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The Gyroscope: Theory and Application

Present guidance requirements have made development of the gyroscope an important technology.

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For the guidance and control of submarines, ships, aircraft, missiles, and spacecraft it is necessary to have certain directional references available. These references, which serve as the basis for obtaining navigational data or for geometrical stabilization of a vehicle or some of its equipment, must be maintained despite various interferences and should be rotatable on command. The device which has proved most successful for the instrumentation of a reference direction is the gyroscope. In 1852 the gyroscope was defined by Foucault (*1*) as a device exhibiting strong angular momentum. Angular momentum (or moment of momentum) is a vector property of any physical body that is spinning, with respect to inertial space (*2*), about an axis. In the absence of an applied torque—a rotational effect of a force about an axis—an angular momentum vector maintains a fixed orientation in inertial space, thereby providing a directional reference. By applying a calibrated torque to the spinning body—that is, to the gyro element—one may command the angular momentum vector to rotate relative to inertial space in a known manner. Measurement of the calibrated torque provides a measure of the rate of inertial rotation of the directional reference.

The gyroscope has a high rate of spin about an axis of symmetry and is so mounted that this spin axis has freedom of angular rotation. The basic parts of a gyroscope are shown in Fig. 1. The gyro element consists of the spinning rotor, its drive mechanism, the spin-axis support, and the gyro-element support or gimbal. Other gimbals may be added to give the gyro element the desired rotational freedom. The torque generator is the component by means of which a calibrated torque is applied to the gyro element. The signal generator measures the angular orientation of the gyro element relative to the supporting structure. The number of gimbals, or the nature of the support, determines the type of gyro. The two-degree-of-freedom gyro has one gimbal, or equivalent support, in addition to the gyro-element gimbal; thus the spin axis has two degrees of rotational freedom. A two-degree-of-freedom gyro without torque-generating equipment is called a "free gyro." The single-degree-of-freedom gyro has no gimbal other than that of the gyro element. Thus, advantage is not taken of one of the possible degrees of rotational freedom, as may be seen in Fig. 2.

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The Gyro Model

The basic properties of gyroscopes have been treated in many texts (*3*) under the heading of "rigid body dynamics" and "the theory of the spinning top," as well as in several texts on the gyroscope itself (*4*). The classical description is very elegant, from a mathematical point of view, but is too complex for practical engineering design.

A major contribution to the practical understanding of the gyro is the simplified vector model developed by C. S. Draper of the Massachusetts Institute of Technology, which permits quantitative analytical study while retaining the geometrical picture. The development of this model is given in the Derivation Summary at the end of this article. Its validity rests on its proved usefulness and accuracy when applied to practical instruments under operating conditions. The basic equation of motion for a practical gyro is given by Eq. 5 of the Summary. The transient or characteristic solution (*5*) describes the nutation of a gyro or spinning top. The forced solution (Eq. 9 of the Summary) describes the precession. (Note that this physically oriented definition of *nutation* and *precession* differs somewhat from the definition associated with the classical treatment of the spinning top, which is based on the mathematical variables involved.) For most practical instruments the nutation is an undesired transient that must be damped out. Consequently, in the engineering design only the simplified, forced solution, which describes the precession, is considered—namely,

$$\overline{W}_{i(ge)} \times \overline{H}_s = \overline{M}$$

where $\overline{W}_{i(ge)}$ is the angular velocity of precession of the gyro angular momentum (effectively, the spin axis) with respect to inertial space; $\overline{H}_s$ is the spin angular momentum of the gyro; $\overline{M}$ is applied torque; and $\times$ indicates a vector (cross) product operation. Physically, this equation means that the spin angular momentum vector, $\overline{H}_s$, precesses relative to inertial space in an attempt to align itself with the applied

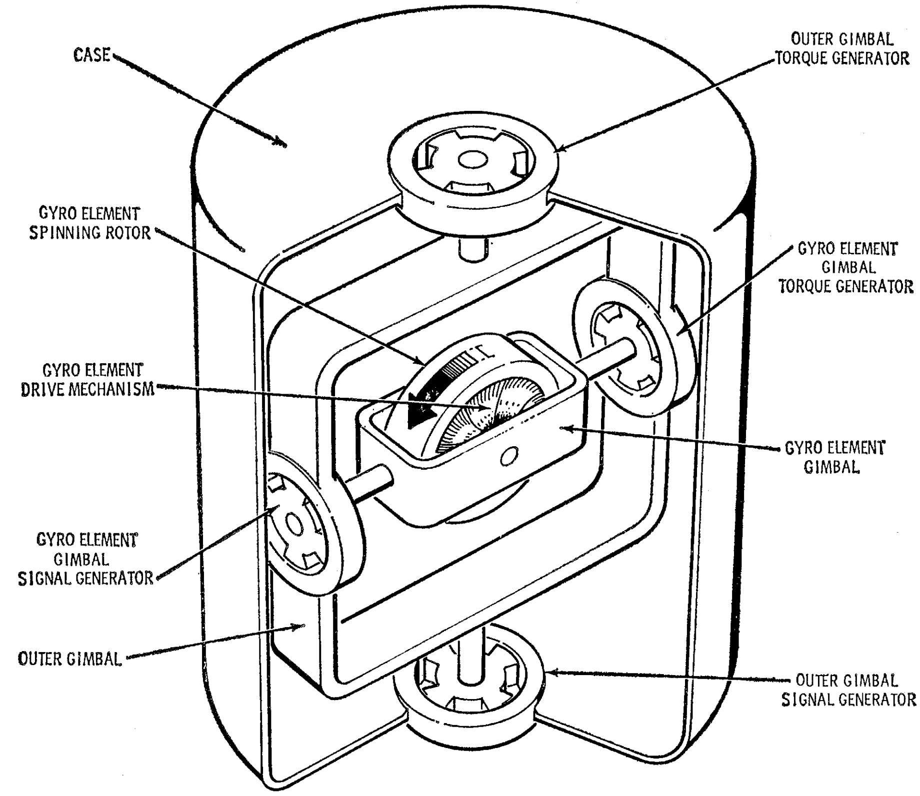


Fig. 1. Essential elements of a two-degree-of-freedom gyroscope.

torque vector, $\overline{M}$. This effect is illustrated in Fig. 3. The relatively simple equation contains all the information essential for analyzing the vector-model gyro.

