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Atmospheric Tides

These oscillations are caused by the gravitational pull of sun and moon and by the sun's thermal effects.

B. Haurwitz

The term *atmospheric tides* refers not only to the atmospheric oscillations produced by the gravitational forces of the moon and the sun but also to the oscillations due to the sun's thermal effects on the earth's atmosphere. This dual meaning is theoretically appropriate, since both gravitational forces and thermal effects induce oscillations which are gravity waves. Also, it is necessary for practical reasons to consider these two effects together, since one cannot separate them in the case of the oscillation of 12-hour period which appears both in the sun's tide-producing gravitational attraction and in the thermal influence of the sun on the atmosphere.

Barograph traces at the earth's surface show that in tropical regions the pressure has two maxima and two minima per day, the maxima occurring at about 10 a.m. and 10 p.m. local mean time, the minima occurring, respectively, 6 hours later. The amplitude of these oscillations is about 1 millibar (*1*). Figure 1 shows the hourly values for barometric pressure for a week at Balboa, Panama. At higher latitudes the nonperiodic pressure changes due to the passage of weather systems are very much larger than they are in the tropics and mask, in general, the 12-hourly oscillation. To find this oscillation a statistical investigation is required, based on a sufficiently long record to permit elimination of the irregular pressure changes.

Figure 1 shows clearly that the pressure oscillation has a period of half a solar day (12 hours), rather than of half a lunar day (12 hours 26 minutes). On the other hand, the gravitational tidal force of the moon is 2.2 times that of the sun. Therefore, one of the main problems of the theory of atmospheric tides is the need to explain why the solar tide is so much stronger than the lunar tide in the atmosphere. Laplace suggested early in the 19th century that the solar tide might be caused by the thermal rather than the gravitational action of the sun on the atmosphere. The daily temperature curve is not a pure sine wave; it contains, in addition to the 24-hourly oscillation, oscillations of higher frequencies—in particular, an oscillation of 12-hour period, discussed later. The amplitude of the 12-hourly temperature wave is smaller than that of the 24-hourly temperature wave, hence it becomes necessary to explain why the amplitude of the 12-hourly pressure wave is greater than that of the 24-hourly pressure wave. Kelvin (*2*) conjectured that the atmosphere may have a free oscillation of period of about 12 hours, and that the 12-hourly oscillation (but not the 24-hourly oscillation) is thus magnified through resonance. This suggestion is now generally referred to as the "resonance theory" of the semidiurnal pressure oscillation. Later I describe this theory and its development and modification in detail, but first it is necessary to discuss the observational information available on the atmospheric tides. This discussion is confined largely to the tidal variation of meteorological parameters, especially of pressure and wind. Only occasionally do I refer to the tidal variations of geomagnetic and ionospheric parameters.

For conciseness and clarity the notations L_n and S_n are used in the discussion of lunar and solar variations of geophysical parameters. The subscript n indicates that the period referred to is the nth part of the (lunar or solar) day. Where necessary, the parameter under discussion is shown in parenthesis. For instance, $S_2(p_0)$ denotes the solar semidiurnal oscillation of the sea-level barometric pressure p_0.

The Lunar Atmospheric Tide

Although the lunar atmospheric tide L_2 is about 15 times smaller than the solar atmospheric tide S_2, it can nevertheless be found, by means of statistical methods, from long series of data (*3*). The results of a determination of the lunar tide (and, with appropriate changes, of the solar tide) are customarily expressed in the form of a sine wave

$$L_n = A_n \sin (15\ n\tau + \alpha_n) \qquad (1)$$

where n is the fraction of the day, τ is the local lunar mean time (in hours), A_n is the amplitude, and α_n is the phase constant of the oscillation. In the case of the lunar tide only the semidiurnal oscillation ($n = 2$) has been found so far in meteorological variables, whereas in the case of the solar tide, S_1, S_2, S_3, and S_4 have been determined. From the phase angle α_n the time of the maximum can readily be obtained, since the argument of the sine must then be 90 degrees. For example, the annual mean value for the

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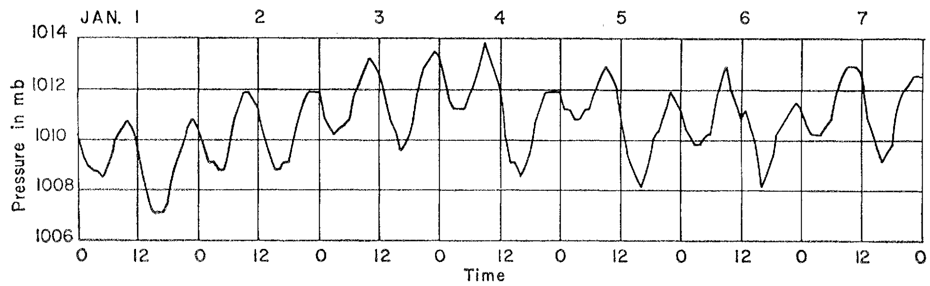


Fig. 1. Surface barometric pressure at Balboa, Panama, from 1 to 7 January 1952.

