Quantized Magnetic Flux in Superconductors

Experiments confirm Fritz London's early concept that superconductivity is a macroscopic quantum phenomenon.

R. D. Parks

When one is caught in the flurry of research activity, both theoretical and experimental, which has been raging in the field of superconductivity for the last 7 or 8 years, it is easy to forget the name Fritz London. This is unfortunate, because, when the storm is over, surely his name will reappear and his precocious and tumultuous contributions to the understanding of superconductivity will be put into proper perspective. Nearly two decades ago, when theorists were blindly groping for a microscopic theory of superconductivity, Fritz London (*1*) cleared the air by defining the problem and setting the ground rules. Fashionable at that time was the concept of a "supermobile electron lattice" in which the electrons were permitted boundless freedom. This notion seemed consistent with the experimentally established fact that a superconductor has infinite conductivity. However, London pointed out that this general approach could never lead to an understanding of the equally mysterious fact that a superconductor completely expels the magnetic field (the Meissner effect). In order to explain the Meissner effect, London introduced a revolutionary concept of the superconducting state. Usually we would assume that an electron at one point in a metal moves independently of an electron at some other point unless the two electrons are very close to each other. In a normal metal this is true, but London conceived of the superconducting state as being rather like the ground state of a very large atom or molecule, in which the motion of quite widely spaced electrons is correlated. This is then a unique ground state, and the whole metal must be treated as a single quantum mechanical system. This ground state is described by a single-valued wave function which is rigid in the presence of a magnetic field. A direct consequence of this concept was the prediction made by London (*2*) about 15 years ago that the magnetic flux threading a superconducting ring must occur only in discrete values called flux quanta. This prediction has now been verified experimentally. In what follows I describe the recent experiments which confirm London's concept of the superconducting state.

London's Prediction

I shall now repeat, in essence, the straightforward arguments used by London to show that the magnetic flux in a superconducting ring should be quantized. In the following discussion I use the Gorter-Casimir (*3*) two-fluid model, in which it is assumed that the conduction electrons can be divided into two groups, the "superelectrons," which are responsible for the "superfluid" properties of the electron gas, and the "normal electrons," which behave like electrons in the normal metal (*4*). If the total number of electrons per unit volume is n and the number of superelectrons is n_s, then the remainder, $n - n_s$, are normal electrons. The quantity n_s varies from zero at the superconducting transition temperature to the value n at 0°K. Since only the superelectrons participate in the supercurrent, the supercurrent density

$$\mathbf{J}_s = n_s e \mathbf{v}_s \qquad (1)$$

where e is the electronic charge and $\mathbf{v}_s$ is the average velocity of the superelectrons.

Now, let us consider the multiply connected superconducting sheet of Fig. 1. If London's concept that all of the superelectrons are in the same quantum state, or can be described by a single-valued wave function, is correct, then we can apply the famous Bohr-Sommerfeld quantization rule to any arbitrary closed path c′ which lies in the superconducting body. This rule states that if the quantum mechanical wave function on some closed path c′ is single-valued, then the line integral of the conjugate momentum appropriate to this wave function changes by integral amounts upon each traversal of the path,

$$\oint_{c'} \mathbf{p}_s \cdot \mathbf{dl} = nh \qquad (2)$$

where $\mathbf{p}_s$ is the conjugate momentum of the superelectrons, $\mathbf{dl}$ is a line element, n is an integer, and h is Planck's constant. We need also, from quantum mechanics, the result that in the presence of a magnetic field described by the magnetic field vector $\mathbf{A}$,

$$\mathbf{p}_s = m\mathbf{v}_s + e\mathbf{A}/c \qquad (3)$$

where m and e are the mass and charge of an electron and c is the velocity of light. Equation 2 now becomes

$$\oint_{c'} m\mathbf{v}_s \cdot \mathbf{dl} + \frac{e}{c}\oint_{c'} \mathbf{A} \cdot \mathbf{dl} = nh \qquad (4)$$

If we employ Eq. 1, transform the second integral to a surface integral according to Stokes's theorem, and employ the relation $\mathbf{H} = \mathrm{curl}\,\mathbf{A}$, (where $\mathbf{H}$ is the magnetic field), Eq. 4 becomes

$$c\Lambda \oint_{c'} \mathbf{J}_s \cdot \mathbf{dl} + \oint_s\oint \mathbf{H} \cdot d\mathbf{S} = \frac{nhc}{e} \qquad (5)$$

where Λ, the London parameter, is defined by $\Lambda = m/n_s e^2$. Thus the left-hand side of Eq. 5, which London called the "fluxoid," must be quantized in units of hc/e. Up to this point we have not explicitly utilized the fact that there is a hole in the superconducting sheet of Fig. 1. If the hole is not there, after transforming the first integral in

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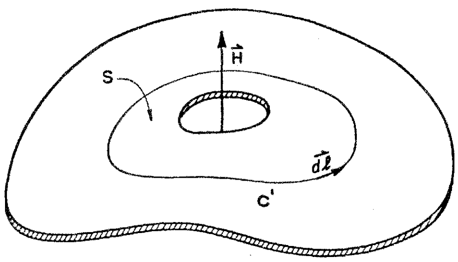

Fig. 1. Multiply connected superconducting sheet in a perpendicular magnetic field.

