atom between protium and deuterium. This difference in vapor pressure would lead one to expect a shift of 21°C in transition temperature for a transition occurring near room temperature. A shift of 12°C in transition temperature has been found between deuterated and ordinary polybenzyl glutamate (*18*) (see Fig. 12). At any temperature the helical form is more stable for the deuterated compound than for the protium compound. This is a consequence of the greater stability of the deuterium bonds than of the protium bonds in hydrogen-bonded species. Similar results have been found with the enzyme ribonuclease. If enzymatic activity is dependent on the structure of the enzyme, deuteration can be expected to have profound effects on enzyme activity in some cases. The small shift in transition temperature can manifest itself in a qualitative difference in enzymatic behavior.

In addition to this effect of catalyst structure, we can expect rates and equilibria in deuterated media to be different from those in ordinary water. Mechanical properties such as viscosity are different in light and in heavy water. All these effects lead to differences in biological activity.

The general findings have been that growth rates are retarded in D_2O as compared with those in H_2O. It has been found possible to adapt lower animals from growth in H_2O media to growth in D_2O media, and vice versa. The change must be made slowly; even so, the morphology is considerably altered (*19*) (see Fig. 13).

In higher animals—for instance, rats—tolerance to 30 percent of D_2O has been established. Even with this concentration of D_2O the animals become quite sick after a month's exposure. The symptoms include considerable impairment of kidney function, anemia, disturbed metabolism, and altered adrenal function (*20*).

Summary

With the exception of the field of chemical kinetics, a brief survey has been presented of the principles of isotope chemistry and their utility in the ever-unfolding panorama of scientific research. We have come a long way since Soddy and Fajans arrived at the concept of chemical twins—isotopes. There are still places to go.

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Galileo's Discoveries in Dynamics

He left a mathematical stamp on nature still felt by physicists and natural philosophers.

Norwood Russell Hanson

Giants are frightening at close range. One views them with detachment only from far off. A giant within the history of science is no different. To give hindsight full play, we need to be 400 years distant from such an Olympian. How else can mere mortals take the measure of Galileo's mixture of imagination, intellect, and courage? (See *1*.) His contemporaries idolized him, or tugged back at his sandals, depending on whether they were frightened or inspired. At our remove fright is no longer likely. But even inspiration must be restrained for complete objectivity. Centuries of scholarship to the contrary notwithstanding, Galileo was not a great experimental scientist. He was no experimental scientist at all, not as we would know one. Nor was he a powerful theoretical thinker—surely not within technical mechanics. But he left a mathematical stamp on nature, the full imprint of which is still felt by physicists and natural philosophers.

Let me urge these theses seriatim: first, that Galileo was no experimentalist.

Experimental Scientist?

My first exposure to Galileo was to Galileo as "the father of experimental science." Apparently he was the first fully to appreciate precise laboratory measurement, careful experimental con-

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trols, and the repetition of all determinations of significant parameters. He was the earliest who, apparently, let the natural world write *its own* description. He did not force scholastic preconceptions on matter; rather, mathematical physics was for him Nature's autobiography. Galileo's laws, such as that $s = \frac{1}{2} at^2$, were generalizations (I was told) from stacks of repeated measurements, trials, observations, and tests—ever finer adjustments, ever more precision and calibration, ever more heaping up of the raw facts. One counterinstance made him cheerfully abandon any thus-discredited "law," so strict were his empirical sensibilities. Galileo was the first, continues the myth, to perceive physics as the factual and essentially observational discipline it really is (*2*).

Purely myth all this surely is. It is a fantasy about Galileo, and a fantasy about past and present physics, dreamed up by commentators who (perhaps) understand too little of both. What does one say of a man, like Galileo, for whom experiments were only *demonstrations* of what reason and reflection and argument have already revealed? Just that he was not really an empiricist at all? No. Such a man may tell the truths of physics better than "dust-bowl experimentalists." Perhaps being an empiricist in the full sense is being more than a fact-grubbing pebble-counter. Galileo was never prostrate before nature's mere minutiae. He was never seduced by the attractions of precision for its own sake, or by the Sangreal of finer, ever finer, tests and measurements. Yet he unraveled some critical factual knots within the history of thought.

For him, laws of nature were not just superdescriptions generated out of observations. Rather, observations were themselves intelligible only insofar as they were informed by laws (*3*). Laws constituted the rationale of nature; Galileo's acute demonstrations were just lively illustrations of that rationale. To comprehend the structural plan of the physical world required not busy elbows, dextrous fingers, and sharp eyes. It required hard thinking about the nature of Nature—about the essential form and format of physical processes and phenomena (*4*).

This is not to suggest that Galileo, like a Romantic Idealist, snubbed the facts of sense evidence. Hardly (*5*). He was ever ready to consider new ideas and unorthodox techniques with which to acquire knowledge. Uncannily, he always hit on observations (often commonplace ones) which probed and provoked immense theoretical issues; he *could* have seen the universe in a grain of sand! That water rises only to 32 feet in suction pumps was a lens for his philosophical vision. But again, it was physical theory which was beheld by such vision, not the making of further experimental lenses for inquiry per se. Granted, Galileo is renowned for his perfection of instruments; the proportional compass (1596), the thermoscope (1602), and, most significantly, the telescope (1609) were perception aids with which he observed what men had never encountered before (*6*). But this remarkable creativity should be charged to technical development, to curiosity, and to the deepest respect for Nature as she is. Nowhere does Galileo gravely pronounce Baconian principles, by the rigorous adherence to which truths might be tortured out of natural subject matters. The idea of *inertia*, although imperfectly formulated by Galileo, was a great conceptual and theoretical achievement (*7*). But as for leaving it to experiment and observation to corroborate that concept, Galileo could never have understood such a suggestion. Factual details (like rococo tracery) confuse the intellect and cloud the imagination; only these latter faculties can apprehend and comprehend the structure of nature beneath the deceptive superficialities which constitute the surface of things (*8*).

