and tried, but it must be said that what cannot yet be done, or done well enough, far outweighs what can be done. So here again there is an immense opportunity for ingenuity and systematic hard work. The putting together of all these devices and techniques into systems will have to wait until we have all the parts and the necessary understanding. That time will surely come—how soon, it is impossible to say.

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Acoustical Thermometer

An ultrasonic interferometer provides a new method for the precise measurement of low temperature.

Harmon H. Plumb and George Cataland

In the last few decades scientific interest in the area of low-temperature physics has been accelerated and the related field of cryogenic engineering has been developed. While some of the engineering applications have been direct products of previous research efforts, considerable impetus has been provided by ventures into outer space. Accumulated knowledge of the properties of hydrogen (in both the liquid and the gaseous states) has been important for propulsion, but additional interests are steadily developing—for example, cryogenic pumping for a variety of applications, the possible use of superconducting phenomena, and creation of environments which provide low noise levels.

Basic to the extension of fundamental research on phenomena at cryogenic temperatures, as well as to engineering development, is the attainment of a practical temperature scale. To be useful such a scale must be reproducible and must reasonably approximate the thermodynamic, or Kelvin, temperature scale. Additionally, there must be a secondary thermometer possessing a temperature-related parameter that can be relatively simply and reproducibly measured.

In this article we discuss methods of achieving a temperature scale which approximates the thermodynamic scale (below 20°K); sources of difficulty in the methods; a new approach (measurement of the speed of sound in helium gas) to precision low-temperature thermometry; the instrument employed; achievement of a new provisional temperature scale; and the means by which the scale is made available to the scientific and engineering public.

Attainment of Absolute Temperature

The absolute thermodynamic, or Kelvin, temperature scale, which is independent of the properties of any substance, results from basic thermodynamic considerations. It is frequently derived (*1*) by operating a reversible engine in a Carnot cycle between two heat reservoirs, which may be characterized by the temperatures T_1 and T_2. The ratio of the heat absorbed (Q_1) and the heat rejected (Q_2) by the engine is equal to the temperature ratio on the Kelvin scale.

$$\frac{Q_1}{Q_2} = \frac{T_1}{T_2} \qquad (1)$$

To fix the size of the degree and thereby complete the definition of the Kelvin scale, an arbitrary number of degrees must be assigned to some temperature interval.

Originally the size of the degree was defined by assigning exactly 100 degrees to the temperature interval between the freezing and the boiling points of water.

In 1954 the Kelvin scale was redefined in terms of a single fixed point —the triple point of water—and the temperature interval between zero of

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the scale and the triple point was assigned the value of 273.16 degrees exactly. Thus, for the reversible engine operating in a Carnot cycle between reservoirs at temperatures T_1 and 273.16°K,

$$T_1 = 273.16 \frac{Q_1}{Q_{273.16}}, \qquad (2)$$

where Q_1 and $Q_{273.16}$ are the heats exchanged at T_1 and 273.16°K, respectively.

It can be shown (*1*) that temperature values in terms of a scale based on properties of an ideal or perfect gas agree with temperature values in terms of the absolute thermodynamic scale provided the same fixed-point value is used to define the size of a degree on the two scales.

Since this article concerns temperatures based on properties of a perfect gas, we first consider such a gas. We define it by the following two laws:

1) Boyle's law: At a definite temperature the product of pressure and volume for a constant mass of perfect gas is constant, or

$$pv = f\ (T) \qquad (3)$$

2) Joule's law: At a constant temperature the energy E for a constant mass of a perfect gas is independent of the volume, or

$$\left(\frac{\partial E}{\partial v}\right)_T = 0 \qquad (4)$$

While a perfect gas does not exist, it can be approximated. There are several methods by which ideal-gas temperatures may be arrived at. Those generally employed are (i) pressure-volume isotherm determinations with extrapolations to zero pressure, to simulate an ideal gas; and (ii) gas thermometry, in which thermometric measurements of a real gas are corrected to give reference to an ideal gas.