Two-Degree-of-Freedom Gyros

The two-degree-of-freedom gyro is characterized by a support which permits the spin axis to have two degrees of rotational freedom. Originally the gyro was gimbaled in a cardan suspension similar to that shown in Fig. 1. In principle, the two orthogonal gimbal axes provide the two required degrees of freedom for the spin axis. By means of the torque motors the spin axis may be made to precess at a calibrated inertial rate. The signal generators measure the orientation of the spin axis relative to the gimbals. This type of gyro, chronologically the earliest, has served satisfactorily for applications in attitude instruments, fire control systems, and shipborne gyrocompasses. As applications requiring higher accuracy have developed, steps have been taken to remove some of the problems associated with the gimbals. When the two inner gimbal axes become aligned, the gimbals can no longer provide the spin axis with two degrees of freedom. To prevent this alignment, outer gimbals have been added whose function it is to keep the inner two gimbal axes orthogonal. The gimbals and the gyro element have often been floated to remove the bearing friction and the gimbal axes and to provide better shock resistance. The result is a much more accurate gyro, capable of inertial-grade performance. Two-degree-of-freedom gyros have been built without gimbals, by utilizing other means of support for the gyro rotor, one of the oldest being flotation, as utilized by Anschutz (*6*). Three newer types of suspension are the gas-bearing (*7*), the electrostatic-suspension (*8*), and the magnetic-suspension (*9*) types. Gas-bearing gyros are supported by the pressure drop in a thin film of gas which surrounds the rotor. Since the gyro rotor must be brought up to operating speed before the hydrodynamic action of the gas is effective, the life of such a gyro is limited by the number of starts and stops. For this reason gyros of this type are sometimes kept running continuously throughout their lifetimes.

The electrostatic-suspension gyro consists of a light sphere suspended in an evacuated spherical cavity by an electrostatic field. The attractive force on the rotor is controlled by making the capacitance between the rotor and its housing part of a tuned circuit which changes the current to the capacitance in such a manner as to keep the rotor centered, and so prevent it from touching the walls. The rotor is brought up to operating speed by means of a rotating magnetic field. The drag on the

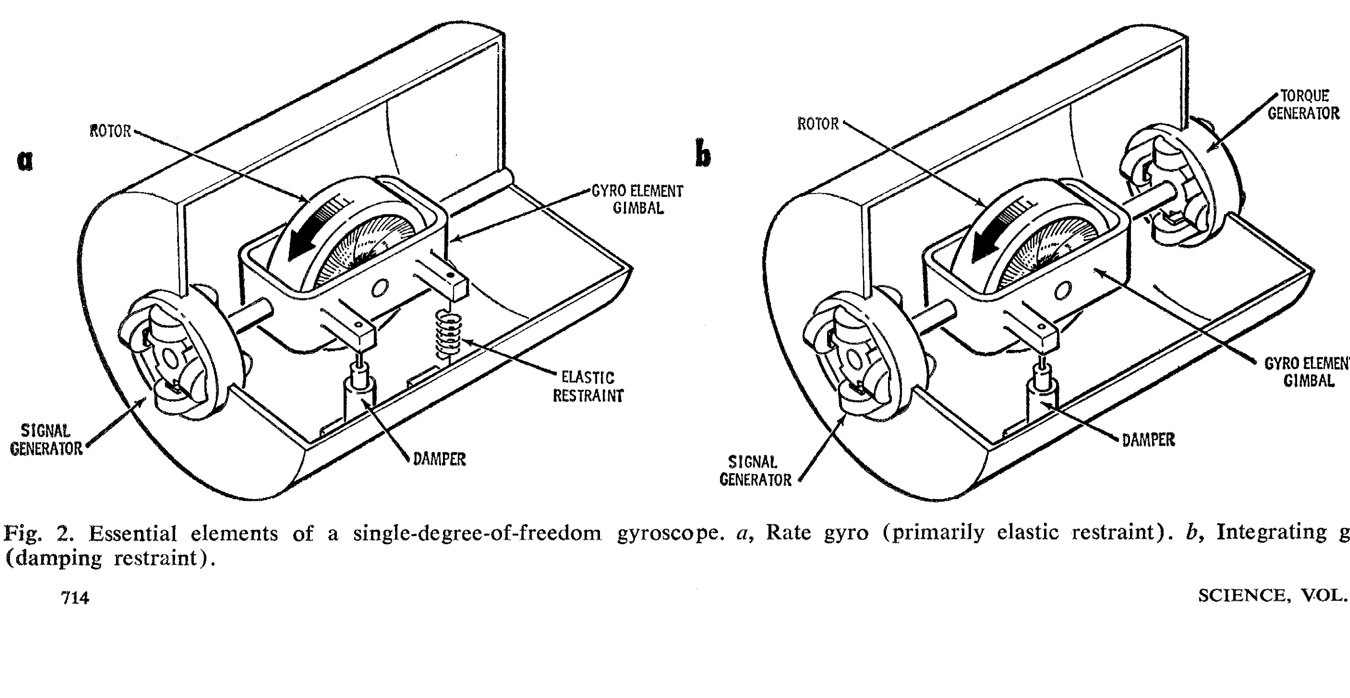


Fig. 2. Essential elements of a single-degree-of-freedom gyroscope. *a*, Rate gyro (primarily elastic restraint). *b*, Integrating gyro (damping restraint).

rotor is so small that it takes over a week for the gyro to run down to a stop. The attitude information is read optically. Protection must be provided to prevent the rotor from ever touching its container walls after a sudden power loss, else the gyro suffers a catastrophic failure.

The cryogenic gyro consists of a superconducting metal rotor suspended by a magnetic field. At temperatures near absolute zero certain metals become magnetic insulators and are repelled by a magnetic field. If the rotor is subjected to such a field it repels the walls of its container and is stabilized in the center of a spherical cavity without the use of any exterior electronics. The only requirement for maintaining the suspension is a continuous supply of liquid helium. The optical readout of the attitude information is something of a problem because the energy used to illuminate the rotor also dissipates heat, which raises the temperature, thereby endangering the continuance of the superconducting state. All these two-degree-of-freedom gyros must have some means of either preventing or damping nutation. Damping relative to the case is inadequate, and some form of inertia damper must be used.