Eq. 5 to a surface integral we can relate $\mathbf{J}_s$ to $\mathbf{H}$ through the powerful London relation

$$c\Lambda \operatorname{curl} \mathbf{J}_s = -\mathbf{H} \quad (6)$$

which holds within a superconducting body, and thus everywhere on the surface S. Then the first integral is just the negative of the second integral, and we have the result that the fluxoid is zero in a simply connected superconductor.

Let us return to the problem of the multiply connected superconductor. I have shown that the fluxoid which is associated with some path c′ which encircles a hole must be quantized. According to Eq. 5, the fluxoid consists of two parts, the magnetic flux passing through the surface S (second integral) and a term involving the supercurrent (first integral). In order to understand the contribution of the latter term, we need to discuss more quantitatively the Meissner effect. When a superconducting body is placed in a magnetic field or when it is cooled from the normal state to the superconducting state in a magnetic field, supercurrents are established which set up magnetic fields that exactly cancel the externally applied fields at all points within the body. These currents flow near the surface within a characteristic distance λ (the London penetration depth), which varies rapidly from infinity at the transition temperature T_c to an asymptotic value λ_0 (which for ordinary superconductors is of the order of 500 angstroms) as the temperature is lowered below T_c. In Fig. 1, at a temperature well below T_c there are supercurrents flowing inside the path c′ within a distance λ from the hole, and outside c′ within a distance λ from the outside edge. If the distance between the hole and the outside edge of the superconductor is large compared to λ, we may choose a path c′ which lies in a region of approximately zero current flow, in which case the first integral in Eq. 5 is negligible compared with the second integral. Thus, if the superconducting ring is thick, as defined by the above criterion, we have the result that the magnetic flux through the surface S is quantized in units of hc/e. If, in addition, the radius of the hole is large compared with λ (5), which is usually the case, then the magnetic flux through the hole itself must be quantized in units of hc/e according to London's model.

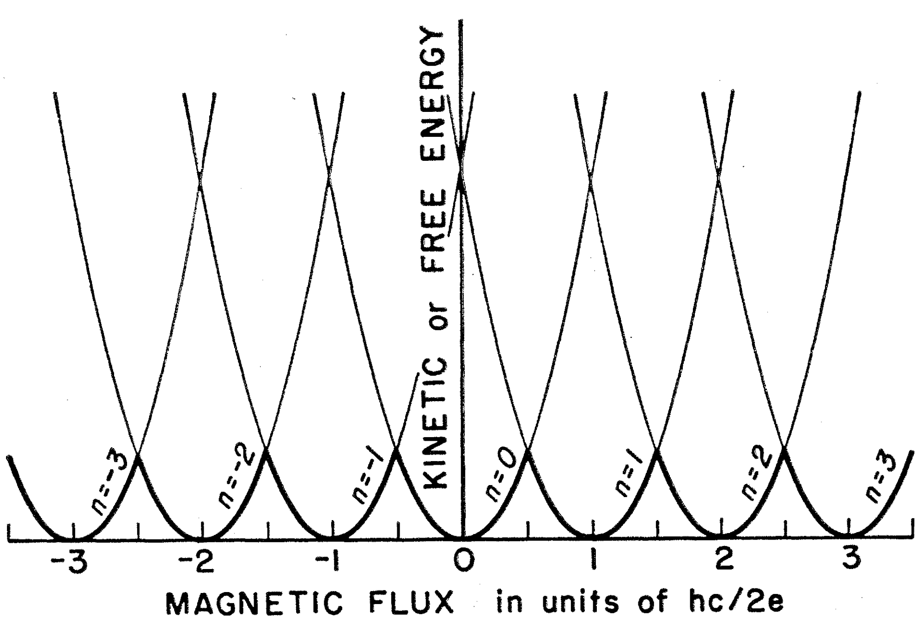

Fig. 2. Kinetic or free energy of the superconducting state of a ring or hollow cylinder in an axial magnetic field in the temperature limit $T \longrightarrow T_c$. The heavy curve represents the lowest possible value of the free energy.

Discovery of Quantized Flux

About 15 years after London made his prediction, two research teams, Deaver and Fairbank (*6*) in California and Doll and Näbauer (*7*) in Germany, reported simultaneously the observation of quantized magnetic flux in hollow superconducting cylinders. Why the result was so long in coming is somewhat mysterious. Certainly, part of the answer must lie in the fact that the experiment is one of considerable difficulty, due to the smallness of London's flux quantum ($hc/e = 4 \times 10^{-7}$ gauss cm^2). But probably more important is the fact that important experiments of moderate-to-extreme difficulty are often, if not usually, by-passed by experimentalists in favor of "quick money" experiments of lesser difficulty. Equally mysterious is the reason why, after 15 years, the result was simultaneously reported by two independent research groups. Perhaps it has something to do with long-range extrasensory perception.