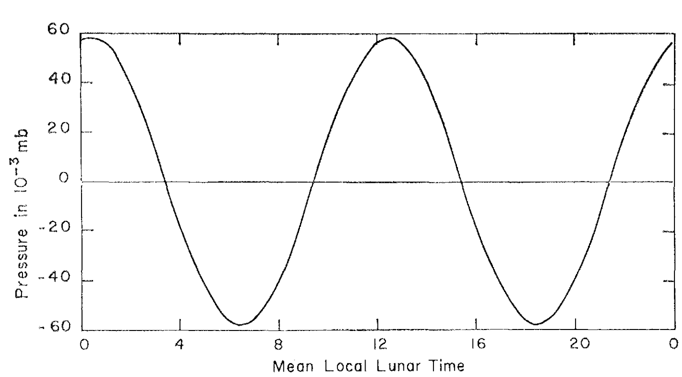
lunar tide in the barometric pressure p_0 at sea level at Balboa, Panama, as determined from data for 17.5 years, is

$$L_2(p_0) = 58.3 \cdot 10^{-3} \text{ mb} \sin (30_\tau + 77.0°), \qquad (2)$$

hence the maximum occurs about half an hour after the moon passes the local meridian. This variation due to the lunar tide is illustrated in Fig. 2 (left). The two numbers characterizing the oscillation are the amplitude and the phase angle. These, and therefore the whole oscillation, can be represented more concisely in a polar diagram, where the distance from the center 0 represents the amplitude, and where the phase constant is plotted so that the time of the maximum can be read off the diagram directly. Figure 2 (right) is a polar diagram for $L_2(p_0)$ at Balboa, the annual mean being represented by point A. (Because all plotted points lie in one quadrant, only this quadrant is shown.) The direction of the line $0A$ shows the phase constant, in degrees (outer scale at the circumference) and as the time of the maximum (inner scale). Since, for a 12-hourly oscillation, this type of figure is similar to a clock face, it is called a "harmonic *dial*."

Because long series of data are required for finding the variation in atmospheric pressure caused by the lunar tide, this variation has so far been determined at only about 70 stations. The determinations have been mainly the work of S. Chapman (*4*). Figure 3, which is largely based on Chapman's results, shows the global distribution of the amplitude of this tide. The amplitude decreases from the equator poleward, as would be expected, since the lunar tidal force decreases toward the poles. In general, the high tide occurs about ½ to 1 hour after passage of the moon through the local meridian. But there are, despite the regular distribution of the lunar tidal force, some very striking irregularities in the distribution of the lunar atmospheric tide. One of these is the smallness of the lunar tide on the Pacific coast of North America, presumably due to the obstructing effects of the mountains on the westward progression of the tide. Another irregularity is the asymmetry of the amplitude with respect to the earth's equator; there are large amplitudes over the eastern part of Africa south of the equator and over the western part of Indonesia. No explanation of this phenomenon has yet been proposed.

If we group the months into three "seasons," designated D (November, December, January, February), E (for Equinoctial) (March, April, September, October), and J (May, June, July, August), we obtain for Balboa (Fig. 2, right) the three points J, E, and D, which are connected by straight lines indicating a pronounced seasonal variation. The circles around these points are probable-error circles (*3*) of $L_2(p_0)$ for each season. A determination of such oscillations cannot be considered satisfactory if the radius of the probable-error circle is more than one-third the amplitude. The determinations of Fig. 2 (right) are clearly satisfactory. The seasonal variation for Balboa is fairly typical: largest amplitudes around the June solstice, smallest amplitudes around the December solstice. Contrary to many meterological variables with a seasonal period, the maxima and minima for L_2 occur at the same time of the year, rather than with opposite phases, in the Northern and Southern hemispheres. Since the lunar tidal force has no seasonal variation, the cause of the seasonal variation of the lunar atmospheric tide must be sought in the varying response of the atmosphere to this force. But no satisfactory explanation has as yet been offered. An understanding of the apparently anomalous responses of the