Nearly a decade ago it became apparent that the National Bureau of Standards should undertake thermometry investigation in the temperature range below 20°K. The need for a basic scale and attendant calibrations between 4° and 20°K had been expressed by both engineering and basic research groups. At approximately the same time temperature scales in the He^4 vapor pressure range (2° to 5°K) were being discussed, with the resulting emergence of the 1958 He^4 Scale of Temperatures (the T_{58} scale) (*2*). Provisional scales maintained by platinum resistors were in existence; they covered the temperature range from 20° down to about 11°K and left an obvious gap from 5° to 11°K.

The decision was made to initiate an investigation that would result in a temperature scale for the range from 2° to 20°K. Then, as now, there was sufficient divergence among the absolute-temperature determinations made by various investigators in both the 2°–5°K and the 10°–20°K ranges to justify a special effort. Since gas thermometry is the method which has been most widely used for achieving temperature values that approach values on the thermodynamic scale, the technique was given serious consideration. But precise gas thermometry is a very painstaking undertaking, especially at low temperatures, where certain difficulties (see *5*) are intensified. These difficulties relate to corrections which must be made because of errors in the gas-thermometry data caused by (i) the "dead" space of gas thermometers (*3*); (ii) erroneous pressure measurements resulting from thermomolecular pressure effects (*4*); (iii) departures of the thermometric fluid from an ideal gas; (iv) uncertainties concerning thermometric volumes resulting from uncertainties in coefficients of expansion; and (v) inability to conserve the thermometric fluid (that is, adsorption of the gas). In addition, the demands of manometric precision in gas thermometry are always stringent.

Thus, while the opportunity existed for developing a gas thermometer of greater accuracy than the existing instruments in the temperature range of interest, the inherent characteristics of gas thermometry left some doubt as to the possible significant gain from such an effort. What was clearly needed was a completely different experimental approach to thermometry, one which gave promise of being at least as accurate as existing methods. We felt that experimental determination of isotherms from measurements of the speed of sound in helium gas as a function of pressure was a promising approach.

The concept of deriving values of absolute temperature from speed-of-sound measurements is not of recent origin. Lord Rayleigh recognized the possibility of such an experiment in 1878 (*6*) in observations on a communication from A. M. Mayer (*7*). More recent manifestations of interest appear in the contemporary literature—for example, in a paper by Hedrich and Pardue (*8*). Van Itterbeek and Keesom (*9*), De Laet (*10*), and Brodsky *et al.* (*11*) have investigated the feasibility of attaining the required accuracy, with varying degrees of success.

It is pertinent to introduce, here, a theoretical discussion which will demonstrate that values of temperature on the absolute thermodynamic scale can be approximated from the experimentally determined quantities, sound-speed and pressure, in helium gas.

Theory: Speed of Sound in a Gas, and Absolute Temperature

When sound is propagated through a gas, if the frequency of the sound is not too high relative to the mean collision frequency of the gas molecules, the propagation is nearly adiabatic, and the speed of sound (*12*) is

$$W = \sqrt{E/\rho} = \sqrt{\frac{-v^2}{M}\left(\frac{\partial p}{\partial v}\right)_S}$$

$$= \sqrt{-\frac{C_p v^2}{C_v M}\left(\frac{\partial p}{\partial v}\right)_T} \qquad (5)$$

where ρ is the density of the medium and E is the adiabatic-bulk modulus defined by the relation

$$E = -v\left(\frac{\partial p}{\partial v}\right)_S = -\frac{C_p}{C_v} v \left(\frac{\partial p}{\partial v}\right)_T \qquad (6)$$

The symbols v and p refer to the volume and pressure of the medium, and the partial derivative is at constant entropy or temperature, as indicated; C_p and C_v are specific heats at constant pressure and volume, respectively. If the conducting medium were an ideal gas, its equation of state would be expressed by

$$pv = RT \qquad (7)$$

and there would result

$$\left(\frac{\partial p}{\partial v}\right)_T = -\frac{RT}{v^2}. \qquad (8)$$

Additionally, the ratio of specific heats would be that of an ideal gas,

$$\left(\frac{C_p}{C_v}\right)_{\text{ideal gas}} = \frac{5}{3}$$

(if the gas is assumed to be monatomic), and Eq. 5 becomes

$$W_0^2 = \frac{5\ RT}{3\ M} \qquad (9)$$

Equation 9 indicates the simple relationship between the absolute thermodynamic temperature T and the speed of sound in an ideal gas W_0. (The gas constant R and the molecular weight M are considered to be accurately known constants.)