As a Theoretical Thinker

Galileo's reflections take one to the edge of imagination. Continually he sweeps us to the consideration of *limits* of rectilinear motion, as traced through media of diminishing density—ultimately complete vacuums. The processes he discusses may be nonterminating; they may, indeed, require an infinite amount of time. This man stretched his readers' minds to the utmost. In thought he dilutes to the vanishing point the accidental colors of objects. He sharpens and straightens their dimensions and edges until they are razorlike. He is ever subtly smoothing their motions, leaving us at last with the ideally abstract case: colorless, tasteless, soundless, frictionless. This abstraction lacks the thousand accidental features that actual phenomena are heir to. Thus does Galileo force us to mark and remark the essential formal aspects of dynamical phenomena. Thus does physics seem increasingly like pure mathematics, plus some additional parameters involving masses, forces, velocities, and so on. Indeed, whatever in nature could not be so managed seemed to Galileo not amenable to proper physical analysis at all. Mere descriptions of accidental, local phenomena suffice for natural history—"bug hunting"—but not for natural philosophy.

Hence, laws of nature could not possibly be descriptive generalizations for Galileo, however precisely generated and carefully culled. Rather, laws set out the conceptual "framework features" of dynamical phenomena. These must be comprehended before their factual embodiments (objects and events) can even be perceived intelligibly (*9*). Thus, the laws must somehow precede our confrontations with phenomena—psychologically, logically, and conceptually—else there could be nothing lawlike in our experience of phenomena. Laws considered as a composite residue from multiple exposures to "mere" phenomena—such a notion would have disturbed Galileo deeply. Rightly so. There are no objects, processes, or facts *simpliciter*; an object is never *just* seen, never *just* experienced in phenomenological isolation, nor *just* known in an epistemological vacuum. It is seen *as* something or other; it is experienced as this or that kind of thing; it is known to have these properties rather than those. Processes are observed to have this direction or that, this development and fulfillment or that particular disintegration (*10*). Facts are always facts about, or with respect to, or set out in terms of, some theoretical framework. Should the framework deliquesce, the objects, processes, and facts will dissolve conceptually (*11*). Where now are the "facts" of alchemy, of phlogiston theory? Or must we grant that no observations ever really supported such frameworks of ideas? Where can one now locate a sample of caloric, or a magnetic effluvium? How easy and doctrinaire to remark these as chimerae, as illusions of fact. They are actually once-descriptive references whose supporting rationale has disappeared. Their articulators were, in their way, dedicated empiricists groping, struggling, to delineate *the facts* concerning intricacies of a near-incomprehensible world. Bu effluvia, caloric, phlogiston, influences virtues, humors, essences, harmonies, at tractions, and powers—these are n longer sustained by laws, as once the appeared to be, and as *our* nov recorded facts, processes, and objec seem so surely to be. But the negativ energy electron of 1928, the luminiferous ether and the planet Vulcan of the 19th century, are not so long departed from the scientific stage. May not the solid acquisitions of our own laboratory performances yet grow pale before the chilling winds of new doctrine—doctrine opposed to our presently accepted theories?

Physics is an open-ended investigation of nature. Yesterday's data, recast within tomorrow's theories, may depict yet a different world even to the closest observers. J. J. Thomson could see everything that Compton saw, but not in the same way (*12*). Kepler espied nothing which Tycho Brahe lacked the capacity to observe, but their observations cohered very differently (*13*). And, save for his telescopic work, little within Galileo's perceptual field had not been perceived, element for element, by earlier natural philosophers (*14*). Yet he perceived what they could not: he discovered connections and relations between known elements of inquiry—he found their organization. Galileo's theoretical vision made him a *better* empiricist than his contemporaries (*15*), better even than our statistics-bound, data-mongering contemporaries. It enabled him to see more of the world than they could, or can. He pierced the surface of dynamical events, conceiving within them a mathematical order analogous to what Euclid had perceived in the space all around him. Euclid geometrized space. Thereby he made its observed properties intelligible. Galileo mathematicized dynamics, and thereby he made its *facts* an object of philosophical study (*16*).

What, then, was *discovery* to Galileo? It was the perception of cohesive, mathematical structure within the buzzing detail of experience. For him, every falling coin, every wind-blown leaf, every new moon was a special kind of anomaly, an occasion for inquiry. Phenomena like these, familiar but not understood, were the windows through which the anatomy of the universe could be witnessed, if one but focused the appropriate mathematical lens. Through lenses of his own design Galileo had seen the moon as terrestrial. So also he viewed dynamical events through algebraic lenses ground by his own intellect. To have perceived that all of nature was visible through such lenses—more, to have urged that its capacity to be so viewed was the defining characteristic of what we are entitled to call "Nature"—*there* is the synoptic discovery of this visionary student of the facts. All his other findings are subordinate to, and supportive of, this one brilliant insight. For now the world lay before Galileo, as it lay before Adam, virtually undiscovered, in facts-as-yet-unseen. Here was the first modern natural philosopher with the eyes of a mathematical physicist. Hence, for him, as for the ancient Archimedes (and for Newton later), to look anywhere—to see and to understand the phenomena before one—was to discover. Galileo had found the code of physical existence—the rest was largely decipherment. It soon came to be recognized that the code was infinitely more complex than Galileo had imagined. But the fundamental insight is not different, as Lagrange, Laplace, and Leverrier, Clerk Maxwell and Willard Gibbs, Dirac, Heisenberg, Pauli, Schrodinger, von Neumann, and many other modern heroes all make abundantly clear. These are the names of empiricists possessed of the compound vision of mathematicians. These men, too, tell the truth about the facts of nature, only, like Galileo, they can see those facts more fully than searchers with mathematically untrained eyes.

