec^{-2}. Whether this is "large" or "small" depends on the standard of comparison, a standard that must, of course, be expressed in the same units. For example, the inverse square of the time for $(2\pi)^{-1}$ revolutions in the first Bohr orbit is of the order of 10^{-33} sec^{-2}, compared with which the cosmical constant is exceedingly small. The acceleration of gravity due to the sun at the distance of the earth divided by the radius of the earth's orbit is 4×10^{-14} sec^{-2}, which is again much larger than the cosmical constant. The expression of the cosmical constant as the inverse square of a time-interval characteristic of the physical situation envisaged shows why, in the Newtonian approximation, this constant manifests itself as a universal force (*5*). If the cosmical constant is negative, the force is a general binding force which, like gravitation, serves to bind matter together on a cosmical scale. If the constant is positive, the force is one of repulsion, and it tends to accelerate the expansion of the universe.

Until recently I had fondly imagined that my method of introducing the cosmical constant as a "constant of integration" in the establishment of Einstein's equations was original. I am indebted to Windsor L. Sherman (*6*) for drawing my attention to the fact that the method had long ago been used by Einstein (*7*). At that time Einstein was in his creative stage and was undoubtedly converting formal mathematics into theory. It was only later that he turned to other methods by which esthetic considerations were to abolish the cosmical constant, or the deity's alleged unwillingness to play games of chance with humanity was to abolish the quantum theory.

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References

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6. W. L. Sherman, *NASA (Natl. Aeron. Space Admin.) Tech. Note D-2601* (1965).
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Reconstruction of Protein and Nucleic Acid Sequences: Alanine Transfer Ribonucleic Acid

Holley *et al.* (*1*) have proposed the complete sequence of Ala-sRNA (*2*) shown in our Table 1 (sequence II). The sequence was reconstructed by an elegant logical process from the most extensive data on fragments of polynucleotides yet published. While reconstructing this sequence from the data with a digital computer we have observed some inconsistencies between the data and the published sequence. These might be considered trivial were it not for the fact that it is possible to construct slightly different sequences which resolve them. Precedent has already been established for the idea that a reconstructed sequence evolves as more extensive and refined data become available (*3–7*). We wish to