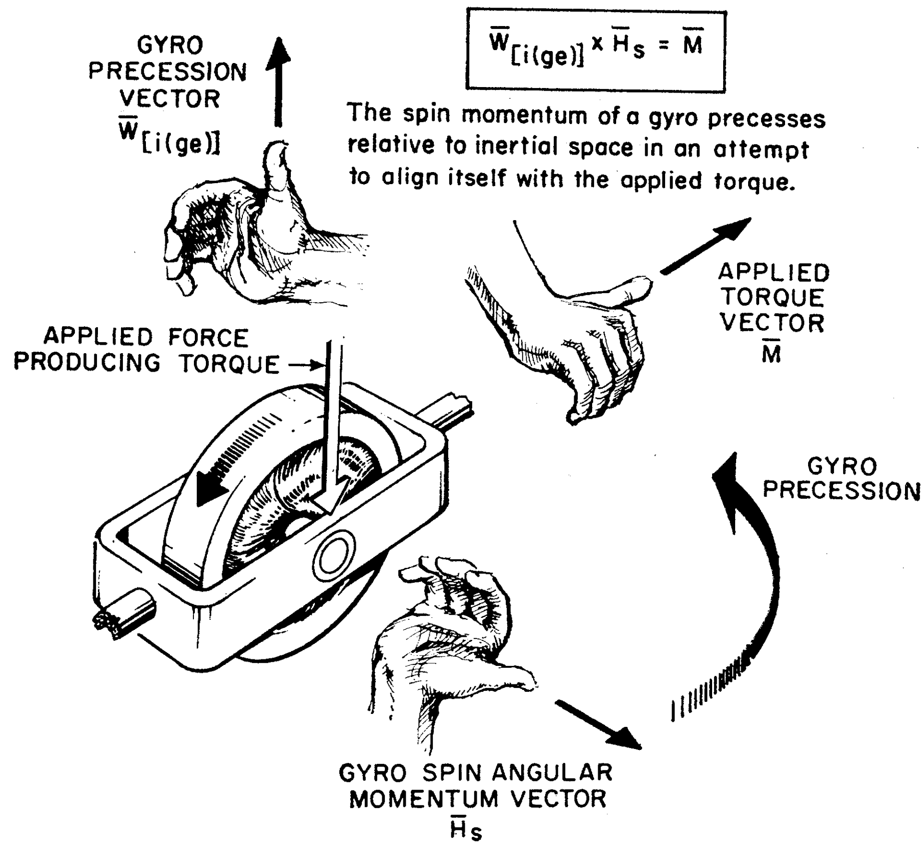


Fig. 3. Representation of the basic law of motion which underlies the operation of a practical gyroscope.

Single-Degree-of-Freedom Gyros

The single-degree-of-freedom gyro has no gimbal other than that of the gyro element. The basic features of these gyros are shown in Fig. 2. Such a gyro operates by nulling the torques applied about its output axis, rather than by the geometrical freedom, as in the case of the two-degree-of-freedom gyro. There are three types of such gyros, distinguished by the nature of the reaction torque employed. The unit is called a rate gyro if the primary restraining torque is elastic. The term *rate* refers to the fact that a steady deflection angle of the gyro element relative to the case is a measure of an inertial angular velocity of the instrument about its sensitive or input axis. The unit is called an integrating gyro if the primary restraining torque is a damping reaction. The term *integrating* refers to the fact that, in this type, the deflection of the gyro element relative to the case is a measure of the integral of the inertial angular velocity of the instrument about its sensitive axis—that is, the change of angular attitude of the instrument (*10*). When the damping-reaction torque is reduced to a very low value through use of a low-viscosity liquid or a gas, the primary restraining torque is the inertia reaction of the gyro element (really a change in momentum). A unit operating on this principle is called an unrestrained gyro. Actually there will always be some damping reaction, so the unrestrained gyro and the integrating gyro are essentially the same instrument with quite different amounts of damping reaction.

The earliest type of single-degree-of-freedom gyro to be used was the rate gyro. It was first used about 1920 as a basic indicator for instrument flying (*11*); later it was used to provide lead-angle data for antiaircraft fire-control sights (*12*), and later still, used in the flight-control systems of aircraft and missiles (*13*). Early rate gyros were mechanically supported on ball bearings, used a linkage indicator, had an air-driven rotor, and depended upon a mechanical spring for elastic restraint. More accurate rate gyros have an electromechanical "spring" formed by feeding the signal output of an integrating gyro back to the torque generator. The gyro element is supported by flotation in the damping fluid, and the rotor is electrically driven.

We owe the perfection of the integrating gyro largely to Draper (*14*). He realized that an excellent way to eliminate many of the performance limitations of the rate gyro was to get rid of the spring. The resulting instrument, an integrating gyro, depends primarily upon the damping torque for restraint of the gyro element. In practice, the integrating gyro is always used with a powered follow-up, which, in response to the signal generator output, rotates the case of the instrument about the input axis to keep the deflection angle between the gyro element and the case at null (*15*). In this manner, base motion effects produced by movement of the vehicle are eliminated, since the inertial rotation of the instrument about its input axis is maintained at zero unless the gyro is commanded to precess by means of its torque generator. Should the gyro be torqued, it will have a calibrated precession rate relative to inertial space about its input axis which will be matched by rotation of the case of the instrument through the powered follow-up. In this manner the integrating gyro is used to give physical equipment very accurately calibrated inertial angular velocities.

Some unrestrained gyros are essentially the same as integrating gyros, differing only in that the low viscosity of their supporting liquid makes the damping torque negligible in the desired regions of operation. Single-degree-of-freedom gyros which are supported by the pressure gradient of a gas are sufficiently unrestrained to be doubly integrating throughout the region of practical operation. An example of this type of unit is the air-bearing gyro developed at Huntsville, Alabama (*7*). The unrestrained gyro, like the integrating gyro, must have a powered follow-up which rotates the case of the instrument so as to null the deflection angle between the gyro element and the case. The engineering design of the servo follow-up must usually be more sophisticated for the unrestrained than for the integrating gyro because the double integration performed by the instrument makes the gyro deflection angle more sensitive to motion of the case.

The combination of damping and inertia reaction within the gyro gives the unit a characteristic response time. For the heavily damped, integrating gyro (viscosity of the damping fluid, a few hundred centipoises) the characteristic time is about 1 millisecond. For the unrestrained gyro with a low-viscosity damping fluid (viscosity, about 1 centipoise) the characteristic time is about 1/10 second. For the unrestrained gyro with a gas support (viscosity, a few hundred micropoises), the characteristic time is about 100 seconds. A short characteristic response time means rapid response with an associated small angular rotation of the gyro element within its case, and vice versa.

Reference Frames

The ultimate use of a gyro is to store directional information about the orientation of a selected reference frame. A reference frame is represented by a right-hand, orthogonal set of three coordinate axes. The orientation of one reference frame relative to another defines the attitude or navigational position of the vehicle. For example, consider the three frames which provide the conventional orientation of an aircraft, as shown in Fig. 4—namely, the navigational frame, body frame, and earth frame. The navigational frame is defined by a set of axes oriented north, east, and vertically down. The body frame is defined by a set of axes oriented forward, out the right wing, and out the bottom of the fuselage; the body frame is fixed relative to the aircraft.