Consider all this as it bears on but one tiny portion of Galileo's work (*17*). Galileo had argued that, when a sphere rolls down an inclined plane on one side of a room and then rolls across the floor and up a plane on the opposite side, it will ascend the second plane to just that height (above the floor) from which it had been released on the first plane. What the sphere acquires in its descent is thus equal to what is needed to drive it up the second slope to its original height.

Incline the second plane less and less steeply relative to the floor. The sphere will still "seek" a height on that second plane equal to that from which it started. As the second plane gets closer to the floor, the sphere travels along that plane "in order to" resume its original height. As the angle between the floor and the second plane gets closer to zero, the distance the sphere will travel along it will increase; it will, indeed, proceed toward an infinite length of travel as the angle of inclination approaches the limit zero (see Fig. 1, top).

Thus a sphere moving on an ideal (frictionless) floor will proceed along a straight line to infinity. Only some-

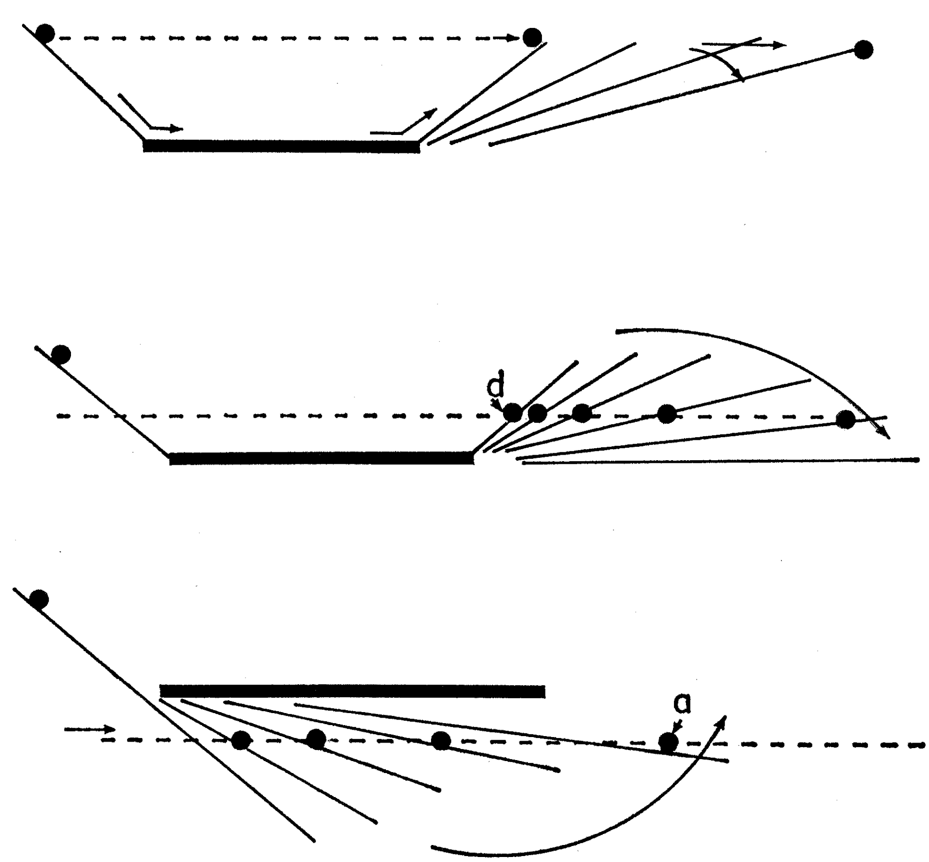

Fig. 1. Galileo's inclined-plane experiment. (Top) Sphere ascends to the same height as that from which it began its descent. (Middle) As the plane-inclination flattens toward the horizontal limit, the deceleration variable (*d*) goes to 0. (Bottom) As the plane-inclination lifts toward the horizontal limit, the acceleration variable (*a*) goes to 0.

thing like our second inclined plane (or like friction, or air resistance) prevents this, by taking from the sphere just what distinguished it from a sphere at rest.

Galileo's reflections (*18*) could have gone a little further. This much is convincing as to the nonterminating, rectilinear character of force-free motion. But that such motion will be *uniform* Galileo assumed to follow qualitatively from his "thought experiment"; either that or it seems to follow from his idea of "force-free motion" (*19*). But why assume what is demonstrable? Think of all those inclined planes which could nest within the angle between our secondary plane and the floor. As the plane is lowered, every intervening angle will have been traversed by the plane, and by the ascending sphere. Imagine a line parallel with the floor (but lower than the original height from which the sphere descended) (see Fig. 1, middle). A line could then be drawn through all those possible inclinations to which I have just referred. Consider the intersection of that line with each of these inclined planes. At each such intersection note the deceleration of the ascending sphere (the rate at which its velocity is falling off as it climbs the secondary plane). Intuitively, we expect the deceleration to be greater on a steeply inclined plane than it would be at the corresponding point on a plane of shallow inclination. As this plane is "flattened" down to where it coincides with the floor, the value of the deceleration variable (as it "moves" along the line) will itself decrease. When the plane does join the floor, the deceleration will be zero (*20*). A similar argument applies for acceleration (see Fig. 1, bottom). But the point is that the uniformity of a body's (force-free) motion can be argued for: it need not be merely assumed (*21*).

The ideas of "rectilinearity," "motion ad infinitum," "uniform," and "force-free" are interdependent conceptions within Galileo's mechanics. One can treat uniform, rectilinear motion ad infinitum as itself built into the notion "force-free" (as part of the latter's semantical content). Thinking of a body free of impressed forces would then just mean thinking of a body either at rest or in uniform, rectilinear motion. But this game can also be played by packing "force-free" into one of the other concepts—uniform, or rectilinear, or motion ad infinitum. And so on.