I was fortunate to be at Stanford during the course of Deaver and Fairbank's experiment, and to overhear the speculative comments on the size of the flux quantum made by various well-known visiting theorists. From those who didn't agree with London's prediction came predictions of zero or of hc/Ne for the size of the flux quantum, where N is the total number of electrons in the sample. A few, of course, agreed with London in predicting a flux quantum of hc/e. Lars Onsager of the "a superconductor is somewhat like a Bose gas" school was the only theorist to predict that it might be ½(hc/e). It turned out that Onsager was correct. One heard a different story from theorists who were questioned after the experiments had been performed. They impatiently explained that $hc/2e$ was an obvious consequence of the Bardeen-Cooper-Schrieffer (BCS) theory (*8*), which predicted a correlation of electrons in zero-momentum pairs (*9*). We may think of the supercurrents as arising from the drift velocity of the pairs. Since the charge carriers are pairs of electrons, we must now, if we are to use London's phenomenological theory, replace e, where it appears, by $2e$, m by $2m$, and n_s by $n_s/2$ (the total number of electron pairs).

In general, this will not change the

predictions of the London theory because the quantities e, m, and n_s usually enter through the parameter Λ (see, for example, Eqs. 6 and 7), which is invariant with respect to this set of transformations. Of course, in Eq. 5, where e appears separately, it does make a difference, and we must make the correction.

I now describe briefly the details of the two experiments. In both experiments the problem was to cool a thick-walled (*10*), hollow metallic cylinder from the normal to the superconducting state in an applied magnetic field, and then to remove the applied field and measure the magnetic flux trapped inside the cylinder. If London were correct, the trapped magnetic flux would be found to exist only in discrete quantum values. The experiments of Deaver and Fairbank and of Doll and Näbauer differed, essentially, only in the techniques used.

Deaver and Fairbank used a sensitive, dynamic alternating-current method to measure the magnetic moment due to the trapped-in flux in a superconducting tin cylinder, which was prepared by electroplating tin onto No. 56 copper wires. The cylinders prepared in this way were approximately 1 centimeter long and 10 to 20 microns in diameter, and they had a wall thickness of 1 to 5 microns.

Doll and Näbauer used an equally sensitive method to measure the magnetic moment of a small hollow lead cylinder with trapped-in flux. This method consisted of measuring the torque exerted by an applied magnetic field on the cylinder, which was suspended by a thin torsion fiber. The lead cylinders were prepared by evaporating lead onto a quartz fiber about 10 microns in diameter and 1 millimeter long.

In both experiments the trapped magnetic flux in the superconducting cylinders was found to exist only in units of $hc/2e \pm \sim 20$ percent.

Before these experiments were made there had been some speculation (*11*) that flux quantization might be a general property of the electromagnetic field. Of course, these experiments seemed to dispel this notion because the flux quantum was found to depend upon the size of the charge carriers, which is certainly a property of the superconductor, not of the electromagnetic field.

Quantum Periodicity in T_c

In a sequel to the papers of Deaver and Fairbank and Doll and Näbauer, Byers and Yang (*12*) discussed flux quantization within the framework of the BCS theory as well as the London model and made the important observation that the free energy of the superconducting state is periodic in the magnetic flux threading a multiply connected superconductor. We can see this readily within the framework of the London theory. For simplicity we consider a hollow cylinder of vanishingly thin wall, of radius R, in a uniform magnetic field H which is parallel to the axis. Since the wall is of infinitesimal thickness, the supercurrents cannot be large enough to distort the applied magnetic field and we may consider the field uniform in the region of the cylinder. Field uniformity would also be realized if we considered a cylinder of finite wall thickness in the temperature limit $T \rightarrow T_c$; in this case, again, the supercurrents are not large enough to distort the magnetic field, since $n_s \rightarrow 0$ as $T \rightarrow T_c$. After we make a correction from hc/e to $hc/2e$ for the size of the flux quantum, Eq. 5, when applied to the geometry of Fig. 2, becomes

$$2\pi c\Lambda RJ_s + \pi R^2H = nhc/2e = n\phi_o \quad (7)$$

where ϕ_o is the flux quantum. Solving for the supercurrent J_s, we obtain

$$J_s = \frac{\phi_o}{2\pi c\Lambda R}\left(n - \frac{\pi R^2H}{\phi_o}\right) \quad (8)$$

From this relation we can now determine the kinetic energy density f_{KE} associated with the supercurrent, which is given by

$$f_{KE} = \frac{n_s m v_s^2}{2} = \frac{\Lambda J_s^2}{2} \quad (9)$$

from Eqs. 1 and 6. Combining Eqs. 8 and 9 we obtain

$$f_{KE} = \frac{\phi_o^2}{8\pi^2c^2\Lambda R^2}\left(n - \frac{\pi R^2H}{\phi_o}\right)^2 \quad (10)$$

which is plotted in Fig. 2. The total free energy density of the superconducting state is a sum of this kinetic energy term plus other contributions which do not have a periodic dependence upon the magnetic flux. Thus, the total free energy will be periodic in the magnetic flux. In order to keep the free energy minimized in the presence of the applied magnetic field, n in Eq. 10 will switch from one quantum value to another as the magnetic field is changed. For instance, if the magnetic field (or magnetic flux) is increased from $H = 0$, the kinetic energy increases quadratically with the field, following the $n = 0$ parabola. Then when the magnetic flux ϕ reaches the value ½ $(hc/2e)$ or ½ ϕ_o, n switches from 0 to 1. As the magnetic field is further increased the kinetic energy then drops. At $\phi = 3/2\ (hc/2e)$, n switches from 1 to 2, and so on. Thus, the kinetic energy is given by the heavy line in Fig. 2.