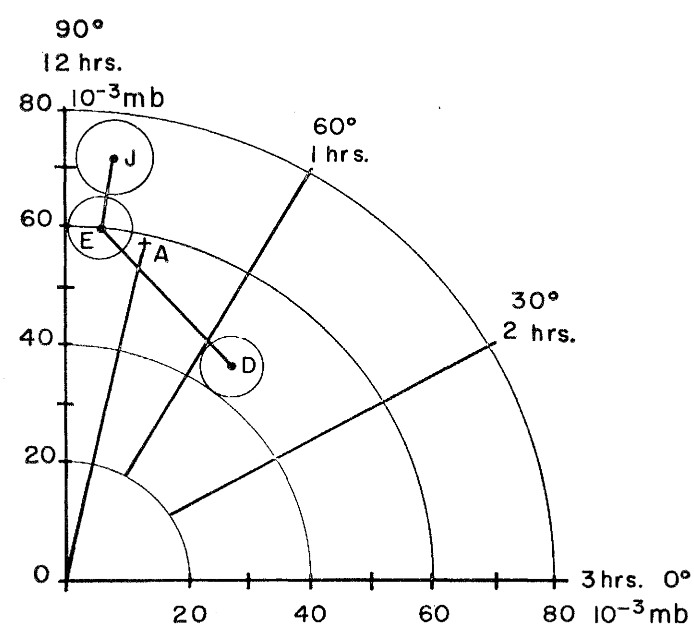


Fig. 2. The lunar tide in the earth's atmosphere at Balboa, Panama, as determined from data accumulated over 17.5 years. (Left above) Variation of pressure at the earth's surface due to the lunar tide. (Right) Harmonic dial showing seasonal variation: (J) summer months; (D) winter months; (E) equinoctial months; (A) annual mean.

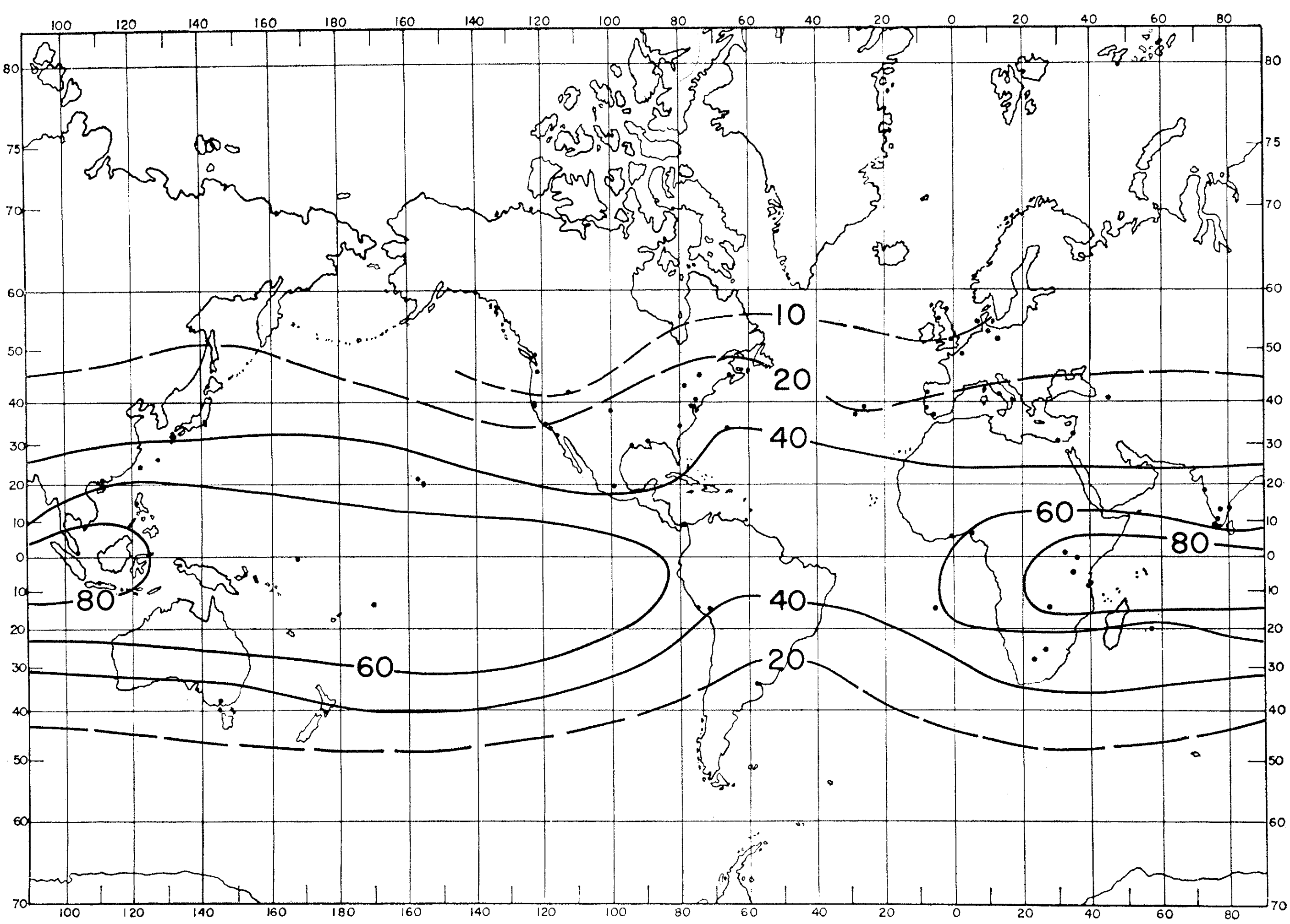


Fig. 3. Lines of equal amplitude for the lunar tide as manifested in atmospheric pressure at sea level. The amplitude is given in units of 10^{-3} millibar. [Map based largely on data from Chapman and Westfold (4)]

atmosphere to the completely regular and well-known lunar tidal force would represent an important advance in atmospheric dynamics.