Thus, one thing learned in trying to understand Galileo's law of terrestrial inertia is that its terms are semantically linked. The meaning of each of its terms "unpacks" sometimes from the others, but sometimes the meaning of these others "unpacks" from the first. Which are "the contained" and which the semantical "containers" affects the exposition of any mechanical theory built thereupon. In this way one can distinguish the mechanical theories of Galileo, of Lagrange, and of Hertz. Archimedes longed for an immovable platform away from which to lever the world; so also every physical theory requires a set of stable, primitive conceptions in terms of which all its other terms can be explicated. Although Galileo, Lagrange, and Hertz gave their energies to the same theory, classical mechanics, they chose different semantical platforms on which to fix their fundamental laws; hence they confronted their further theoretical problems in different conceptual postures.

"The Law of Inertia" is thus really a family of schemata, and this is so even in Galileo's formulation. The essentially algorithmic function of the law is contained in this fact; the theoretician can trace whatever genealogy of concepts he chooses from little more than his first decision to invest Galileo's law with *this* semantical structure rather than *that*.

I have considered a typical Galilean statement of the law. I have noted also the reflections which made it seem plausible to Galileo and his successors, as well as the semantical decisions which will, in different formalizations of the theory, generate different meaning-relations between the constituent terms within the law: sometimes *A*, *B*, and *C* will be semantically primitive and *D* will be derived; sometimes *D* will be primitive. One function of Galileo's law consists in such interrelating of mechanical concepts. Hence, a Mach and a Hertz may interrelate these terms differently; the law does different work within their theories—different even from what it did within Galileo's reflections.

Discoveries in Dynamics

I have also characterized as mythical the oft-heard claim that Galileo was a great experimentalist. His ingenuity with instruments must not be confused with the repetitive data-gathering typical of genuine empirical science. Indirectly, all this is clear from his attitude toward laws of nature; for Galileo these are never summaries of observed phenomena. Rather they are the "pure cases" through which the observations become intelligible—this, even though some "non-observables" may figure in the law-statements themselves.

These "non-observables," in Galileo's laws of dynamics, concern the numerical *limits* of sequences (considered in spatial or temporal contexts, or both) and the physical *limits* of processes (such things as pure vacuums and instantaneous velocities). References to infinities and infinitudes, "sharp" instants and geometrical points, planes without edges, spaces of indefinitely great dimensions—these references abound in the "Dialogues" and in the "Discourses." These make nature tractable and mathematical, without the insignificant local "accidents" of faded colors, uneven textures, odors, impurities, crude calibration, and lopsided carpentry. Understanding thus the structure of dynamical phenomena, Galileo came to perceive the formal framework within which such events occurred—fully to comprehend which required "Euclidean" treatment, founded on abstract, "pure," nonfactual and wholly general "limit-ideas." Thus, just as "ad infinitum" dominates Euclid's fifth axiom, so also "ad infinitum" figures essentially in Galileo's law of terrestrial inertia. He imparted a geometrical denouement to the simplest demonstration—the famous (or infamous) inclined plane "experiment." At one "end" this revealed the law of freely falling bodies (when the plane was tipped up vertically), and at the other "end," the law of terrestrial inertia (when the plane was lowered level with the earth's surface) (*22*).

Let us consider this last law again, in even more detail. We have seen what it meant as a discovery for Galileo. What did it mean in the discoveries of his successors? What *must* it have come to mean? After all, expressions like "rectilinearity," "motion ad infinitum," "uniform," "force-free," and "frictionless," although clear to one who sees nature through a geometer's eyes, must yet be "cashed" in the actual observations of this world. An expression lacking operational interpretation is indistinguishable from jabberwocky, at least in the hallowed halls of the history of physics.

Galileo's law of terrestrial inertia is not a linguistic translating device. It is not concerned only with the substitutability of terms inside a mechanical game. It originated in factual opposition to an alternative claim, itself quite amenable to observational tests. The ancients' contention that continued application of force was necessary for motion to continue meant simply that, without it, motion would cease (*23*). All terrestrial bodies would therefore come to rest (sooner or later) were no further motive power applied. As a description of what we observe, as engineers, physicists, and travelers, the ancients' claim seems not only substantiable, but substantiated in fact. Its negation ought also to be vulnerable to factual inquiry; indeed, since we take that negation as a physical basis for much of the science of the last 400 years, Galileo's law of inertia should also be substantiated in fact, at least as directly and plausibly as the ancient "law." It was that observational plausibility which made the older view basic to Aristotle's scientific work. Despite so many initial observations to the contrary, Galileo's law of terrestrial inertia should also be referable to demonstrable facts, which, however nonobvious, will nonetheless anchor our science of mechanics in an observational foundation of physical truth. In the "Discourses," as we saw earlier, Galileo begins the demonstration.

If, however, we restate the law in its most transparent form, it will read as follows: *If* there were a particle free of unbalanced, external forces, *then* it would either remain absolutely at rest or would manifest uniform rectilinear motion ad infinitum.

Here the meaning-content of the law is clearer than it is anywhere in the "Discourses." The law is what logicians call "an unfulfilled hypothetical" or "a counterfactual conditional." We have no reason for supposing that particles free of unbalanced external forces do exist. But the law tells us what would obtain with such particles *if* they did. This has the doubly awkward consequence that (i) we (like Galileo) cannot investigate the properties of such bodies, and (ii) the law, being hypothetical, cannot be shown to be false. It cannot even be shown to be falsifiable, something which many take as a necessary condition for meaningfulness within science. An unfalsifiable claim—one which is compatible with anything—is termed "insignificant." Thus the claim that the universe shrank last night, being now one billionth part smaller than it was yesterday ("the universe" being taken to include ourselves, measuring instruments, theodolites, micrometers, diffraction gratings, the wavelengths of standard radiation, elementary particles, and so on)—this claim, since *ex hypothesi* it cannot be falsified, is physically meaningless. Does the counterfactuality of the law of inertia put it into this same class? Not quite. When linked with a network of other physical assumptions, the law does have testable consequences (*24*). This is not true of the "shrunken universe" claim. Given any two moving bodies, one demonstrably freer of external forces than the other, that one body will approximate, more closely than the other, to uniform, rectilinear motion ad infinitum, although it can never proceed to the formal limit of perfect inertial motion. Semantical difficulties arise, however, in this idea of pushing an approximation "to its formal limit"; how can one body move "closer to" infinity than another? The remark is unintelligible as it stands. But now another perplexing difficulty obtrudes.