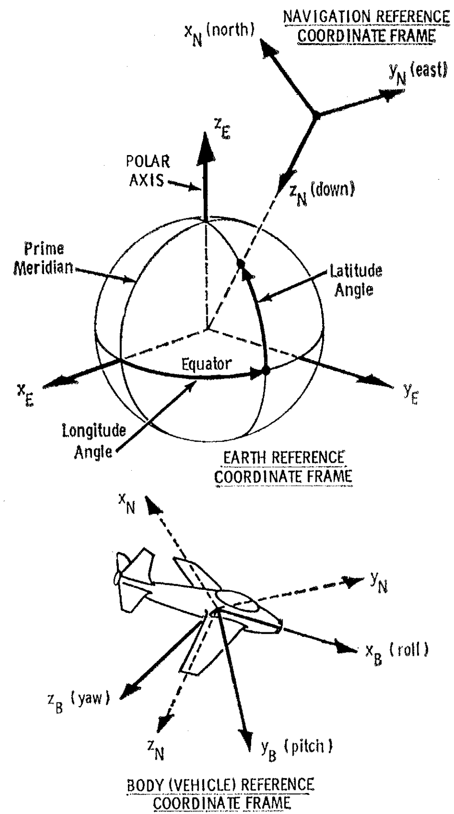


Fig. 4. Directional reference frames.

The orientation of the body frame relative to the navigational frame is sufficient to uniquely define the attitude of the aircraft. The attitude would be expressed in terms of aircraft heading relative to the north axis of the navigational frame, and of aircraft pitch and roll relative to the vertical axis of the navigational frame. The earth frame is defined by an axis parallel to the earth's polar axis, an axis normal to the plane of the Greenwich meridian, and an axis parallel to the intersection of the plane of the Greenwich meridian and the equator. These axes are nonrotating relative to the earth, and therefore rotate relative to inertial space exactly as the earth does. The orientation of the navigational frame relative to the earth frame is sufficient to uniquely define the position of the aircraft in terms of latitude and longitude. Instrumentation of all three frames leads to a complete on-board determination of attitude and position. Instrumentation of these frames, either by the geometric orientation of a physical platform or through the storage of signals which characterize the orientation of one frame with respect to another, is the basis of inertial navigation.

Attitude Instruments

Historically the gyroscope found many early applications as an attitude indicator in aircraft. When the horizon is not visible an aircraft cannot be flown safely without such instruments. A human pilot is incapable of detecting through his natural sense of balance the attitude of an aircraft when the vehicle is accelerating. Attitude instruments require instrumentation of the navigational frame. Indication of the vertical—that is, the direction of gravity—is sufficient for determining pitch and roll. Indication of north is sufficient for determining the heading. The accuracy required is usually of the order of about half a degree.

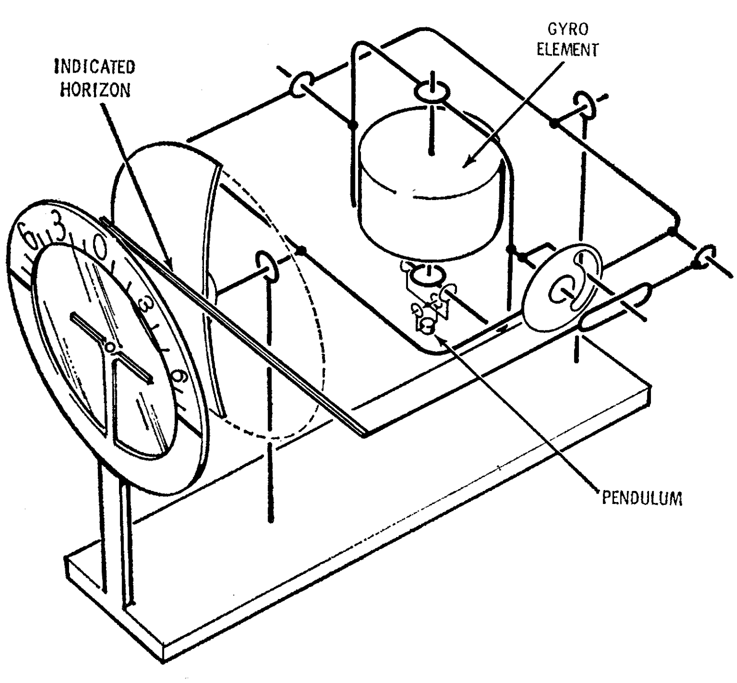
The most successful instrument so far developed for indicating aircraft pitch and roll is the "artificial horizon." It combines a two-degree-of-freedom gyroscope, for inertial space reference and mechanical smoothing, with a pendulous device for gravity tracking. Such a device is shown in Fig. 5. The apparent vertical (the combined effect of gravity and acceleration) is the direction indicated by a simple pendulous device. Such a device therefore does not correctly indicate the direction of the true vertical (gravity) at any given instant, because of vehicle accelerations. However, the average direction of the apparent vertical over a period of several minutes will approximate the direction of gravity well enough to provide an attitude reference. The artificial horizon operates by attempting to torque the gyro spin axis into alignment with the direction of the apparent vertical at an extremely low precession rate—for example, 1 degree per minute. The low precession rate causes the gyro to act effectively as a low-pass mechanical filter. The filtering action keeps the spin axis aligned with an average orientation of the apparent vertical, which is sufficiently representative of the true vertical for an attitude reference. In existing artificial horizons the torque-producing mechanism has many forms. Early air-driven gyros were torqued by means of air jets that were controlled by pendulous flippers. Newer methods include the use of rotating steel balls, pendulous eddy-current motors, and pendulous mercury switches with electric torque motors. The outputs of the artificial horizon are the angles that the gyro axis makes with the case of the instrument, which is fixed to the aircraft. These angles represent the measured pitch and roll of the aircraft and are usually presented to the human pilot as the orientation of a bar, or line on a ball, which he is meant to visualize as the orientation of the true horizon. The bar may be mechanically linked to the gyro gimbals, or the output may be picked off electrically and displayed by means of synchro repeaters. The pitch-and-roll information may also be used as input into an autopilot or a bombing- or fire-control system.