The Solar Atmospheric Tides

Among the atmospheric oscillations that proceed according to solar time the one with a 12-hour period, S_2, is largest. This oscillation, as manifested in the atmospheric surface pressure p_o, is distributed quite regularly over the globe, its amplitude decreasing from the low latitudes toward the poles. The amplitude distribution (5) is shown in Fig. 4. Between approximately 50° north and 50° south latitude the maximum occurs at about 09:45 a.m. and 09:45 p.m. local mean time, giving two waves whose crests precede the sun in its westward motion by about 2 hours and 14 hours, respectively. At higher latitudes the data available for the determination of S_2 are quite sparse. They show, nevertheless, a standing oscillation whose maximum occurs everywhere at the same Universal time—roughly, at about 11 a.m. and 11 p.m. An approximate expression for S_2, which consists of these two superposed oscillations, is

$$S_2(p_o) = 1.16 \text{ mb} \cos^3 \phi \sin (2t + 158°) + 0.06 \text{ mb} \cos (180° \sin \phi) \sin (2t - 2\lambda + 303°) \quad (3)$$

Here ϕ is the geographic latitude, t is the local mean time in angular measure, and λ is the geographic longitude; thus, $t - \lambda$ is the Universal time. The first term on the right-hand side of Eq. 3 represents the migrating component of $S_2(p_o)$, the second term represents the standing component.

Because of the superposition of these two oscillations there are four points, two in the Northern Hemisphere and two in the Southern Hemisphere, where the amplitude of S_2 is vanishingly small. At these points the lines of equal phase, which may also be called "co-tidal lines," as in oceanography, intersect. These co-tidal lines (5) are plotted in Fig. 5. Here the phase constant is shown in hours of Universal time in the upper and lower part of each co-tidal line, and in degrees in the center of the figure. The lines show clearly the manner in which the semidiurnal wave crest progresses westward. The co-tidal lines sweep around the four points of vanishing amplitude, which may be called "amphidromic points," as in oceanography.

The diurnal oscillation, S_1, as manifested in the barometric pressure at the earth's surface, is much more irregularly distributed and less strongly developed than S_2. The topography of the earth and the unequal distribution of water and land clearly disturb the distribution of S_1 more than they disturb that of S_2, giving rise to many local irregularities. Nevertheless, the diurnal temperature wave must produce a worldwide or "planetary" diurnal pressure wave, traveling westward with the sun, on which the more local disturbances are superimposed. From an analysis of all the available data, the following approximate expression is obtained

$$S_1(p_0 = 0.59 \text{ mb} \cos^3 \phi \sin (t + 12^\circ) \quad (4)$$

where the meaning of the symbols is the same as in Eq. 3. The magnitude of this oscillation is thus about half that of the semidiurnal oscillation, and its maximum occurs shortly after 5 a.m. local mean time. S_1 and S_2 undergo seasonal variation. Even for S_2 these variations differ greatly at different stations.

The higher harmonics, S_3 and S_4, as well as the standing component of S_2, can be produced only by the thermal action of the sun, since the sun's tide-producing gravitational force does not contain any corresponding terms. S_3 is considerably smaller than S_1 and S_2. Its maximum amplitude of about 0.2 mb occurs at about 30° north and 30° south latitude. Despite its smallness it is very regularly distributed over the globe. An interesting feature of S_3 is its pronounced seasonal variation with phase reversals between January and July and minimum amplitude at about the time of the equinoxes; this corresponds to the seasonal variation of the 8-hourly temperature variation. S_4 is even smaller than S_3, but it also has a fairly regular distribution over the earth, and this makes it interesting from the standpoint of theory.

Atmospheric Tides at Higher Levels

The tidal oscillations near the earth's surface are very small as compared to the mean for low levels of the atmosphere; the oscillations are very much larger at high levels. The first, indirect, evidence of the importance of the atmospheric tides at great heights was furnished by the daily variations (according to both solar and lunar time) of the geomagnetic parameters. An explanation of these variations is given by the "dynamo" theory. According to this theory a current is induced in the ionized layers of the high atmosphere when they are moving in the geomagnetic field because of the atmospheric tides. At the surface, the effects of this current are observed as the daily variations of the geomagnetic parameters.