Reflect on this: Not only has no one ever encountered a force-free body, but also the expressions "uniform" and "rectilinear," to have operational significance and physical meaning, must be coordinated with measuring techniques. How do we establish a motion as rectilinear and uniform? We set up coordinates by reference to which a point's translation from x, y, z at t (to x', y', z' at $t + \Delta t$) may correspond to a rectilinear and uniform Cartesian translation within the spaces defined. This is not an exercise in geometry; it is no exploration of some abstract space. Quite the contrary. It requires setting up physical coordinates, determined by actual objects; when these are assumed to be fixed, they allow the relational, intra-geometrical distinctions necessary for describing the point's trace within the resulting frame of reference.

Suppose the universe consisted of one and only one punctiform mass (*25*). Of its mechanical behavior nothing could be said. To claim of that mass that it moves uniformly along a rectilinear path, it is necessary to fix physical coordinates by assuming other masses to be anchored (*26*). How many others? One at the zero point and three others out along the coordinates. Without these other "absolutely immobile" masses, the motion of our original particle could not be described as uniform and rectilinear; it could not be said to be in motion at all. To so describe it would appear to require at least five particles. Any particle one describes as moving uniformly and rectilinearly must be but one particle in a universe containing five—the specimen-particle and the four coordinate-fixers. But no particle within such a universe can be free of external, unbalanced forces—a simple inference from the law of universal gravitation (*27*).

So the counterfactual character of Galileo's law stands not merely as an observation that no bodies *are* found to be force-free but, rather, as a consequence of there being no body whose motion is uniform and rectilinear which could possibly be force-free. Any alternative crushes the gravitational cornerstone of mechanics. Appraisals of the law's logical status are pierced by this point. The law thus refers to entities *not* such that, although never observed, they remain observable but, rather, entities that are unobservable in physical principle. This is a conceptual truth, not a factual one. Either the law conflicts with our conception of physical meaning or it conflicts with other laws of mechanics. Either way, it is difficult to understand.

Concerning the number of particles needed to fix coordinates, it might be argued that four is too many. With a zero-point particle and with two coordinates determined by two further bodies, a third dimension is easily "mapped away" from the plane so defined. But this eradicates also the remaining two particles; thus we can always determine from the zero point two perpendiculars normal to each other. But then only one of these last two need remain; the first coordinate line can be laid out in any arbitrary direction from the zero-point particle, it being easy to construct "imaginary" perpendiculars on that. But there the reduction halts. For the zero point must, as a matter of physical intelligibility, be construed as a fixed particle, out from which a reference frame may be constructed (*28*). Our earlier arguments remain unaffected, therefore. In order to say of any particle that its motion is uniform and rectilinear, it must be one member of at least a two-particle universe. It remains logically impossible, then, both for that particle to be force-free and for the rest of mechanics to be true (*29*). Either dynamical theory is false, or it is meaningless to suppose Galileo's law could be other than hypothetical and counterfactual in principle. The function of the law, within physical theory, is thus difficult to assess. Is it "true" although counterfactual? What does it do?

The rectilinearity part of Galileo's law is thus an operational puzzle. Operationally to ensure that the motion is *uniform*, one further needs a measuring rod in our barren two-particle universe. A metric must now be fixed within the reference frame constructable upon the zero-particle. For how can we establish a motion as uniform other than by determining that it traverses equal spaces in equal times, which is just what "uniform motion" means (*30*). The obvious way of ensuring that the spaces are equal is to lay a measuring rod first against one translation segment and then against another. If the ends of the segments coincide with the ends of the rod, then the spaces are equal. But in saying this one assumes that the rod suffers no deformation when transported from one trajectory segment to another. The assumption cannot rest on other reasoning inside classical mechanics—within matter theory, kinetic theory, and elasticity theory; this would be circular. These are derivative subsections within classical mechanics, which itself depends on the meaning of Galileo's law. So that law cannot be substantiated by considerations which themselves rest on the assumption that the law *is* substantiated, an assumption built into all these derivative disciplines. No—that our measuring rod does not expand or contract during translation is itself fixed by convention. This suggests that even to understand the meaning of "uniform" in Galileo's law of inertia, a conventional appeal must be made to another cornerstone of mechanics—namely, that mere transportation does not alter the physical properties of a body. For ordinary objects, we substantiate this principle within classical mechanics. But assumptions about the ideal rod cannot be supported by such derivative disciplines, since the law is fundamental to classical mechanics as a whole, and hence to all its subdisciplines. The "uniformity" part of the law, therefore, requires for its understanding a sophisticated appeal to the conventionality thesis. Such an exploration is necessary even to comprehend the meaning of the first law of motion.

These reflections have exciting consequences for the theoretical development of Galileo's law. Once it is apparent that every physical reference frame is, and must be, itself accelerated (that is, that none is free of impressed forces), it follows that the concepts of *rectilinearity* and *infinity* must either be defined physically in terms of such physical frames or else they, and the law, must assume an Absolute Space. If *rectilinearity* and *infinity* are anchored to the properties and behavior of actual objects, then these terms cannot be understood (as Galileo understood them) in their original geometric and number-theoretic manner. Their entire significance must depend on the local peculiarities of particular physical spaces and processes. Yet it was Galileo's objective to bring precision into studies of spaces and processes by translating geometrical ideas from mathematics into physics. Were that attitude fully to control our understanding of Galileo's law of inertia, however, we would *have* to take pure geometrical space, Absolute Space, as a force-free framework within which all particles and processes reside. Without this one cannot comprehend geometrically founded statements about the inertial motion of particles. Either the meaning of *rectilinearity* and of *infinity* comes through to us (as it did to Galileo) from pure geometry and number theory (in which case Absolute Space is *the* envelope for all mechanical subject matters), or else these terms must be defined through possible physical configurations (in which case the law is in principle a counterfactual conditional, since it denotes entities which are nonobservable).