In actuality an ideal gas does not exist, so Eq. 9 must be modified for application to real gases. Of the two gases, He^3 and He^4, which can be used for sound measurements at temperatures between 2° and 20°K, He^4 has been selected for experimental measurements. Its equation of state can be assumed to be represented by the empirical relationship (*13*)

$$pv = RT\ (1 + \frac{B}{v} + \frac{C}{v^2} + \ldots) \qquad (10)$$

where B and C are functions of the temperature called the second and third virial coefficients, respectively. If the quantity $(\partial p/dv)_T$ is evaluated from Eq. 10 and the ratio C_p/C_v is evaluated for the real gas, Eq. 5 can be written

$$W^2 = \left(\frac{C_p}{C_v}\right)_{\text{ideal}} \frac{RT}{M} (1 + \alpha p + \beta p^2 + \ldots) \qquad (11)$$

where

$$\alpha = \frac{1}{RT}\left[2B + \frac{4T}{3}\frac{dB}{dT} + (4/15)T^2\frac{d^2B}{dT^2}\right]. \qquad (12)$$

β is a term that includes higher orders of the virial coefficients B and C. From Eqs. 9 and 11 we obtain

$$W^2 = W_0^2 (1 + \alpha p + \beta p^2 + \ldots). \qquad (13)$$

It is now apparent that, if one determines an isotherm of W^2 as a function of pressure, the intercept W_0^2 of the extrapolated isotherm readily affords a means of calculating the isotherm temperature on the absolute scale.

In the foregoing relationships we note that precise volume or changes in volumes, conservation of thermometric fluid, nonideal representations of the gas, and "dead" spaces are extraneous considerations. And since the pressure measurement is a second-order quantity (in Eq. 13 αp is small as compared with 1 and the pressure terms of higher order are even less significant), the precision with which it must be determined is less stringent than in the case of gas thermometry; thus, for practical purposes, the need for thermomolecular-pressure corrections is entirely eliminated.

In order to obtain values for the speed of sound as a function of pressure, one must determine the wavelength and frequency. Several methods are possible, but our work is based upon the creation of standing waves in helium gas by means of an apparatus with a fixed frequency and a variable path. Other investigators, in attempting a similar type of thermometry (that is, thermometry based on the creation of standing waves in helium gas), have employed apparatus in which the path length for the sound is fixed and the frequency is variable (*9–11*). We readily acknowledge that other methods may be as feasible as the method we have used.

The instrument we have used is called, in the field of acoustics, an ultrasonic interferometer, but because of its particular application we generally refer to our model as an acoustical thermometer (*14*).

Qualitative Method

A method (often illustrated by Kundt's tube) for measuring the speed of sound in a gas is shown schematically in Fig. 1. In Fig. 1, Q is a quartz crystal energized, by an outside circuit, at its resonance frequency f. The voltage across Q is continuously monitored. Energy radiating from the crystal is transmitted through helium gas; after reflection from the plane surface R, the energy again traverses the gas and returns to the source crystal. The impedance at the electrical terminals of Q, and therefore also the magnitude of the voltage (the input current is held constant), depends on the length of the gas column. If the electrical circuit is properly designed, the magnitude of the voltage at Q peaks sharply for certain distances between R and Q; each of these distance values differs from the next by a half-wavelength.

One measures the displacement of R through n half-wavelengths, which are visually observable on the voltmeter; the displacement divided by $n/2$ yields the wavelength of sound in helium gas. Since the resonance frequency f of the quartz crystal is continuously measured, the experimental value for the speed of sound in helium gas may be readily calculated from $W = \lambda f$. This value of W is, of course, the phase speed corresponding to whatever mode exists in the tube of gas. It is supposed that this mode is sufficiently like the plane wave mode to allow identification of W, for our purpose, as the free-space value. The mode may not be pure but, if this is the case, its components are spaced so closely that the interferometer does not resolve them.

Experimental Instrument

The most important requirement in the design of an ultrasonic interferometer concerns the mounting-relationship between the crystal source and the reflecting surface. It is essential that plane parallelism exist between these surfaces, regardless of the displacement of one surface with respect to the other. Our instrument is so constructed that plane parallelism is maintained to within three light fringes, even though the reflection surface is either displaced by 2 centimeters from the crystal source or completely rotated. This parallelism was maintained at room temperature, and because of the design a comparable result is expected at operational temperatures (20°K and below).