In "thick-walled" cylinders such as the ones used in the experiments of Deaver and Fairbank or Doll and Näbauer, the "quantum-switching" process can take place only at temperatures very close to T_c, where the Meissner effect is incomplete. When the temperature is lowered, even slightly, below T_c the Meissner effect becomes complete, and it is no longer possible for flux to freely enter or leave the cylinder, which is necessary if n is to change. Thus, the superconducting cylinder "decides" at T_c what its final quantum state will be. This decision will of course depend upon the value of the applied magnetic field at T_c. After the temperature has been lowered below T_c and the magnetic field has been removed, one can determine which quantum state the superconductor chose at an earlier time by measuring the trapped flux inside.

Since the superconducting cylinder (or ring) "does all of its thinking" at T_c, where it is free to select the quantum state of its choice, it seems both important and interesting to examine the cylinder at T_c in an effort to observe this "thinking process." At first sight such an effort seems hopeless, because the magnetic field inside the cylinder is equal to the applied magnetic field, as discussed before. However, from Eq. 8 or Eq. 10 we know that the supercurrents, and therefore the kinetic energy in the cylinder wall, are periodic functions of the applied magnetic field. Shortly after Deaver and Fairbank had made their experiments at Stanford, Little made the important observation that the transition temperature T_c should itself be a periodic function of the applied magnetic field. This follows directly from the arguments given above, because T_c depends only upon the difference in free

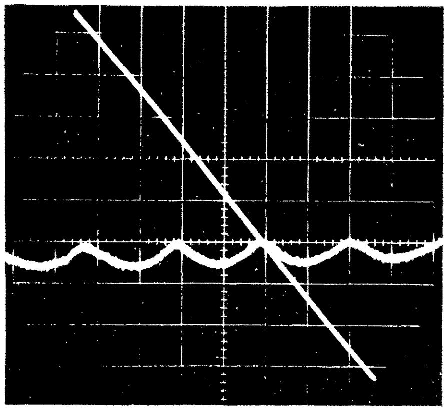

Fig. 3. (Lower trace) Variation of resistivity of an aluminum cylinder with magnetic field, at its transition temperature; (upper trace) magnetic field sweep.

energy between the superconducting and the normal states (*13*); when the free energy is the same for the two states, the sample becomes superconducting.

Little and I then tried to devise an experiment to measure the periodicity in the transition temperature of a thin-walled superconducting cylinder. A rudimentary calculation indicated that ΔT_c (the amplitude of the oscillations in T_c) should vary inversely as the square of the radius of the cylinder and should have the value $\Delta T_c \simeq 10^{-5}$ degree Kelvin for a cylinder of 1-micron diameter (*14*). This, of course, is a very small effect. Even assuming that we could make a hollow superconducting cylinder 1 micron in diameter (which is about 1/50 the diameter of a human hair), we were faced with the necessity of measuring the temperature accurately to 10^{-6} degree Kelvin to obtain a measurement of the effect accurate only to within 10 percent. The first problem, that of making the small cylinder, proved to be easily surmountable. "Hollow" superconducting cylinders of approximately 1-micron diameter were prepared by evaporating the appropriate metal (tin, lead, and so on) onto the circumference of a fiber made from General Electric Company varnish GE 7031.

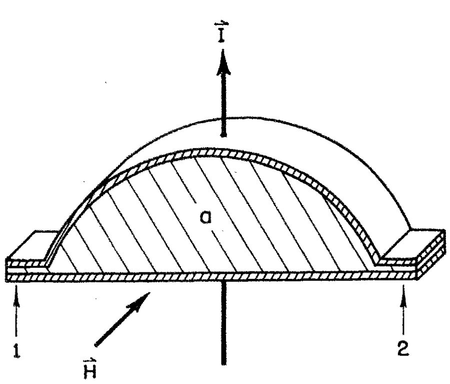

Fig. 4. Doubly linked superconducting circuit. The Josephson junctions 1 and 2 consist of very thin dielectric layers separating the two superconducting strips. The region of cross-sectional area *a* between the junctions is also filled with a dielectric material. The directions of the total direct-current Josephson current **I** and the magnetic field **H** are indicated.