Geometrical meaning, or physical meaning; Absolute Space, or counterfactual conditionality—which is it to be? Our earlier discussion concerning Galileo's credentials as an empiricist now has consequences. His attitude toward natural philosophy was that of a geometer; it would have been the geometrical meaning for his law of inertia that would have drawn him toward the Newtonian concept of Absolute Space. Operationalist objections to this formulation might well have mattered to a genuine empiricist; it seems unlikely that they would have deterred Galileo in the slightest (*31*).

I have suggested that every operational difficulty which attaches to local spaces whose geometrical coordinates are not fixed by particles—and this is how Galileo's presentation proceeds—is magnified when reference is made to Absolute Space itself. The complex later discussions by Leibniz, Euler, Laplace, Gauss, Hertz, and Mach, as well as Neumann's attempt to fuse pure geometry with "impure" physics by his postulated "body alpha"—all this lies implicitly in Galileo's "Discourses." The insight of Galileo and of the giants of the Scientific Revolution was that the world, and its constituent processes, can be viewed as built on geometrical-mathematical lines.

Thus physics only "got off the ground" when it was mathematicized. Yet it often appears that mathematics continues *its* ascent only when its research is elevated on the unsolved problems of modern physics. Philosophical attitudes toward Galileo's law of inertia have oscillated accordingly. Either its meaning seems to be imported into physics from pure mathematics, or else that meaning is determined by physical considerations and then sent back to mathematicians for further formal development.

Let mathematics and physics be construed as but different aspects of one comprehensive discipline, a kind of mathematics-physics, the Galilean ideal. The law of inertia then comes into physics with predetermined geometrical meaning. It serves as a paradigm for all physics. But stop: now attend to the logical differences between mathematics and physics, differences clear enough to us, but rather obscure to Galileo. Physical statements are true only in that their negations, although consistent, do not describe facts. The hunt must then be on for the *physical* meaning of "uniformity," "rectilinearity," and "infinity," not to mention terms like "equals," "is proportional to," "is commutative," "is of the second order in time," "is divergent," and so on. Only by finding physical meanings for such expressions can one use them in making factually true physical statements. Mathematics and physics are logically different disciplines; the former can only occasionally solve the latter's problems, via a kind of analogical transfer of formal operations to physical processes. Galileo's definition of physics in terms of mathematics is no longer fully acceptable.

Neumann's "body alpha" is a fascinating half-way house—something to which Galileo might have assented. Yearning for definitions of "uniformity," "rectilinearity," and "infinity" which would *not* be subject to overhaul within each new physical reference frame, Neumann invented *alpha*. Through this one body Neumann sought to fulfill Galileo's ideal, to convey the absolute definitions of Euclidean geometry into observational physics. One result would then have been that in a two-particle universe one *could* recognize motion as rectilinear or otherwise, simply by reference to this physically fixed but gravitationally inconsequential body. However, physical unintelligibility is the upshot, both for alpha and for its relationship with the other particle (*32*). What sense is there in describing alpha as "gravitationally inconsequential"? Alpha must then also be geometrically inconsequential for the purposes of physics. After Neumann, physicists have in unison pronounced, "Let no man join what nature hath sundered"—to wit, the formal creation of spaces and the physical description of bodies. Not even a giant like Galileo ever *really* joined these sundered disciplines.

Operationalism

We have scrutinized Galileo's real discoveries in dynamics. They stemmed from his primary "discovery" that nature could be viewed as from a draftsman's table—that calculations with symbols could capture the essence of phenomena. This primary discovery was not something "tripped over," as experimentalists sometimes trip over the unexpected. It was thought over. And it has been *fought* over ever since. For every Lagrange who, Galileo-like, sought to fuse physics with function-theory, there has been a Mach nearby and "at the ready," insisting that each mathematical-physical concept be "cashed" fully in operational terms, or else banished thenceforth from the province of proper physics. Modern physicists discovered "the problem of physical meaning" when reflecting on Galileo's discovery that nature could be geometrized. Here, too, modern philosophy has made a discovery—a derivative discovery about meaning and semantics generally. For just as Galileo's pronouncements provoked Mach and Einstein and Bridgeman to formulate what comes to us as "the operational criterion of meaning," so also philosophers have looked more searchingly at the logical credentials of that criterion.

Should we demand that all concepts within physical theory be at once "negotiable" in terms of operations and observations, or else be manacled to mere metaphysics? The history of physics suggests this demand to be therapeutically valuable. Criticisms of the *F* in Newton's law of gravitation, and of the muddled ideas about gravitational "attraction" acting across immense empty distances, led to a less anthropomorphic, a kinematical, restatement of the law, this restatement being a partial condition for Einstein's theory of general relativity. Similarly, there was Newton's Absolute Time and Space, within which "simultaneous" events many light-years apart might easily be thought of as occurring; Einstein again demanded the operational "cash value" of such a conception, and lo, we were made thereby to see how dependent on such operations as space measurement and clock synchronization our ideas of the universe really are. The moral? Any intratheoretical conception lacking operational translation signaled nonempirical elements (hence scientifically suspect and improper elements) within the theory.

We have seen how "being empirical" has a narrow and a broad meaning. Galileo was not an empiricist in any narrow sense; his metier was not the constant reiteration of accurate tests, statistical summaries and probability parameters, precise experimental design, and an elaborate theory of errors. Yet, in the broad sense—the sense in which an empiricist is anyone who tells the truth about matters of physical fact because that alone is his objective—Galileo was an empiricist. He did not *invent* nature's properties in an act of divine geometrical creativity. Rather, he discerned profound analogies between mathematical analysis and analytical mechanics. Thus he articulated the dynamical facts as no one ever had before.