Another requirement pertains to the purity of the gas being investigated. Because measurements are being made at low temperatures, nearly all the possible impurities are frozen out. Nevertheless, to maintain an operationally clean system and also to exclude the few gases which might be contaminating, the entire enclosed gas system was made vacuum-tight. Additionally, the entire system was repeatedly flushed with He^4 gas during routine preparations and operations. Outgassing at a temperature that is high as compared with ambient conditions is, for our apparatus, impractical and seems unnecessary, since, as mentioned, nearly all the possible contaminants are frozen out at temperatures below 20°K.

One of the most difficult design problems is associated with the measurement of the relevant displacement of the reflecting surface relative to the crystal surface. The difficulty arises from the possible expansions and contractions of material in existing extreme temperature gradients—which are not necessarily constant. The design must not only provide for actuation of the reflecting surface but must also permit accurate measurement of the translational motion.

Another requirement concerns the need for making precise measurements of electrical resistance. If the ultrasonic interferometer is to serve as a primary thermometer, we must be able to calibrate secondary thermometers against the scale of temperature that is established. Germanium resistors can be used as secondary thermometers, but their use presents certain measurement problems: the occurrence of stray joule heating at radio-frequencies; the problem of making resistance measurements at low power levels; and the difficulty of attaining thermal equilibrium.

Certainly not least among the design requirements is the necessity of accurately maintaining the essential components at any desired temperature between 2° and 20°K for long periods. Considerations of primary and secondary thermometry are paramount, for it is hoped that the instrument will be at least competitive with the best existing techniques of low-temperature thermometry.

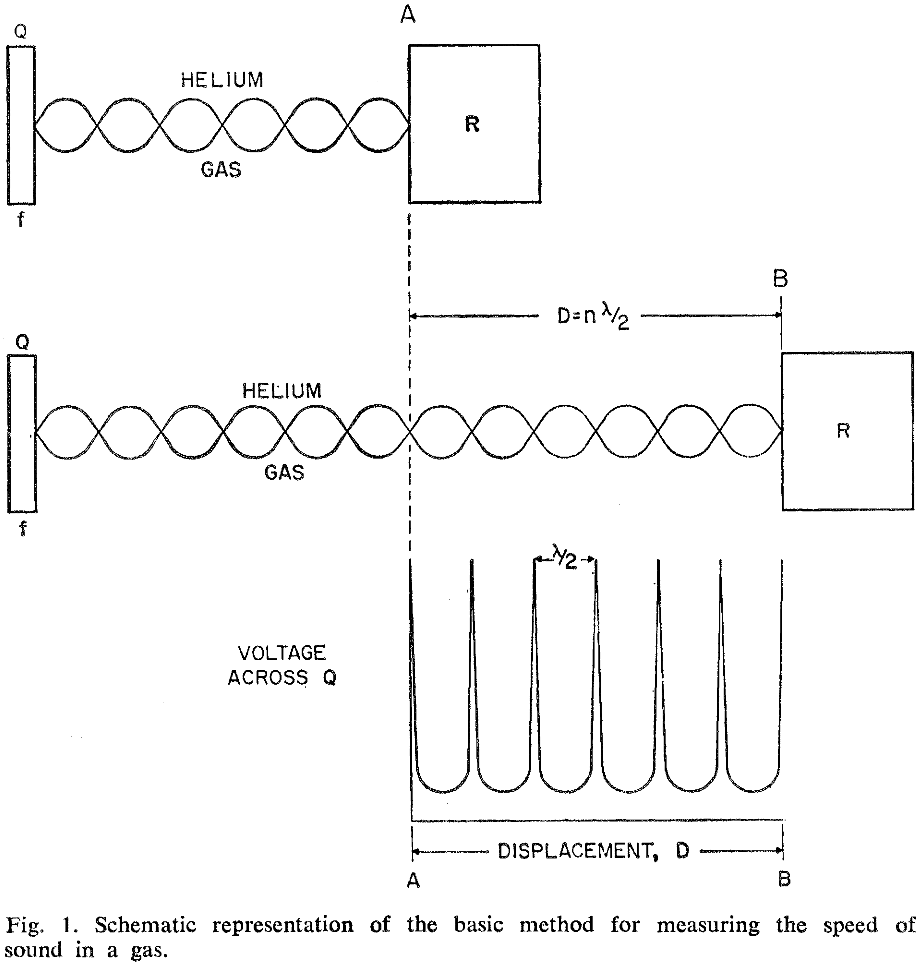

Fig. 1. Schematic representation of the basic method for measuring the speed of sound in a gas.