Gyro directional instruments are devices for maintaining an established horizontal reference such as the direction of north. The spin axis of a two-degree-of-freedom gyro is mounted horizontally, as shown in Fig. 6. A leveling mechanism keeps the gyro spin axis close enough to the horizontal to give satisfactory indication of heading. (The leveling device might be a single-axis artificial horizon.) A slaving torque is applied to produce azimuthal precessional control. Depending on the type of slaving-torque control, one of three different azimuth-indicating devices is used. (i) A device called a directional gyro is used if the slaving torque compensates only for the angular velocity of the reference meridian, with respect to inertial space, that is due to the vertical component of earth's daily rotation. The directional gyro was the first of the gyro directional instruments to be used in aircraft. The earliest models were air-driven and set to the correct heading by the pilot, from magnetic-compass data. The slaving-torque control was effected by a mass unbalance of rather low precision due to an off-center weight, and the correct heading had to be reset about every 15 minutes of flight. The resetting process required that the aircraft be flown straight and level long enough for the magnetic compass to settle down to an accurate reading. (ii) If the slaving torque compensates for the angular velocity of the reference meridian, with respect to inertial space, that is due to both the vertical component of earth's daily rotation and vehicle velocity with respect to earth, then the device is called a velocity compass. Whereas the directional gyro is a low-precision device that has to be reset often, the velocity compass is a high-precision device that maintains its reference direction for a long period by using accurate components. Components able to produce such accuracy are said to be of inertial quality. (iii) If the slaving torque operates to continually maintain the spin axis in alignment with the average direction of indicated magnetic north as sensed by a magnetic compass, then the device is called a gyromagnetic compass. It operates with respect to magnetic north in the same manner that the artificial horizon operates with respect to the direction of gravity, smoothing the input information to maintain the average value.

It is possible to use any or all three types of slaving-torque control on one horizontal gyro and to offer the navigator his choice of two or three different operational modes. This is the trend in modern aircraft.

The Gyrocompass

The gyrocompass is a navigational instrument which accurately seeks the direction of true north. Its function is to indicate attitude or direction, but its principle of operation requires gyro performance superior to that normally needed in other attitude instruments. The gyrocompass can really be considered the forerunner of inertial navigation (guidance by means of self-contained devices that respond to inertial forces) because the development of the gyrocompass led to the introduction of the first inertial-grade gyroscopes. As originally constructed the gyrocompass had a two-degree-of-freedom gyroscope with a mass unbalance that gave it a pendulosity at right angles to the spin axis. The principle of operation is shown schematically in Fig. 7 for the situation in which the instrument is fixed relative to the earth. The spin axis of the gyro indicates true north by settling on the intersection of the horizontal plane and the local meridian. However, when the spin axis is to the east of the meridian the rotation of the earth causes it to incline at a steeper angle, like the line-of-sight to a star. If there were no pendulosity, the tip of the spin axis would describe a circle



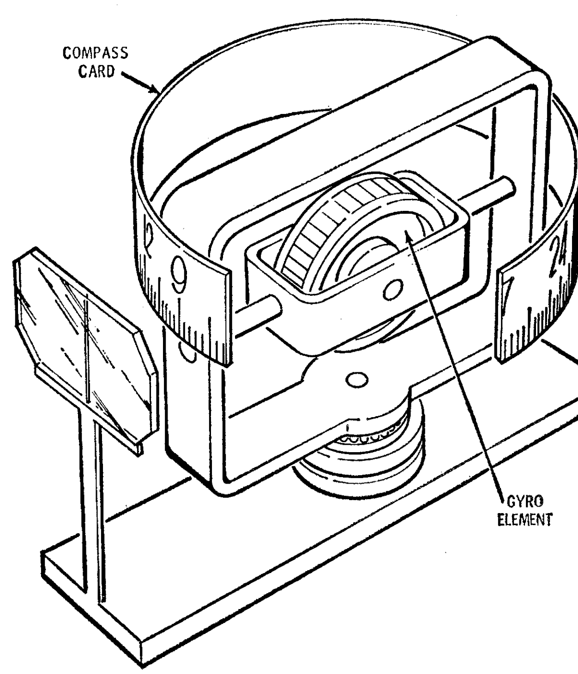


Fig. 5 (left). Schematic diagram of the artificial horizon. Fig. 6 (right). Schematic diagram of the directional gyroscope.

for an earth-observer in 1 sidereal day. The pendulosity, however, produces a torque that causes the spin axis to precess to the west when its tip is above the horizon and to the east when its tip is below the horizon, the tip thereby producing an elliptical pattern about true north; for a given gyro the period of oscillation decreases with increased pendulosity. This oscillation is damped out by applying an anti-pendulous torque caused by the flow of a viscous fluid through a constriction in a tube; the flow responds to the tilt of the gyroscopic element. The viscosity and the direction of flow through the constriction are such that the torque is applied in the proper phase for damping. The combined action of pendulous and damping torques causes the spin axis to spiral in toward the meridian. In the steady state the spin axis is on the meridian, with its north end tilted up slightly to provide the torque necessary to cause the gyro to precess at a rate equivalent to the vertical component of the earth's daily rotation. The heading accuracy provided by early shipborne gyrocompasses was of the order of ½ degree. This meant that the drift of the gyro had to be a small fraction of the earth's rate of rotation.

On a moving vehicle, the inertial angular velocity of the local navigational frame consists of the sum of the inertial angular velocity of the earth and the angular velocity of the local navigational frame relative to the earth. The horizontal component of the inertial angular velocity of the navigational frame defines the direction of "dynamic north." At ship speeds at navigable latitudes the angle between true and dynamic north is generally less than 4 degrees, due to latitude rate (the rate of change of latitude). Because the gyrocompass tracking dynamic north is on a moving vehicle, an accurate measure of ground speed is needed in order to determine true north. The pendulosity that produces the north-seeking property responds to vehicle accelerations as well as to gravity. A major contribution by Schuler (*16*) was the discovery that when the period of oscillation is made equal to

$$2\pi\left(\frac{\text{earth radius}}{\text{gravity}}\right)^{\frac{1}{2}}$$

(that is, when the instrument is tuned), the heading precession of the gyroscope spin axis due to acceleration is exactly that required to produce alignment with the changing direction of dynamic north. The resulting period of oscillation is about 84 minutes. This "Schuler tuning" is what made the use of the gyrocompass on a moving vehicle practical.

A gyrocompass may also be constructed with single-degree-of-freedom gyroscopes. This was the first step toward a complete inertial navigation system. Inertial navigation has made the gyroscope big business today. It is a complete subject in itself, and is described by several authors (*17*). Briefly, the gyroscope furnishes the directional references, and departure of these references from the corresponding correct values by 1 minute of arc per hour can lead to an error of 1 knot in the indicated velocity. It follows that high precision is required in inertial-grade gyros. The location of gyros in a typical inertial navigation system is shown in Fig. 8.