Direct evidence of the tides in the high atmosphere has been obtained only fairly recently, through observation, by means of radio techniques, of the drift of meteor trails. When a meteor penetrates the atmosphere down to a level of about 80 to 100 kilometers it produces an ionized trail during its disintegration as it collides with the air molecules. This trail drifts along with the air. Radio signals are reflected from these meteor trails, and the radial-drift velocity, with respect to the observer, can be found by means of Doppler techniques. The wind velocity can be determined from these radial velocities, because in a reasonably short time a sufficient number of radio meteor echoes from different directions can be observed to compute this velocity.

Such data have, so far, been obtained mainly at two stations—at Jodrell Bank, near Manchester, England, and at Adelaide, Australia. The Jodrell

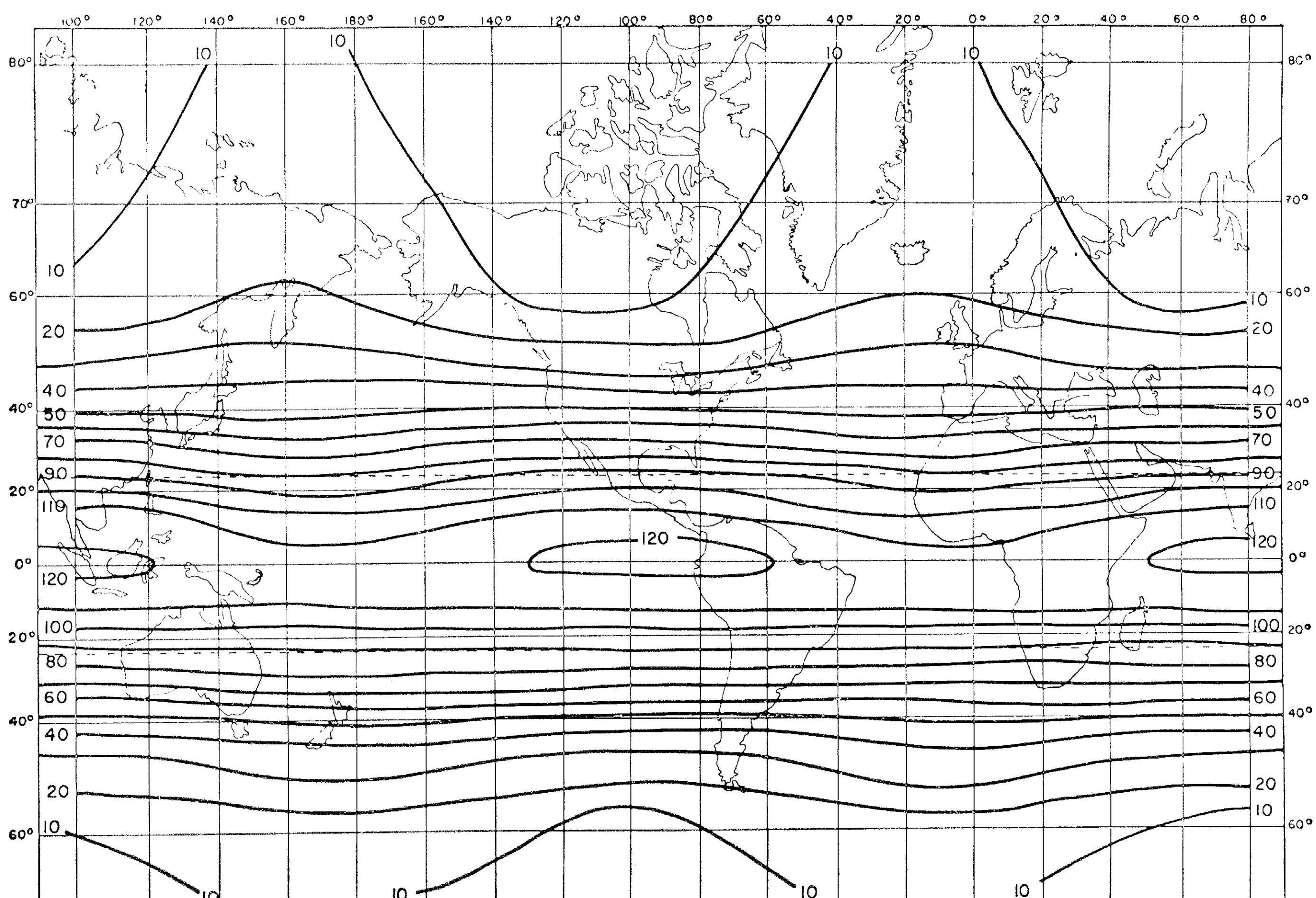


Fig. 4. Lines of equal amplitude for the semidiurnal solar tide as manifested in atmospheric pressure at the earth's surface (see *16*). The amplitude is given in units of 10^{-2} millibar.