Construction Details

During experimentation, the lower portion (Fig. 2) of the apparatus is maintained in a liquid helium bath; the upper portion (Fig 3), which is necessary for effecting and measuring displacements of the reflecting piston *A*, is at room temperature.

Copper piston *A* is capable of being translated smoothly, without seizing, within the copper cylinder *B*. The quartz crystal ($f = 1$ Mcy/sec) is seated against the lower surface of the cylinder *B*; the lower assembly *D* not only maintains the crystal in position but also accommodates the electrical lead which energizes the crystal. The piston's upper end is connected to a fused quartz rod by a ball joint designed to permit reasonably simple assembly and a degree of self-adjustment during thermal expansion or contraction and to constrain the piston to follow axial displacements of the fused quartz rod. The rod extends upward about 1.2 meters to enter ambient conditions above the indicated neoprene O ring vacuum seal (Fig. 3). At its upper end the rod is smaller in diameter than it is throughout the rest of its length, and at the point where this change in diameter occurs it is secured to a plate, *P*, that is constrained to move in translation only along the axis of the quartz rod. Fixed pins which pass through holes drilled in the plate prevent rotation of the plate. The motion of the hand-operated driving mechanism is limited to rotation about the axis of the quartz rod. Plate *P* is threaded into the driving mechanism, hence a rotation of the driving mechanism produces a translation of plate *P* along the axis of the quartz rod. Because the quartz rod and the piston undergo the same translation, the position of the reflecting surface (Fig. 2) with respect to the quartz crystal is controlled by manual turning of the driving mechanism.

The thermometric gas He^4 is contained within cylinder *B*, assembly *D*, copper piece *C*, and the Invar and quartz tubings which lead to the base plate *L* (Figs. 2 and 3). From the base plate there is gas communication through a side arm to a manifold that affords connection with a manometer, a large gas-ballast volume, and gas evacuating or filling systems. This arrangement permits vacuum leak detection, gas filling, and pressure control and measurement.

Since the entire assembly of Fig. 2 is maintained in liquid helium during experimental measurements, a large temperature gradient exists along the fused quartz tubing—from 4°K at the top of the vacuum can to 300°K several centimeters below the base plate. Fluctuations in the gradient could have a critical effect on the displacement measurement from which the speed of sound is calculated. Precautions have been taken to minimize the changes in the thermal gradient; more important, fused quartz was selected for the tubing because of its relatively small coefficient of expansion.

The temperature of the helium gas between piston *A* and the quartz crystal is of paramount importance; it must be controllable so that it can be maintained for long periods at any selected temperature in the range of interest, 2° to 20°K. This is best accomplished by controlling the temperature of *C*, *A*, *B*, and *D*, which are in intimate contact with the thermometric gas. When the apparatus is in operation there should be no discernible temperature difference for these

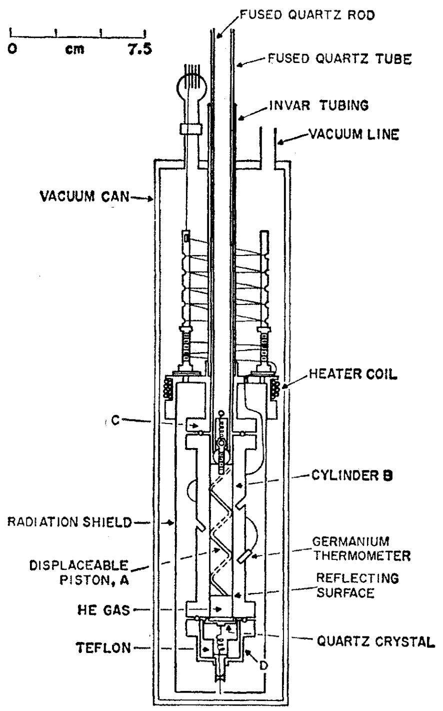

Fig. 2. Acoustical-thermometer components which are maintained at low temperature during the experiment.