We got around the second problem—that of measuring temperatures accurately to at least 10^{-6} degree Kelvin—in the following way. We made use of the fact that the resistivity transition of a thin film superconductor (or, for that matter, a bulk superconductor) has a finite spread in temperature. In an isothermal measurement, at some point in the transition region—say, at a temperature T_0—if the resistivity changes because of a change in some parameter such as the applied magnetic field this corresponds to a shift to a new transition curve. For instance, an increase in the resistivity at constant temperature corresponds to a decrease in T_c (or to transferral to a new transition curve which has a higher value of resistance at T_0). Thus, by measuring the modulations in resistivity as a function of magnetic field, we would in effect be measuring modulations in T_c. To relate the two we need only know the slope of the resistivity curve relative to the T curve at $T = T_0$.

The problem was to keep the temperature of the sample constant at some point in the resistivity transition, then sweep the applied magnetic field and look for a modulation of the resistivity. The results of a measurement made in this way are shown in Fig. 3, which is an oscillogram showing the resistivity of a hollow aluminum cylinder of 1-micron diameter as a function of the applied magnetic field. The "scallops" in the resistivity, and therefore in T_c, have a periodicity of $hc/2e$ in the magnetic flux through the cylinder. From the sharpness of the parabolas it is evident that this method is highly precise. The accuracy in the value obtained for the flux quantum ($hc/2e$) for the samples studied (tin, lead, aluminum, indium, and various tin alloys) was approximately ± 10 percent. This accuracy was limited only by the error in the measurement of the diameter of the cylinders. In these experiments we were also concerned with obtaining more detailed information, such as the absolute value of ΔT_c and its dependence upon parameters such as the mean free path of the sample. Results of these more detailed studies, which are at least in qualitative agreement with the theory, are presented elsewhere (*15*).

Josephson Tunneling

Recent experiments by Jaklevic, Lambe, Silver, and Mercereau (*16*) on tunneling in superconducting circuits have complemented the experiments described above and have demonstrated in a very striking way the long-range order which occurs in the superconducting state. Before presenting these results I think it appropriate to mention the brief history of tunneling in superconductors, a field which has literally exploded since the first experiments of Giaever in 1960.

Giaever (*17*) discovered structure in the current-voltage characteristics of a junction formed by a normal metal and a superconductor separated by a thin insulating layer. This non-ohmic behavior can be explained in the following way. At very low temperatures the probability that electrons will tunnel through such a barrier (and therefore the tunneling current) depends upon the density of electronic states on the two sides of the barrier. Application of a direct-current voltage to the junction shifts the Fermi level of the metal on one side with respect to the metal on the other side. Thus, by measuring the tunneling current as a function of the voltage, one is able to measure the difference in the density of electronic states between the two metals as a function of energy. In a superconductor there is a gap in the density of states at the Fermi surface because of the energy gap. This results in the anomalous behavior in the tunneling characteristics which was first observed by Giaever. Electron tunneling has become a powerful tool for measuring the value of the energy gap in superconductors.

In the tunneling experiments discussed above, only the "normal electrons" (or single-particle excitations) in the superconductor participate in the tunneling current. In 1962 Josephson (*18*) proposed that, in addition to the tunneling of "normal" electrons, the possibility existed that a pair of electrons from the superconducting condensate could tunnel through a barrier from one superconductor to another. This tunneling would result in a direct current in the absence of an applied voltage across the barrier. The size of the current would depend upon the energy gaps of the two coupled superconductors. Observation of the Josephson current was first reported by Anderson and Rowell (*19*).

From a microscopic analysis, the details of which are too long and complicated to be presented here, Josephson (*18*) has shown that the current density, j, for the direct-current tunneling is given by

$$j = j_o \sin \phi \qquad (11)$$

where ϕ is the phase difference between the quantum mechanical wave functions of the two superconductors. In the presence of a magnetic field **H** it is necessary to add to ϕ the gauge term

$$-\,2\pi/\phi_o \int_1^2 \mathbf{A} \cdot d\mathbf{l} \qquad (12)$$

where the integral is taken along a curve joining the two superconductors. If we now integrate j over the effective area of the junction, we find that the total Josephson current is reduced to a minimum whenever the junction contains an integral number of flux quanta. This prediction was verified by Rowell (*20*), who observed sharp minima in the Josephson tunneling current of a lead-insulator-lead junction at integral values of the applied magnetic field.

Jaklevic, Lambe, Silver, and Mercereau (*16*) extended this concept to include multiple links in a superconducting circuit. They examined the direct-current tunneling characteristics of two parallel Josephson junctions in a superconducting circuit, which is illustrated schematically in Fig. 4. If there is a phase coherence over macroscopic distances in a superconductor, we should expect that the phase difference across junction 1 is not independent of the phase difference across junction 2. This dependence would lead to a quantum mechanical interference between the currents flowing through the separate junctions and the prediction of two periodicities of the total current with respect to the magnetic field—one inversely proportional to the magnetic flux threading one of the Josephson junctions (if the two junctions are identical) and the other inversely proportional to the magnetic flux threading the area a between the junctions (Fig. 4). The experimental results of Jaklevic *et al.*, shown in Fig. 5, indeed verify these predictions and offer striking evidence for the existence of long-range order in the superconducting state.