Bank data were obtained by Greenhow and Neufeld; the Adelaide data, by Elford (*6*). The data show that the amplitudes of the periodic variations of the wind components, both S_1 and S_2, are similar in magnitude to the daily mean wind, and that in both cases the amplitudes are about 100 times greater in the high atmosphere than at the ground. The behavior of the periodic part of the wind is illustrated in Fig. 6. Here the speed and direction of the wind are plotted as hodographs, which show the end point of the periodic wind vector $S_1 + S_2$ for the *D* months. The numbers from 0 to 22 adjoining the marked points on the hodograph curve give every second hour in local mean time. Thus, in Fig. 6 (left) for Jodrell Bank, the wind at local midnight (0 hours) blows nearly toward south (a "north wind" in meteorological and everyday terminology) with a speed of about 16 meters per second. The wind vector turns clockwise and describes two complete rotations during 24 hours, demonstrating the presence of the semidiurnal oscillation. Because the 24-hourly oscillation is superposed, the two circuits do not coincide. At Adelaide (Fig. 6, right) the wind vector turns counterclockwise, as it should in the Southern Hemisphere. The hodograph curve does not show two complete loops, because here S_1 has a larger amplitude than S_2. It is impossible to say, on the basis of presently available data, whether in general S_2 or S_1 is larger at these high altitudes, or whether systematic regional differences exist. The amplitudes and phase constants of both S_1 and S_2 undergo large seasonal variation, very much larger at these high levels than at the earth's surface.

The oscillation also changes with elevation, the amplitude increasing, and the time of the maximum being delayed, with ascent from 80 to 100 kilometers. But the observations are not yet numerous enough to provide a basis for reliable quantitative determinations.

Unfortunately, very few data are available for the layers intermediate between the surface level and the meteor levels (80 to 100 km). Harris *et al.* (*7*) have published data on the diurnal and semidiurnal pressure and wind oscillations for Lajes Field at Terceira, Azores, based on balloon observations. These data are for levels lower than 30 kilometers. Up to this altitude no very pronounced vertical changes in S_1 and S_2 are found.

Like the solar tide the lunar tide must increase with elevation. But the data are not sufficient for determining it at meteor levels. One can only say that the winds due to the lunar tide must be less than 2 meters per second. At the surface the lunar tidal winds have a velocity about 100 times less than this value. Higher up, in the ionosphere, the lunar tidal oscillation can clearly be seen in various observable quantities, such as the geomagnetic parameters referred to earlier (*8*). The lunar oscillation has also been demonstrated to exist in other parameters, such as the virtual height of the *E* layer.

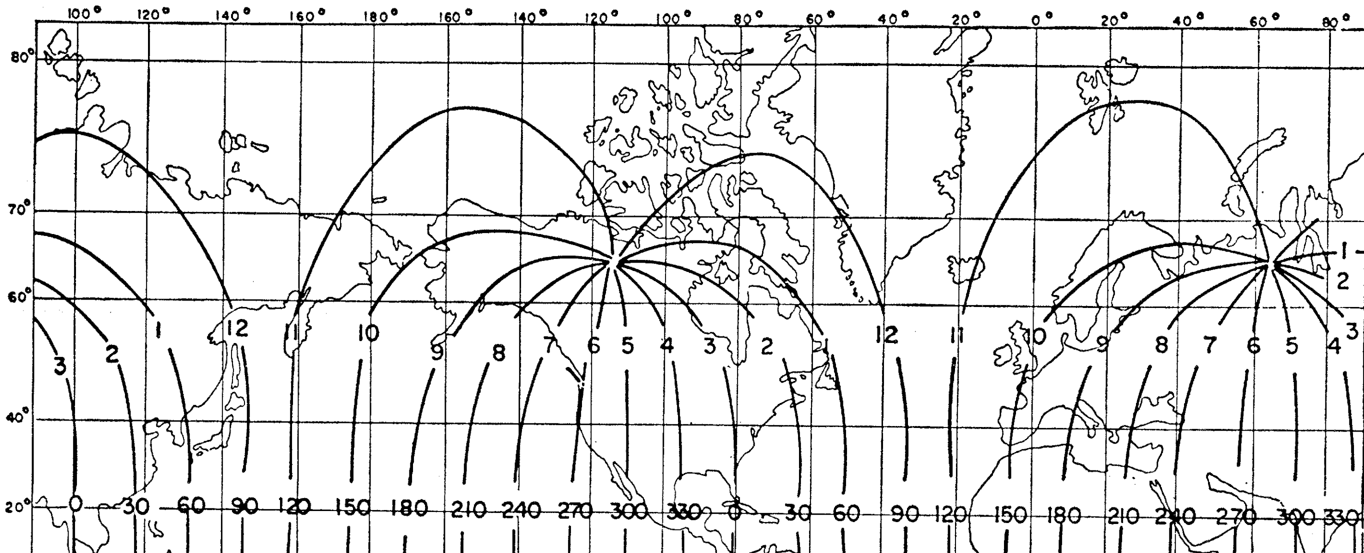


Fig. 5. Lines of equal phase for the semidiurnal solar tide as manifested in atmospheric pressure at the earth's surface (see *16*). The phase constants are given (near the center line of the map) in degrees, and (above and below the center line) in time (U.T.) of maximum pressure.

