Book Reviews

Two Approaches to Mathematics

University Mathematics. vols. 1 and 2. Jack R. Britton, R. Ben Kriegh, and Leon W. Rutland. Freeman, San Francisco, Calif., 1965. vol. 1, xiv + 662 pp., $9.50; vol. 2, xii + 650 pp., $9.50.

An Introduction to Modern Mathematics. Nathan J. Fine. Rand McNally, Chicago, 1965. xviii + 509 pp. Illus. $8.50.

The two books being reviewed were designed for two different sets of readers; as one might expect, there are numerous and considerable differences between them, but the nature of these distinctions is occasionally surprising.

University Mathematics was "written with students in engineering and the sciences in mind" (from the preface) and is intended to provide the basis for the first 2 years of college mathematics for such students. All of the usual analytic geometry-calculus material is here—and then some. The first seven chapters of volume 1 include topics from elementary algebra and trigonometry that are frequently omitted from college courses, whereas the last two chapters of volume 2 are about, respectively, Laplace transforms and probability; and early in volume 2 there appear several chapters on linear algebra with applications to two- and three-dimensional geometry. Vectors are introduced in chapter 4 of volume 1, and several chapters of volume 2 deal with vectors and vector functions.

There are numerous good features. The notation and point of view are up to date, the exposition is straightforward and clear, and the authors have been successful in their attempts to provide a large amount of motivation (see, for example, the discussion of vector product and the preliminary material on the Laplace transform)—frequently a new concept is introduced by means of an example. The use of neighborhoods adds clarity to the treatment of limits. The section on integration by substitution is well done.

There are also some causes for displeasure, mainly lapses of rigor. For example, in chapter 12 of volume 1, after some preliminary discussion about the problem of defining area, area under a curve is simply calculated by an integral without being defined. Similarly, the concept of volume goes undefined. The continuity of a differentiable function is used, without reference, before the statement and proof of this property. In deriving the expression for the derivative of the inverse sine function, it is implicitly assumed that this derivative exists. The definitions of the double and triple integrals are not precise. And the absolute value signs are omitted from the Jacobian in the theorem on page 314 of volume 2.

Clearly, the authors intended to be moderate in their approach to rigor. For example, some of the basic theorems on continuous functions are stated without proof, and so are a number of other theorems throughout the book. In my opinion, the text would be stronger if the fine motivation were accompanied by greater rigor.

Fine's book is of a different breed. It is, to quote from the preface, " . . . for social science and liberal arts students. It is also suitable for advanced high-school students and prospective teachers. . . . For terminal students, the text is designed to give an insight into the nature of mathematics, plus an acquaintance with some of the major achievements in the field." Such a book is one of the most difficult of mathematics books to write: the decisions that have to be made with respect to the topics covered, and the placing of emphases, force the author to commit himself to a small portion of a broad spectrum of tastes and prejudices.

The selection of topics in *Modern Mathematics* is easily within one standard deviation of the mean: logic, sets, axiom systems (concentrating on Boolean algebra, a good choice), the real numbers, linear algebra, analytic geometry, calculus, and probability. The clear, intuitive discussion of logic and sets in the first two chapters is by no means window dressing: the ideas and notation introduced are used heavily throughout the book. The same is true of the use of axiom systems; for example, the first chapter on calculus begins with axioms for a ring of sets and a measure defined thereon.

With regard to emphasis, Fine has taken a firm stand in favor of rigor. He does not talk down to the student, and, except for the chapter on linear algebra, almost all theorems are proven. The novel proof of the integrability of a continuous function has as a by-product the uniform continuity (not mentioned in *University Mathematics*), boundedness, and attainment of maximum of a function continuous on a closed interval. A prominent feature of the view of mathematics that Fine gives his reader is the dependence of mathematics on logic and the precision of language and argument required.

There are, however, some curious omissions, not necessarily dictated by the approach discussed above. In the chapter, "Measure, area, and integration," there is a long, careful discussion of area and measure of area, with theorems on the area of parallelograms and rectangles, as a preliminary to the integral. But after the integral is introduced almost nothing is done about applying it to problems of area or anything else. Instead, a number of abstract properties of the integral are derived. Also, the derivative is introduced in terms of approximating a function at a point; its enormous utility as a measure of instantaneous rate of change is touched on only tangentially.

The authors of *University Mathematics* give the instructor many opportunities to "tighten up" the presentation; Fine has left considerable room for the teacher of his book to add motivation.

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Prehistory

Southwestern Archaeology. John C. McGregor. University of Illinois Press, Urbana, ed. 2, 1965. x + 511 pp. Illus. $9.50.

Southwestern Archaeology is a revised edition of McGregor's 1941 publication of the same title. This is the best available reference book on the