components, and their temperature should exceed that of the surrounding liquid helium bath. In order to achieve the best temperature control and also maintain a reasonable rate of evaporation for the surrounding liquid helium, the controlled components should be thermally isolated from the helium bath. Complete isolation is both impractical and undesirable (*15*); hence thermal losses to the bath must be controlled to minimize undesired thermal gradients. Accordingly, the instrument design is such that there is a heat flow from *C* toward the vacuum-can cover, which is exposed to the helium bath temperature. A vacuum is maintained in the can, which surrounds *C*, *B*, and *D*; a radiation shield surrounding *B* and *D* is thermally coupled to *C*; secondary-thermometer electrical leads are thermally anchored to *C*; and the heater coil for supplying power to maintain *C*, *B*, *A*, and *D* at the desired temperature is wound on *C*, as shown in Fig. 2. Small secondary thermometers (germanium resistors imbedded in the outer wall of cylinder *B*) and the acoustical-wave generating system supply heat to the system (*A*, *B*, *C*, and *D*) and therefore could conceivably establish an undesired gradient. However, experimental variation of the power applied to these small heating sources has not revealed any such gradients. In effect, the components below *C* are "dead-ended" thermally, so that, for all practical purposes, the imbedded secondary thermometers are in thermal equilibrium with the excited helium gas.

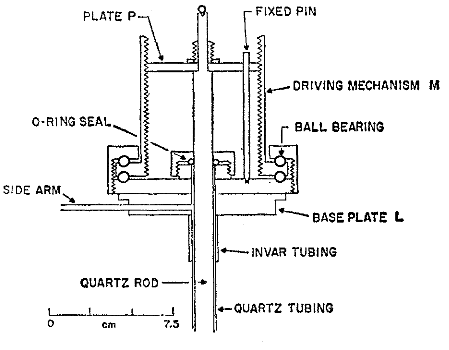

Fig. 3. Mechanism for driving the acoustical-thermometer piston.

Precautions were taken to avoid thermal oscillations (*16*) of the helium gas in the annular space between the quartz tube and the rod. But another serious problem was encountered. When the temperature-controlled section was maintained at a temperature which exceeded the bath temperature by several degrees, convection currents were set up in the gas that occupies the annular volume between the thin-walled Invar tubing and the quartz rod. The cycle transmitted heat in an erratic manner from component *C* to the region of the vacuum-can cover. This necessitated a larger heat input to the heater coil and resulted in the loss of temperature stability for the chamber components. The difficulty was overcome by introducing nylon material, formed into a spiral, to fill the annular volume; the spiral broke up the convection path but permitted the necessary pressure communication between the helium gas chamber and the manometer and ballast volume.

In measuring piston displacements it is assumed that the upper end of the quartz rod (Fig. 3) undergoes the same translation as the piston's reflecting surface. This is correct within the limits of our measurement error. Actually, with the lower assembly in thermal equilibrium at 10°K, if the displacement is 1 centimeter upward and thermal equilibrium is again attained, the end of the quartz rod will have been displaced by approximately 1 centimeter. The departure from this 1-centimeter measurement will be the change in length of a centimeter of fused quartz when its temperature is raised from 10°K to room temperature. A report by Gibbons (*17*) indicates that this correction might be less than a micron, which is approximately the accuracy of our measurements.

A stainless steel ball is seated on the upper end of the quartz rod (Fig. 3), and above it a calibrated micrometer head is mounted. The micrometer head mounting and base plate *L* are both referenced to a rigid plane. All components of the length-measuring system, which are in both the room-temperature and the low-temperature parts of the apparatus, are made of either Invar or fused quartz to minimize errors caused by temperature fluctuations. An electrical system is used to indicate when the micrometer-head barrel makes contact with the stainless steel ball. Quite obviously, the micrometer is never operating under a load, since the piston activation is effected by the driving mechanism.

In Fig. 2, one of four germanium resistors is shown mounted in the cylinder wall. Three of the resistors are used to indicate the temperature stability of the thermally controlled apparatus. Since they are also calibrated against the acoustically determined absolute temperatures, they "retain" the temperature scale and can be used as secondary thermometers in future calibrations. The fourth resistor serves as a temperature sensor. It connects with outside controls that automatically regulate the heater-coil power to achieve necessary temperature control of various components.