The Vector Potential (A)

The vector potential **A**, as well as the scalar potential, was introduced in classical electrodynamics as a mathematical convenience for making field calculations. However, a few years ago Aharanov and Bohm (*21*) pointed out that in quantum mechanics the potentials themselves have physical significance; they are not merely mathematical artifacts. In quantum mechanics the effects of these potentials enter through the quantity

$$\oint \mathbf{A} \cdot d\mathbf{l} = \int\!\!\int \mathbf{H} \cdot d\mathbf{S}$$

which is of great importance in the problem of the multiply connected superconductor (see Eqs. 4 and 5). This quantity can be expressed in terms of the field **H** inside a circuit; however, as pointed out by Aharanov and Bohm, the "latest state of the art" in relativistic electrodynamics specifies that all fields must interact only locally. Thus, in an experimental situation where we have a zero magnetic field in the metallic region of a circuit but yet have a nonzero value for the quantity

$$\oint \mathbf{A} \cdot d\mathbf{l}$$

in the circuit, we have the interesting situation in which electrons in a zero magnetic field are (or should be) affected by the vector potential (**A**).

This effect was recently demonstrated by Jaklevic *et al.* (*22*) in a double tunneling experiment identical to the one discussed in the preceding section, except for the following modification. The magnetic flux through the area a (of Fig. 4), instead of being supplied by a uniform magnetic field, was supplied by a solenoid contained within the superconducting circuit. In this

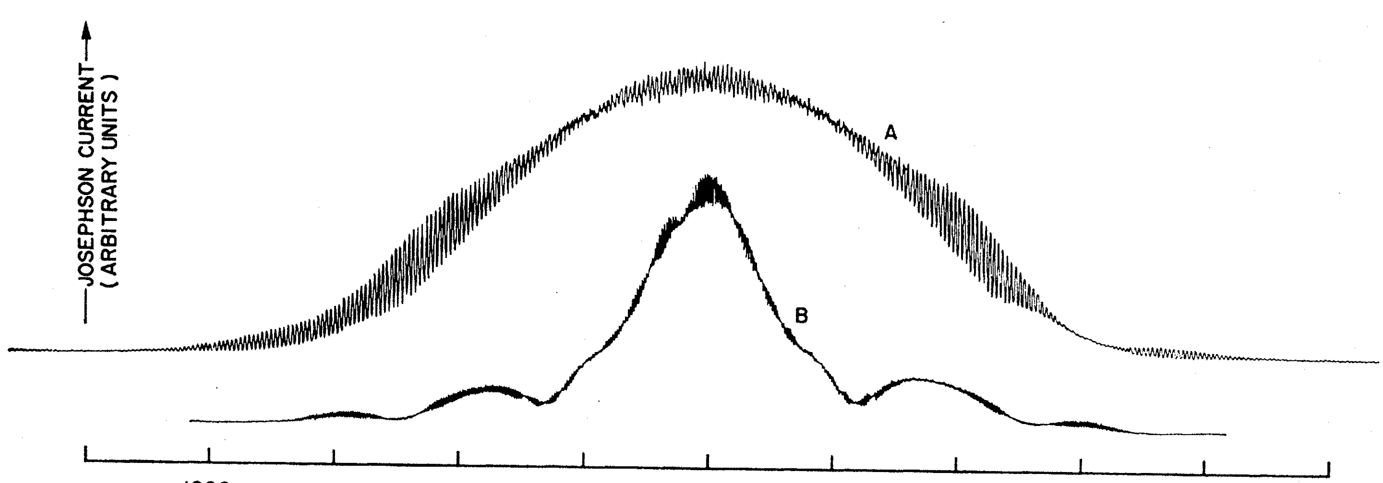

Fig. 5. Curves for Josephson current relative to applied magnetic field for two samples (A and B) of the type shown in Fig. 4 (see *16*). The frequency of the long-period oscillations is proportional to the number of flux quanta contained in one of the Josephson junctions, while the frequency of the short-period oscillations is proportional to the number of flux quanta contained in the area (a) between the junctions.

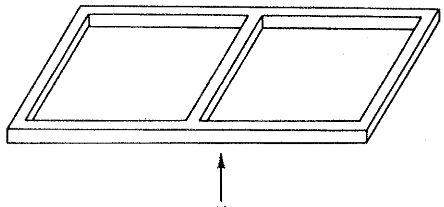

Fig. 6. Double-loop superconducting net in a perpendicular magnetic field.

way a flux was made to link the circuit, with no significant magnetic field at the superconducting elements. Measurements of the maximum supercurrent flow through the circuit showed the expected periodicity of the current with flux, whether the flux was produced by a uniform external field or by the enclosed solenoid. This is the expected result which verifies the Aharanov-Bohm hypothesis (*23*).

What Is Left To Be Done?