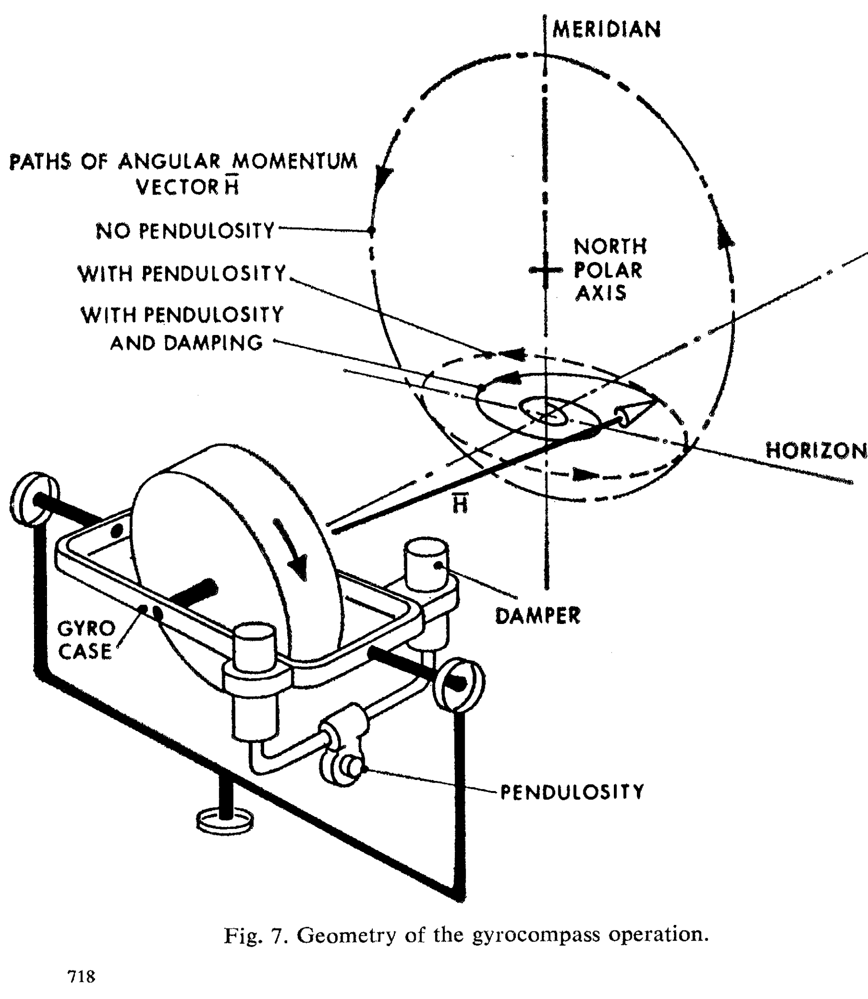


Fig. 7. Geometry of the gyrocompass operation.

Gyro Technology

The history of gyro technology is a story of methods, materials, engineering skill, and perseverance rather than of scientific breakthrough or new physical principles. A case study is the development of the single-degree-of-freedom, floating, integrating gyro, shown in Figs. 9 and 10. Of primary concern to the gyro designer is elimination of torque uncertainty about the output axis of the gyro, because such uncertainty is the direct cause of gyro drift—that is, failure to maintain the inertial reference direction. Early single-degree-of-freedom gyros were supported by ball bearings about the output axis. The first major step toward reducing the friction due to this support was to float the gyro element and replace the ball bearing by a jewel (sapphire "doughnut") bearing for centering it. The most practical flotation fluid is fluorolube (a fluorocarbon with a density a little less than twice that of water); it fills an annular gap a few thousandths of a centimeter wide between the gyro element and its surrounding case. The gyro element may be perfectly floated at only one temperature, so a heating jacket is added on the outside of the gyro, and the temperature is carefully controlled. The development of a dense, noncorrosive Newtonian fluid was an important step. Even with the fluid supporting most of the weight of the gyro element, the small bearing friction of the jewel was found to be excessive for inertial performance of the gyro. Use of a passive magnetic field suspension, which keeps the gyro element centered in its case and supports the residual weight not compensated by flotation, solved the centering problem. Use of plastic potting compounds, which seal off the electric elements, permits machining of the torque generator and signal generator to tighter tolerances, and so helps prevent spurious magnetic torques. A change in position of anything whatsoever inside the gyro element, even though of microscopic size, may produce a torque when the instrument is acted upon by gravity or accelerations.

The rotor itself must be restrained from shifting along its spin axis for more than a few millionths of a centimeter. This necessitated years of research and the development of ball bearings that operate under a load, retain their lubricant, and give stethoscopically smooth operation. Self-lubricated gas bearings with microscopic clearances have recently been developed for spin-axis operation. Spattering of lubricant (from the rotor bearings, for instance) had to be eliminated; this was accomplished by the development of a special nylon bearing retainer which would hold the lubricant like a sponge until it was needed.

One of the most important problems of construction and assembly is the absolute elimination of dirt. A small piece of dust weighing a thousandth of a gram can introduce a drift rate equivalent to 1½ kilometers per hour in an inertial-grade gyro by shifting from one side of the gyro element to the other. Consequently, gyros are assembled in "clean rooms," in which the air is filtered and hospital-operating-room cleanliness is maintained. Prior to final assembly, gyros are inspected with a microscope for dirt. The rotor rim is now made of a tungsten alloy instead of steel, for increased moment of inertia and consequently increased angular momentum. The gimbal float is now made of beryllium instead of aluminum, for lightness and strength. After the gyro has been assembled it is tested to determine its drift characteristics. Any drift characteristics which are constant with time may be compensated for by the system in which the gyro is used, so long as the characteristics are known. Additional operating and statistical techniques to compensate for, or average out, the unknown random gyro drift are currently under development.