Experimental Operation

After precautionary flushings with helium gas, the acoustical-thermometer gas space (which includes the ballast volume, the manometer, and the manifold) is filled to a pressure slightly below atmospheric pressure. A small amount of helium exchange gas is admitted to the vacuum can, and the low-temperature portion of the apparatus is cooled by means of liquid nitrogen. After 4 hours, a liquid-helium transfer is initiated. When the transfer has been completed, the pressure of the helium bath is placed under manostatic control and maintained above ambient pressures.

When the bath has become stabilized and the electrical circuits have been monitored, the temperature desired for measurements is approached. The resistance of the germanium sensor is dialed on the Wheatstone bridge, and the temperature regulator assumes control of the system. For a number of possible reasons a slight loss of control can occur, but this can be offset by infrequent manual corrections to the Wheatstone bridge setting, made in accordance with the temperature indicated by the germanium secondary thermometers. In this manner the interferometer temperature is easily controllable to ±0.002°K for hours. After an operating temperature has been achieved, the pressure of the thermometric helium gas is gradually increased or reduced to a desired value. Next, the quartz crystal is energized at its resonance frequency (*18*), and its voltage is monitored. When the piston is advanced at a uniform rate toward the quartz crystal, the voltage rises to a sharp maximum each time a standing wave exists in the gas.

The Invar spindle (*M* of Fig. 3) is then turned to raise the piston to a point where its lower reflecting-surface is about 1.7 centimeters above the opposing quartz-crystal face, and a standing wave exists in the gas. Since this operation is accompanied by a flow of gas into the volume between the quartz crystal and the reflecting surface (Fig. 2), the temperature will change and the experimenter must wait several minutes until it returns to the temperature control point. Then by means of the micrometer head the position of the stainless steel ball that is atop the quartz rod is determined. At least two independent readings are made at each measured resonance point: after the initial reading the micrometer contact-surface is raised, the piston is intentionally slightly displaced, the piston position for resonance is redetermined, and another micrometer reading is made. Following this, the piston is moved toward the crystal through a definite number of resonance points (an interval of 40 half-wavelengths is often used); this downward motion is terminated at a resonance point. The micrometer head is then advanced downward to determine the new position of the stainless steel ball. The piston is caused to approach the quartz crystal in incremental steps, and other micrometer measurements of the position of the ball are made. From these measurements an average wavelength may be calculated and, consequently, the speed of sound through the gas also, since the resonance frequency of the quartz crystal is known from continuous measurements. The gas pressure is also systematically observed and recorded during this procedure, so a value for average pressure is obtained. The values for speed and corresponding pressure determine one point on an isotherm. To obtain an additional point, the pressure is changed, in general a slightly different frequency is ascertained (the resonance frequency of the quartz crystal varies slightly with the pressure of the surrounding gas), and the measurement proceeds as described above.