The question now arises: What is left to be done in the study of quantized flux in superconductors? My philosophical and categorical answer to this question is: *Much*, because quantized flux is a fundamental and important phenomenon, and people have been working in the field for only 3 years. Actually a few skeptics posed the trite question, "What can you possibly learn from that [a proposed] experiment?" before and during the experiments of Deaver and Fairbank and Doll and Näbauer. These experiments, in addition to verifying London's concept that the superconducting state is a long-range single quantum state, conclusively demonstrated that the pair interaction of the BCS theory is probably the important one, at least for tin and lead. From the work that Little and I did, the list can be expanded from tin and lead to include aluminum, indium, and various alloys of tin with indium, gold, and silver. Since then, Meyers and Little (*24*) have found that the flux quantum in tantalum is also $hc/2e$. The question remains: Must the flux quantum be $hc/2e$ for all superconductors? The answer, of course, is: Not necessarily. While Bardeen, Cooper, and Schrieffer used a truncated Hamiltonian representing a simplified pair interaction, it is possible that a more detailed calculation may reveal the importance of correlations between four or even a higher number of electrons. Since different superconductors behave uniquely in some respects, a reasonable and expedient approach to the above question is to look at different and exotic superconductors. Perhaps in the case of lanthanum and uranium, for which the probable importance of a mechanism different from the electron-phonon interaction has been postulated (*25*), one might find a value other than $hc/2e$ for the flux quantum.

We might ask: To what accuracy can the flux quantum $hc/2e$ be measured? Again, skeptics say: How could the numerical coefficient in the denominator be anything other than 2.000 . . . ? The same skeptics would probably have said: Why should the g-value of a free electron be anything other than 2.000 . . . , and therefore why explore the question experimentally? Now, considerable effort is being made to measure the quantity g (which is not exactly 2) to eight significant figures (*26*), instead of the present six, because the increased accuracy might enable theorists to better understand the structure of the electron. It seems to me important to improve upon the accuracy of the measurement of $hc/2e$, an accuracy which now stands at $\pm$ 10 percent (for the superconductors studied).

Another interesting question concerns what might happen in a multiply connected superconducting net. The simplest type of net, other than a single ring, which might be interesting is the double-loop structure shown in Fig. 6. Relevant to the problem is the theorem of Byers and Yang (*12*), which states, "the flux through any surface whose boundary loop lies entirely in superconductors is quantized in units of $hc/2e$." Consider, now, the following experiment. We cool the double loop, in a magnetic field perpendicular to the plane of the loop, to a temperature below T_c, remove the magnetic field, and then measure the trapped flux in each part of the double loop. The question is: What quantum state shall we find the system in if the original magnetic field produced a magnetic flux ϕ of magnitude $1/2(hc/2e) < \phi < (hc/2e)$ through both holes (or through the whole structure)? If the middle link were not there we would expect to find the system in the $n = 1$ quantum state appropriate to the outside loop (see Fig. 2). However, if the middle link *is* there, this quantum state is not allowed by the Byers and Yang theorem, which permits only the following possibilities: (i) $\phi = 0$ through both holes, or the degenerate choices of (ii) $\phi = hc/2e$ through the left loop and $\phi = 0$ through the right loop, or (iii) $\phi = 0$ through the left loop and $\phi = hc/2e$ through the right loop. It can be shown that any of these three possibilities is energetically expensive, if we consider the quantum selection process near T_c. I believe that, in order not to have to pay this energy, the system will perform the following trick: a part of the middle link will revert to the normal phase, and this in effect will convert the double loop to a single loop. The system can now choose the $n = 1$ quantum state appropriate to the outside loop of Fig. 2, energetically a much cheaper solution than the other possibilities mentioned. Within the formalism (or philosophy) of the Ginzburg-Landau theory of superconductivity (*27*), this seemingly unlikely kind of thing can happen if there is enough energy to pay for it, and I believe that in this case there is.

At the University of Rochester we are now trying to prepare samples that have the double-ring symmetry, as well as more complicated net structures, with microphotographic techniques. It is interesting to speculate on what might happen in an "infinite superconducting net" (one with a very large number of cells of equal size).

Summary

The observation that the magnetic flux in a hollow superconducting cylinder is quantized in units of $hc/2e$ has confirmed Fritz London's prediction that superconductivity is a macroscopic quantum phenomenon and that the superconducting state is a single-valued quantum state. It has also conclusively demonstrated that the pair interaction of the Bardeen-Cooper-Schrieffer theory of superconductivity is the important one, at least for the few superconductors studied. A superconducting ring or hollow cylinder must "decide" at the transition temperature what final quantum state it will be in when the temperature is lowered. This "thinking process" has been observed in experiments which demonstrate that the free energy of a hollow superconducting cylinder at the transition temperature is periodic in the magnetic flux. Very recently quantum-mechanical interference effects have been observed in the Josephson tunneling char-

acteristics of multiply linked superconducting circuits. These experiments complement the other experiments on quantized flux and provide perhaps the most elegant proof of long-range order in the superconducting state.