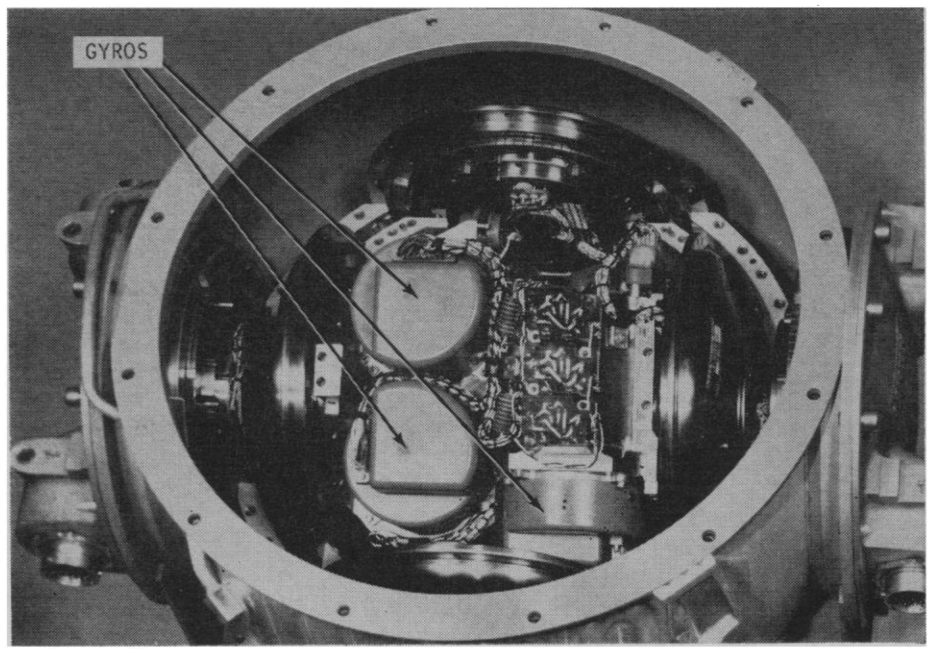


Fig. 8. Location of gyroscopes in a typical inertial guidance system.

In the testing of inertial-grade gyros the earth itself is utilized as a turntable; in fact, the testing of inertial-grade gyros is almost a profession in its own right (*18*). Shock effect and vibration effect (both linear and angular), the effect of large force fields produced by centrifuges, and environmental effects are among the factors studied in evaluations of a gyro's ability to do its job. Only the force-free environment of space is not reproducible in our earth-bound test facilities.

Continous efforts are being made to improve the performance of gyros, as well as to achieve present performance with smaller, lighter, less expensive units. At the same time efforts are being made in many laboratories to find new means of obtaining an inertial reference. These efforts are generally based on one of the following basic principles.

1) Different support for conventional angular momentum devices. In this category are the electrostatic and cryo-

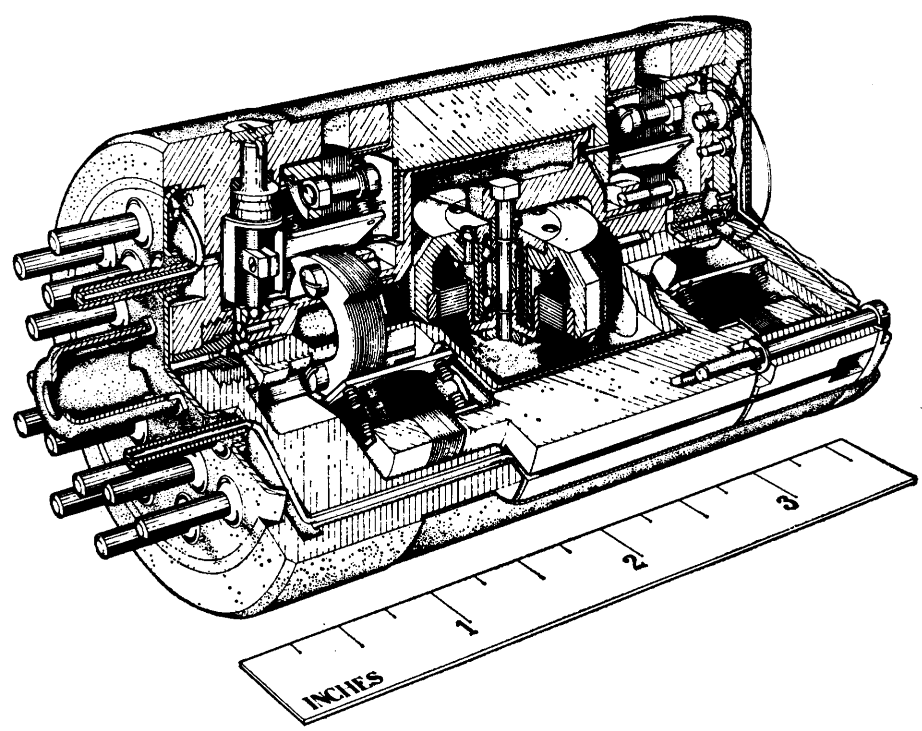


Fig. 9. Cutaway diagram of a single-degree-of-freedom gyroscope.