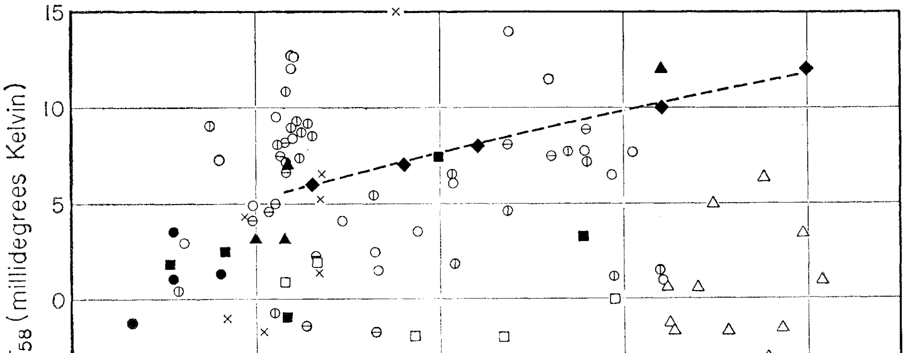

Fig. 4. A comparison, with values of the T_{58} scale, of temperatures derived from (i) pressure-volume isotherm measurements, (ii) gas thermometry measurements, and (iii) acoustical-thermometer measurements (*19*). (Open circles, plain or bisected) Gas-thermometer values [Schmidt and Keesom (*20*)]; (solid circles) gas-thermometer values [Kistemaker (*21*)]; (crosses) isotherm values [Kistemaker (*21*)]; (open triangles) gas-thermometer values [Berman and Swenson (*22*)]; (open squares) He^4 isotherm values [Keller (*23*)]; (solid squares) He^3 isotherm values [Keller (*23*)]; (solid triangles) preliminary acoustical-thermometer values [Cataland *et al.* (*24*)]; (solid diamonds) acoustical-thermometer values obtained in the work described. T_x represents actual values of temperature as determined by the experimenters.

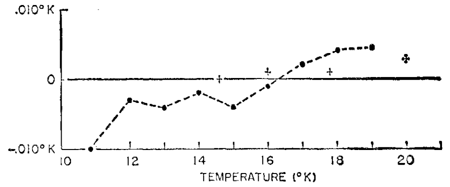

Fig. 5. A comparison between the acoustically derived temperature scale [NBS Provisional Scale 2-20 (1965)] and the NBS (1955) temperature scale (*25*). The difference in the values of the two scales, as indicated by two germanium resistors, is plotted as a function of the Kelvin temperature.

Results and Conclusions

The acoustical thermometer has been used to determine isotherms of He^4 gas at approximately every degree from 2° to 20°K. While the reproducibility of points on an isotherm is ±0.003°K, we are not proposing a value for the accuracy (with respect to the thermodynamic temperature scale) of temperature values derived from the isotherms. Rather, a comparison has been made between the temperature values obtained with our apparatus and those given on provisional scales, in regions where there is an overlap. The regions of overlap are 2° to 5°K, with the T_{58} He^4 vapor pressure scale (*2*), and 11° to 20°K, with the National Bureau of Standards Provisional Scale (*19*). The comparison was afforded by secondary germanium resistance thermometers calibrated against the three scales. The results are given in Figs. 4 and 5.

From the comparison with the experimental values for temperature upon which the T_{58} scale is based—that is, values in the range 2° to 5°K—it appears that the speed-of-sound approach to thermometry is at least competitive in accuracy with other methods that have been employed in this temperature range.

From the comparison with values in the higher temperature range (11° to 20°K) (Fig. 5), it appears that the deviation of our values from those of the NBS (1955) scale is, in general, within the limits of accuracy (from ±0.01° to ±0.02°K) stated for that scale. The lowest point, at 11°K (a difference of −0.01°K), may indicate an actual deviation between the two scales.

The acoustical thermometer has made a worthwhile contribution to absolute thermometry in regions below 20°K. It has resulted in the establishment of a provisional scale [NBS Provisional Scale 2-20 (1965)], which covers a hitherto uncharted range of temperature (5° to 10°K); calibrations against NBS Provisional Scale 2-20 (1965) have been performed for nearly a year. It is expected that the instrument will find more extensive absolute-thermometry uses and will be valuable in the investigation of other problems in the field of low-temperature research.

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Research in Music with Electronics

Effects of modern scientific ideas and technology on the analysis and composition of music are surveyed.

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The juxtaposition of the words *research* and *music* may seem curious to many scientists. On the other hand, as one of us (L.H.) has remarked (*1*, p. 11), "music has been a subject of considerable fascination to philosophers and mathematicians, even often being classified as a form of mathematics and thus considered to reveal natural law in terms of mathematical logic. . . . [If] this attitude strikes many of us as a bit peculiar today, perhaps it is . . . largely a consequence of the rather extreme views developed during the period of the nineteenth century Romantic tradition." In effect, we can say that music is more than a form of entertainment, that it can be subjected to rational investigation, the results of which contribute significantly to our knowledge of human communication.

This coupling of the concepts of music and research seems to be on the increase today; this has come about particularly since the end of World War II, when electronic techniques began to have their first really strong impact on music considered as a creative activity. At present this growing congruity of concepts is not only changing the thinking of creative people in the field of music but also attracting professional people from other fields—notably technical ones like electrical engineering and mathematics.

Research in music today extends well beyond historical documentation, the traditional subject of musicological research. It includes new work in acoustics and music theory and incorporates applications of new theo-

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