References and Notes

1. F. London, *Superfluids* (Dover, New York, 1961), vol. 1.
2. This prediction appears in a footnote on page 152 of F. London, *Superfluids* (*1*).
3. A discussion of the Gorter-Casimir two-fluid model may be found in nearly every review article or book on the macroscopic theory of superconductivity.
4. In the microscopic theory of superconductivity the "superelectrons" are "particles" included in the ground-state wave function, and the "normal electrons" are quasi-particles excited from the ground state. However, if care is taken, the Gorter-Casimir nomenclature and approach are correct and useful in describing some problems, such as the one at hand.
5. This criterion is necessary because the magnetic flux does not fall abruptly to zero at the edge of the hole but penetrates into the superconductor over a distance of the order of λ.
6. B. Deaver and W. Fairbank, *Phys. Rev. Letters* **7**, 43 (1961).
7. R. Doll and M. Näbauer, *ibid.*, p. 51.
8. J. Bardeen, L. N. Cooper, J. R. Schrieffer, *Phys. Rev.* **108**, 1175 (1957).
9. In the case of zero current flow the center of mass momentum of each pair is zero; however, it is the center of mass motion of the pairs, which is responsible for supercurrents.
10. The cylinder was thick-walled according to the criteria discussed.
11. L. Onsager, in *Proc. Intern. Conf. Theoret. Phys., Kyoto-Tokyo, Japan, 1953* (1954), pp. 935–936.
12. N. Byers and C. N. Yang, *Phys. Rev. Letters* **7**, 46 (1961).
13. The free energy of the normal state is probably independent of the magnetic field, at least for the small fields considered here.
14. More refined calculations made later reduced this value by nearly one order of magnitude.
15. W. A. Little and R. D. Parks, *Phys. Rev. Letters* **9**, 9 (1963); ———, in *Proc. Intern. Conf. Low Temp. Phys., 8th* (1963), p. 129; R. D. Parks and W. A. Little, *Phys. Rev.* **133**, A97 (1964).
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19. P. W. Anderson and J. M. Rowell, *Phys. Rev. Letters* **10**, 230 (1963).
20. J. M. Rowell, *ibid.* **11**, 200 (1963).
21. Y. Aharanov and O. Bohm, *Phys. Rev.* **115**, 485 (1959).
22. R. C. Jaklevic, J. Lambe, A. H. Silver, J. E. Mercereau, *Phys. Rev. Letters* **12**, 274 (1964).
23. Descriptions of other experiments related to the Aharanov-Bohm effect can be found in R. G. Chambers, *Phys. Rev. Letters* **5**, 3 (1960); G. Mollnstedt and W. Bayh, *Naturwissenschaften* **49**, 81 (1962); H. Boersch, H. Hamisch, K. Grohmann, *Z. Physik* **169**, 263 (1962).
24. L. Meyers and W. A. Little, *Phys. Rev. Letters* **11**, 156 (1963).
25. D. C. Hamilton and M. A. Jensen, *ibid.*, p. 205. In view of the recent theoretical work by Garland (*ibid.*, p. 111), there is no a priori reason for expecting some mechanism other than the electron-phonon interaction to be responsible for superconductivity in superconductors other than lanthanum and uranium.
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The Outlook for World Population

Population control has begun to receive serious attention from governments and other organizations.

J. Mayone Stycos

There are at least two remarkable and unprecedented aspects to the population problem today—the first is the rate of population growth, the second is the growing inclination on the part of national governments to manipulate this rate.

Rapid population growth was characteristic of most European countries in the past century, and much of the excess population found its way to the New World. But rates of growth in underdeveloped areas today, ranging from about 2 to 3½ percent per year, are about twice those of European countries during the period of their most rapid growth. A population growing at the rate of 3 percent per year will double in 23 years, and one growing at the rate of 2 percent in 35 years. Since the population bases in the underdeveloped areas today far exceed those of Europe, the implications in sheer numbers of a rapid rate of growth are truly impressive. For example, if India alone were to grow for the next century somewhat more slowly than it is growing now, it would still have millions more inhabitants than the entire world has today.

The basic ingredients of this growth are by now well known. Low death rates, which it took European countries a century to a century and a half to achieve, are being approached in underdeveloped areas in a fifth of the time, but birth rates, which it took Europe 60 to 70 years to bring down to modern levels, show little sign of decline.

Various kinds of concern are expressed about the "population explosion." Some people seem concerned about sheer physical space and cite figures to show that there will be "standing room only" at some future date. Others see the increase as outrunning food resources or as hastening the end of our nonrenewable resources. Some are convinced that the increase spells genetic disaster, others are esthetically revolted by human crowding, and still others see it as a cause of wars. All such arguments, while they may have some truth, have serious limitations and in any event have had little impact on policy makers in underdeveloped areas. But there is one general line of reasoning which is having a major impact on leaders in the underdeveloped areas: it is demonstrable that current rates of population growth are slowing down economic development and that a reduction in the rate of growth would have substantial salutary consequences for the economy. This argument does not imply that population control is a substitute for the usual ingredients of modernization—education, industrialization, technological development, and so forth—but that it will enable underdeveloped countries to take full advantage of such developments and make it possible for them to add to their per capita wealth and productivity.

The recent upsurge of interest in the relation between economic development

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