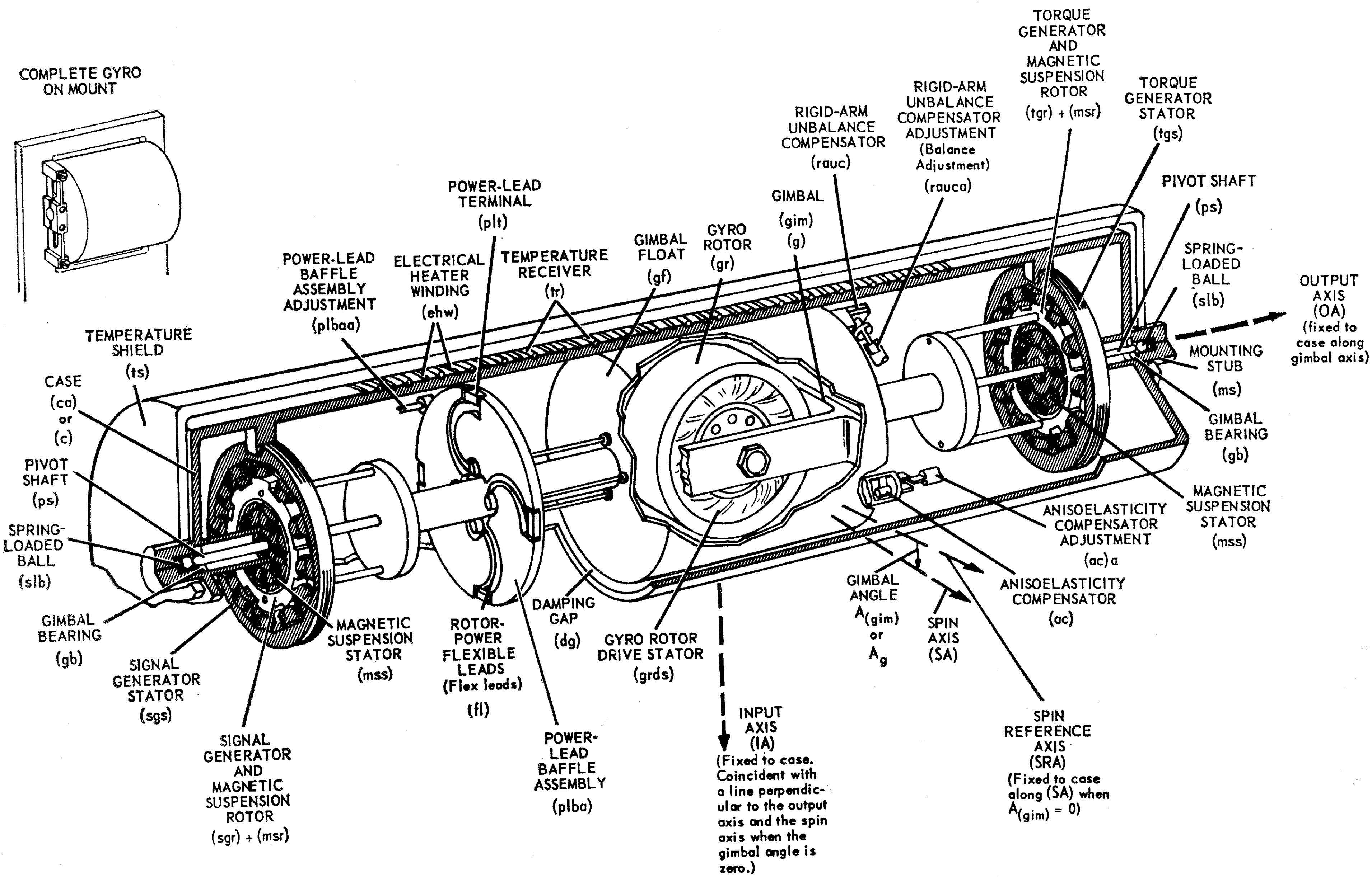


Fig. 10. Schematic diagram of a single-degree-of-freedom gyroscope.

genic gyros, as well as various off-center devices with which it is hoped to produce a modulated signal and so use communications filtering techniques to obtain a higher accuracy.

2) Other forms of angular momentum, achieved, for example, through vibration or use of a fluid.

3) The angular-momentum effect of atomic particles or nuclear particles, or both.

4) The difference in propagation time of oppositely directed beams of light. The ring-laser gyro is based on this principle.

Summary

The present-day inertial-grade gyro has been brought to its present state by the need for self-contained, all-weather navigation. Military requirements for high accuracy and reliability have made gyro technology big business. The major drawback has been the great expense of producing high-performance gyros. If the industry can meet this challenge at relatively low cost, the gyro will find additional application in the guidance and navigation of nonmilitary vehicles. The use of inertial navigation by commercial aircraft is just now developing. The use of inertial systems as an aid in oceanographic and geodetic surveying is being investigated. The gyroscope has become an important instrument and will grow even more important as progress is made toward still higher accuracy, greater reliability, and lower cost.

Derivation Summary

Application of Newton's second law of motion to rotation gives

$$p_i \overline{H}_{ge} = \overline{M} \tag{1}$$

where $\overline{H}_{ge}$ = angular momentum of the gyro element, $\overline{M}$ = torque applied to the gyro element, and $p_i = d/dt$ with respect to inertial space. From the equation of Coriolis, we obtain

$$p_i \overline{H}_{ge} = p_{ge} \overline{H}_{ge} + \overline{W}_{i(ge)} \times \overline{H}_{ge} \tag{2}$$

where $p_{ge} = d/dt$ with respect to the gyro element and $\overline{W}_{i(ge)}$ = angular velocity of the gyro element with respect to inertial space. In a gyro

$$\overline{H}_{ge} = \overline{H}_s + \overline{H}_{ns} \tag{3}$$

where $\overline{H}_s$ = angular momentum of spin and $\overline{H}_{ns}$ = angular momentum other than spin (nonspin).

From Eqs. 1–3 we obtain

$$p_{ge} \overline{H}_s + p_{ge} \overline{H}_{ns} + \overline{W}_{i(ge)} \times \overline{H}_s + \overline{W}_{i(ge)} \times \overline{H}_{ns} = \overline{M} \tag{4}$$

In a practical gyro, by definition,

$$\overline{H}_s >> \overline{H}_{ns}$$
$$p_{ge} \overline{H}_s = 0$$

From this we obtain, for the vector form of Euler's equation,

$$p_{ge} \overline{H}_{ns} + \overline{W}_{i(ge)} \times \overline{H}_s = \overline{M} \tag{5}$$

In matrix form, with the z-axis the spin axis, Eq. 5 becomes

$$\begin{bmatrix} I_x p & H_s & 0 \\ -H_s & I_y p & 0 \\ 0 & 0. & I_z p \end{bmatrix} \begin{bmatrix} W_x \\ W_y \\ W_z \end{bmatrix} = \begin{bmatrix} M_x \\ M_y \\ M_z \end{bmatrix} \tag{6}$$

where I_x, I_y, and I_z are the principal moments of inertia of the gyro element about the x, y, and z axes, respectively. For a two-degree-of-freedom gyro, this reduces to

$$\begin{cases} \left[\dfrac{I_x I_y}{H_s^2} p^2 + 1 \right] W_x = -\dfrac{M_y}{H_s} + \dfrac{I_y}{H_s^2} p M_x \\ \left[\dfrac{I_x I_y}{H_s^2} p^2 + 1 \right] W_y = \dfrac{M_x}{H_s} + \dfrac{I_x}{H_s^2} p M_y \\ I_z p W_z = M_z \end{cases} \tag{7}$$

Undamped natural frequency

$$W_n = \frac{H_s}{\sqrt{I_x I_y}}$$

is "nutation"; forced response to M_x, M_y, and M_z is "precession."

For a single-degree-of-freedom gyro the gyro element is constrained to have independent motion about only one axis. Let the x axis be the input axis, the y axis be the output axis, and the z axis be the spin-reference axis. The motion of interest is that about the unconstrained output axis. The component equation of interest in Eq. 6 becomes

$$I_y p W_y = H_s W_x + M_y \quad \ldots . \tag{8}$$

When the transient can be ignored, Eq. 5 becomes

$$\overline{W}_{i(ge)} \times \overline{H}_s = \overline{M} \tag{9}$$

This is the generally useful expression for gyro analysis.

References and Notes

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