Similarly, there may be a narrow and a broad interpretation of "the operational criterion of meaning." *F* in classical gravitation theory could never be set off against a list of physical operations from a consideration of which the real meaning of *F* would emerge. Gravitational forces cannot be cut, mechanically amplified, focused, insulated against, or modulated by frequency controls. Our evidence in support of *F* is precisely what made us "invent" *F* to begin with! The "stress tensor" of general relativity theory (a replacement for *F*) fares little better in terms of strict and unrelieved operationalism. That is, its physical significance—its extra-geometrical interpretation—is far from unambiguous. Moreover, the physical meaning of $\sqrt{-1}$ (as in classical thermo-dynamics), of Schrödinger's ψ function, and Dirac's δ function (as in contemporary microphysics)—these also are somewhat suspect in rigorously operational terms. Such expressions cannot be "cashed" into observational currency, not without much philosophical advocacy of one rendition as against others.

What does this matter, however? In less direct ways, stress tensors, ι, ψ, and δ allow us to tell the truth about regions of the natural world as puzzling, intricate, and remote from ourselves as were "inertia" and "acceleration" puzzling for, and remote from, Galileo. Yet this great man, by invoking apparently nonempirical ideas like "limits," "infinities," and "instants," was able to discover for us the essence of dynamical events. What better test for their legitimate use in physics? Similarly, the bewilderness of symbols which tumble out of pure function theory into our partial, nonlinear differential equations today are often, through near-interminable chains of intricate inference, ultimately descriptive of physical facts. What better test for their operational utility? We do not insist that automobile parts themselves be tiny automobiles, nor do we require that houses be made of bricks and beams which are themselves small homes. Why, then, demand of entire physical theories —which, as a whole, *should* be operationally responsible—that each component also be operationally interpretable, and in the same way? Words are not small sentences; cries are not petite propositions; theoretical terms are not tiny theories. Demanding the full operational significance of δ is like demanding to be told whether *the* is true or false.

We return full circle here to the vision of Galileo, perhaps now clearer for us than it could have been for him. "Telling the truth" about nature requires telling the whole truth and nothing but the truth; it requires, that is, a comprehensive explanatory account of a subject matter, no part of which is factually false. It consists in more than reciting tiny correlations and stuttering streams of statistical data. Galileo told the truths of dynamics through the languages of mathematical analysis; his objective justified his choice of technique. Had that objective been unattained—had he failed to tell the truth while yet continuing to press mathematics upon nature—then we could, and should, dub him "nonempirical." Galileo's attunement of simple examples and pellucid mathematics to the then-obscure facts of dynamics was always effected with the sureness of a virtuoso. He was the conceptual master of nature, and his techniques were masterfully employed.

So, too, ψ and δ can be masterfully employed in telling the truth about microphysical nature. They are not directly observational, perhaps, but they remain indispensably inferential. Hence they are operationally respectable conceptions, in just the way that Galileo was himself empirically respectable. This does not mean that the theoretical terms of modern physics are operationally tractable in the narrow sense of Mach and Ostwald; nor does it mean that Galileo was an empiricist in the narrow sense of Locke and Mill.

The doctrine of operationalism, then, needs lubrication and re-jointing, as do the "received" views of Galileo's empirical discoveries in dynamics. For, so far as facts in the history of physical science are concerned, extremism in defense of a philosophy is no virtue.

References and Notes

1. Galileo Galilei was born in Pisa in February 1564—the year in which Michelangelo, Vesalius, and Calvin died and in which Shakespeare was born.
2. See, for example, R. J. Seeger, *Am. J. Phys.* (Mar. 1964): "Galileo always begins with experimental premises."
3. See A. R. Hall, *From Galileo to Newton* (Harper, New York, 1963), p. 38: "It seemed that the Laws of Motion *could not be false*, that is, it was inconceivable that any alternative propositions could be valid."
4. See A. R. Hall (*ibid.*, p. 55): "*A priori* geometrical reasoning seemed to make experiment superfluous."
5. See Galileo's *Sidereus Nuncius* ("The Sidereal Messenger") (1610): "One may learn with all the certainty of sense evidence that the moon is not robed in a smooth and polished surface. . . ."; see also *Letters on Sunspots* (Rome, 1613): "If his [Aristotle's] knowledge had included our present sensory evidence [he too would have granted our conclusions]."
6. In 1586 he wrote his first scientific paper, *La Bilancetta* ("The Little Balance"), concerning a hydrostatic balance for directly reading analyses of gold-silver mixtures. He made 100 telescopes, the most powerful achieving a magnification of 33. He optically resolved the Milky Way into component stars. He saw Jupiter's moons, the mountains on our moon, and the phases of Venus—all for the first time in man's history.
7. Galileo clung to the Aristotelian distinction between naturally occurring motions (uniform and circular) and forced ones (accelerated and rectilinear).
8. See R. J. Seeger, *Am. J. Phys.* (Mar. 1964): "Credit is also due Galileo for his discoveries with the telescope. Nevertheless, all that was really required here was primarily sight." How ironic, also, that just before he was struck blind Galileo's final discovery (1637) was of the moon's librations. But even without vision he remained a natural philosopher without equal.
9. He who lacks all understanding of why 2 + 2 equals 4 can hardly be on guard against contingencies such as a couple of apples being added to a couple of pears and thereby generating *five* pieces of fruit! To know what's wrong here does not require a polling of fruiterers; it requires knowing in advance that 2 + 2 = 4.
10. N. R. Hanson, *Patterns of Discovery* (Cambridge Univ. Press, London, 1958).
11. See W. Whewell, *Astronomy and General Physics* (London, 1834), p. 211: "If we in our thoughts attempt to divest matter of its powers of resisting and moving, it ceases to be matter, according to our conceptions and we can no longer reason upon it with any distinctness."
12. See Thomson's articles published in *The Philosophical Magazine* between 1926 and 1928.
13. J. Kepler, *De Motibus Stellae Martis* (Prague, 1609).
14. For example, Joannes Philoponus noted in A.D. 533 that two objects of vastly different weights would nonetheless fall with little difference in the times. Simon Stevin and Johan de Groot made the experiment in 1586.
15. See A. R. Hall, *From Galileo to Newton* (Harper, New York, 1963), p. 75: "Rarely did Galileo claim that the Aristotelians erred in their facts because they had not experimented."
16. Galileo's *Dialogo dei due massimi sistemi del mondo*, known as the "Dialogues," was published in Florence in 1632; his *Dialoghi delle nuove scienze*, known as the "Discourses," was published in Leiden in 1638. The "Dialogues" concern rectilinear motion primarily, inasmuch as Galileo there confines himself to the small scale [see A. R. Hall, *From Galileo to Newton* (Harper, New York, 1963), p. 52].
17. The portion in question is described primarily in the "Discourses," but the passage paraphrased in the paragraphs that follow is anticipated in the "Dialogues."
18. *Discourses Concerning the Two New Sciences* [English translation of *Dialoghi delle nuove scienze*], Crew and de Salvio, transl. (Northwestern Univ. Press, Evanston, 1939), pp. 242–248.
19. This is evident whenever the first law is characterized as being but a limiting case of the second law; that is, when $\Sigma F = 0$ it follows that $a = 0$ (whether $m = 0$ or > 0). That is, $d^2x/dt^2 = 0$; $d^2y/dt^2 = 0$; $d^2z/dt^2 = 0$: the second time derivatives of coordinates x, y, z vanish when a body is force-free. From this much, however, one can infer the rectilinearity of an inertial path only by building that concept *into* "force-free." See also *Discourses Concerning the Two New Sciences* (*18*, pp. 215, 216): "But along a horizontal plane the motion is uniform since here it experiences neither acceleration nor retardation. . . ."