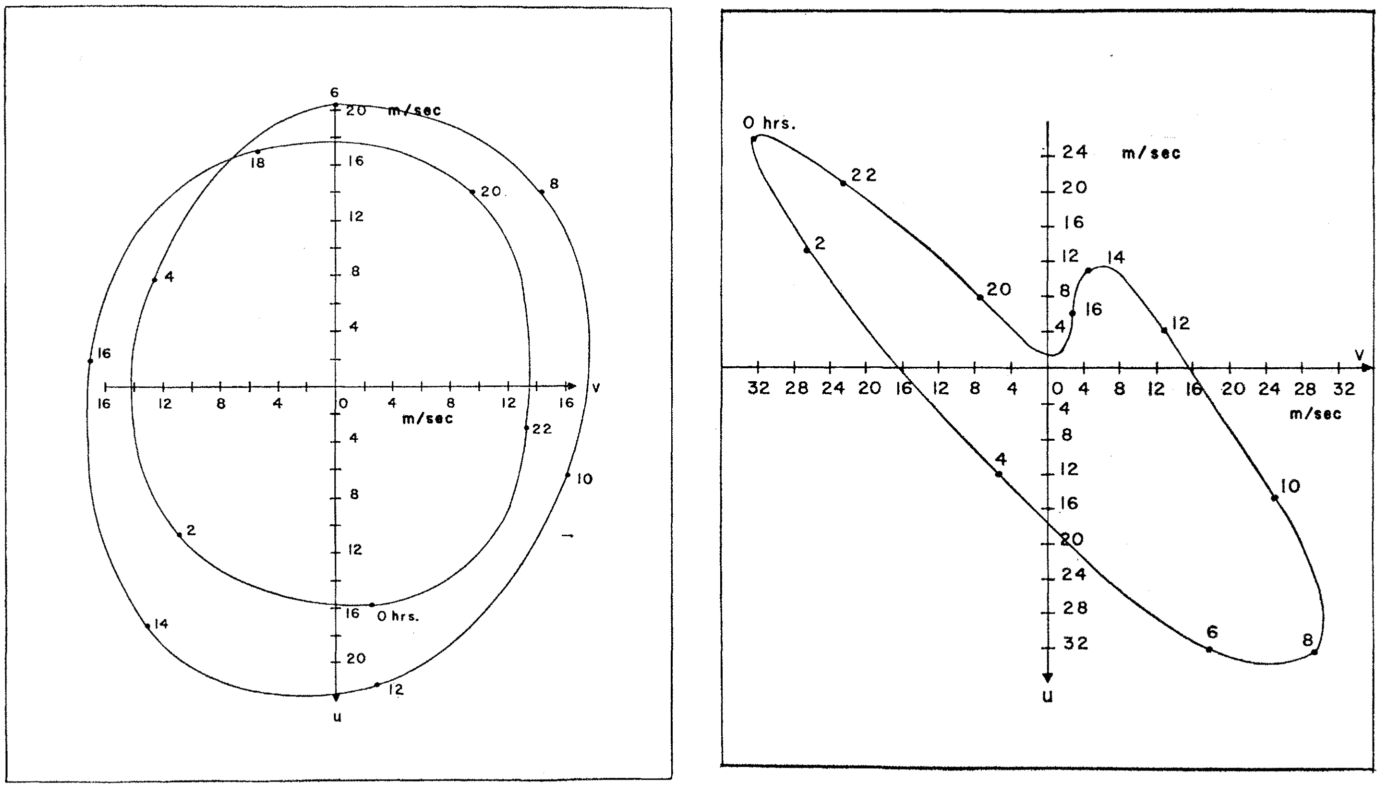


Fig. 6. Hodographs of the periodic components ($S_1 + S_2$) of the wind vector at altitudes of 80 to 100 kilometers during the *D* months (November through February), in meters per second, at (left) Jodrell Bank, England and (right) Adelaide, Australia. The curves represent the end points of the periodic wind vectors. The numbers adjoining the curves are hours, in local mean time.

Daily Temperature Variation

As I have explained, the solar tidal oscillations, including S_2, are probably due to the thermal rather than the gravitational action of the sun on the atmosphere. Therefore, let us consider at this point, as a manifestation of this thermal action of the sun, the daily temperature curve with its various harmonics, in order to understand how this daily temperature wave can produce a semidiurnal effect.