   "Furthermore we may remark that any velocity once imparted to a moving body will be rigidly maintained as long as the external causes of acceleration or retardation are removed, a condition which is found only on horizontal planes; for in the case of planes which slope downwards there is already present a cause of acceleration, while on planes sloping upward there is retardation; from this it follows that motion along a horizontal plane is perpetual; for, if the velocity be uniform, it cannot be diminished or slackened, much less destroyed. Further, although any velocity which a body may have acquired through natural fall is permanently maintained so far as its own nature (*suapte natura*) is concerned, yet it must be remembered that if, after descent along a plane inclined downwards the body is deflected to a plane inclined upwards, there is already existing in this latter plane a cause of retardation; for in any such plane this same body is subject to a natural acceleration downwards. Accordingly there we have the supposition of two different states, namely, the velocity acquired during the preceding which if acting alone would carry the body at a uniform rate to infinity, and the velocity which results from a natural acceleration downwards common to all bodies. It seems altogether reasonable, therefore, if we wish to trace the future history of a body which has descended along some inclined plane and has been deflected along some plane inclined upwards, for us to assume that the maximum speed acquired during descent is permanently maintained during the ascent. In the ascent, however, there supervenes a natural inclination downwards, namely, a notion which, starting from rest, is accelerated at the usual rate."
20. For now the plane and the line "intersect" only at infinity.
21. See Mach, *The Science of Mechanics* (Open Court, La Salle, Ill., 1893), pp. 168, 169; and see again the first line of the quoted passage in (*19*).
22. And by suitable combinations of both references one generates the laws of parabolic trajectories in ballistics.
23. The principle was expressed as follows: *cessante causa cessat et effectus*.
24. See A. R. Hall, *From Galileo to Newton* (Harper, New York, 1963), p. 63: "Only by imagining an impossible situation can a clear and simple law of fall be formulated, and only by possessing that law is it possible to comprehend the complex things that actually happen."
25. This very supposition is internally inconsistent for anyone who accepts Mach's kinetic definition of mass. Since *ex hypothesi* a single particle cannot interact with other particles, it is idle to discuss its mass in Machian terms. Are there other meaningful terms?
26. This already wrecks the supposition.
27. According to this law, "Any two particles in the universe are such that they attract each other directly as their masses, and inversely as the square of the distance between them."
28. This zero-point particle is identical to Neumann's "body alpha."
29. See G. Berkeley, *De Motu*:

   "If every place is relative then every motion is relative and as motion cannot be understood without a determination of its direction which in its turn cannot be understood except in relation to our or some other body. Up, down, right, left, all directions and places are based on some relation and it is necessary to suppose another body distinct from the moving one . . . so that motion is given in relation to which it exists, or generally there cannot be any relation, if there are no terms to be related.

   "Therefore if we suppose that everything is annihilated except one globe, it would be impossible to imagine any movements of that globe.

   "Let us imagine two globes and that besides them nothing else material exists, then the motion in a circle of these two globes round their common centre cannot be imagined. But suppose that the heaven of fixed stars was suddenly created and we shall be in a position to imagine the motion of the globes by their relative position to the different parts of heaven."
30. See *Discourses Concerning the Two New Sciences* (*18*): "By steady or uniform motion, I mean one in which the distances traversed by the moving particle during any equal intervals of time, are themselves equal."
31. See Galileo, "Dialogues": "And when is it that we are supposed to test by experiment whether there is any difference to be discovered among these events of local motion . . . if the Earth remains forever in one or the other of these two states?"
32. Compare B. Russell, *The Principles of Mathematics* (1903): "It seems evident that the question whether one body is at rest or in motion must have as good a meaning as the same question concerning any other body; and this seems sufficient to condemn Neumann's suggested escape from absolute motion."