The solid curve of Fig. 7 shows, as a typical example, the daily variation in air temperature near the ground at Potsdam, Germany, during February; it is based on the average for data accumulated over 60 years. The temperature rises sharply from a minimum at about sunrise (about 7 a.m.) to a maximum at about 2 p.m. From there it decreases, first rapidly, then, especially after sunset (about 5 p.m.), much more gradually to its minimum. Thus, the diurnal temperature curve is not a simple sine curve with a 24-hour period but is asymmetrical. This asymmetry is due to the different processes determining the warming and cooling of the atmosphere. During the daytime the amount of radiative energy received depends largely on the altitude of the sun above the horizon, which follows, very approximately, a sine function; after sunset this energy flux is zero. The heat loss due to radiation is fairly constant throughout the day and night, so it is only throughout the daylight hours that the temperature curve can be expected to have a form approximating a sine curve. The sine curve with 24-hour period which gives the best fit (within the meaning of the method of least squares, the curve being obtained by harmonic analysis) is shown by the dashed curve (to which the daily mean has been added) of Fig. 7. To reproduce the more gradual temperature decrease at night, the delay of the minimum until the time of sunrise, and the more rapid rise and fall of the temper-

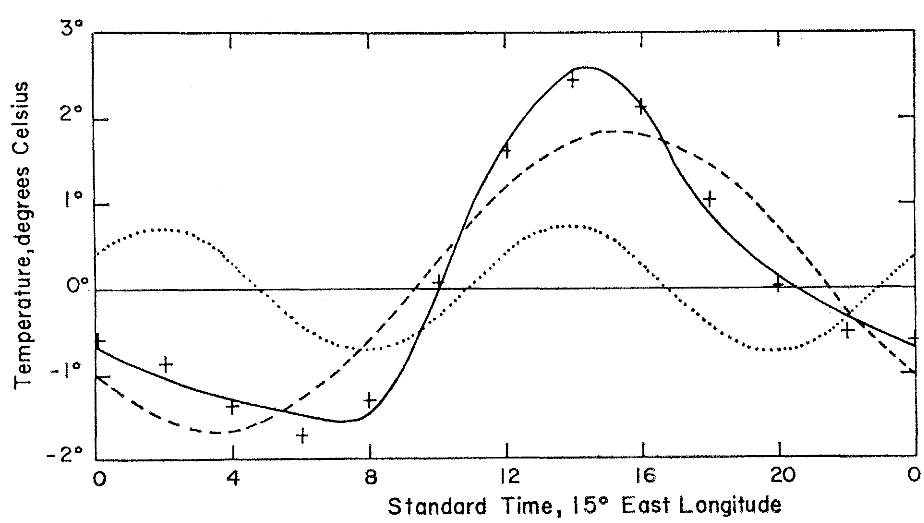


Fig. 7. (Solid curve) Daily variation of the air temperature at Potsdam, Germany, during February; (dashed curve) 24-hourly temperature wave; (dotted curve) 12-hourly temperature wave; (crosses) sum of these two waves.

ature during the daylight hours, one must add a sine curve with a period of 12 hours (the dotted line of Fig. 7). The superposition of the daily mean and the first and second harmonics is represented by the crosses for every second hour. The second harmonic of the daily temperature curve is thus a result of the asymmetry of the effects producing the temperature curve.

The remaining small difference between observed and computed temperatures can be largely represented by a third harmonic, a wave of 8-hour period; addition of this third harmonic corrects essentially for differences in the relative lengths of day and night throughout the year. It is zero or nearly zero around the equinoxes and changes its phase by 180 degrees (or nearly 180 degrees) between winter solstice and summer solstice, just like the corresponding pressure oscillation. A further improvement of the theoretical temperature curve can be obtained by adding a fourth harmonic, which has an amplitude even smaller than that of the third harmonic but which makes itself felt nevertheless as a corresponding small pressure oscillation of 6-hour period.

The breakdown of the daily temperature variation into a number of oscillations of different periods gives, of course, a purely formal result. The atmosphere, as an oscillating system, responds to the total excitation provided by the daily temperature oscillation. But the response depends on the tuning of the atmosphere to the different harmonics of the exciting force. Thus the amplitude ratios of, say, the pressure waves may be quite different from those of the temperature waves.

The Resonance Theory

Now let us return to the problem of the relative magnitudes of S_2 and L_2. Since L_2 is smaller than S_2, it is surmised that S_2 is probably largely attributable to the thermal, rather than to the gravitational, action of the sun. But the diurnal term in the daily-temperature curve is about 2.5 times the semidiurnal term (*9*), while the diurnal surface-presure oscillation is only about half the semidiurnal oscillation. To remove this difficulty Kelvin (*2*) advanced the resonance theory, according to which the atmosphere has a free oscillation with a period of about 12 hours. A great amount of theoretical

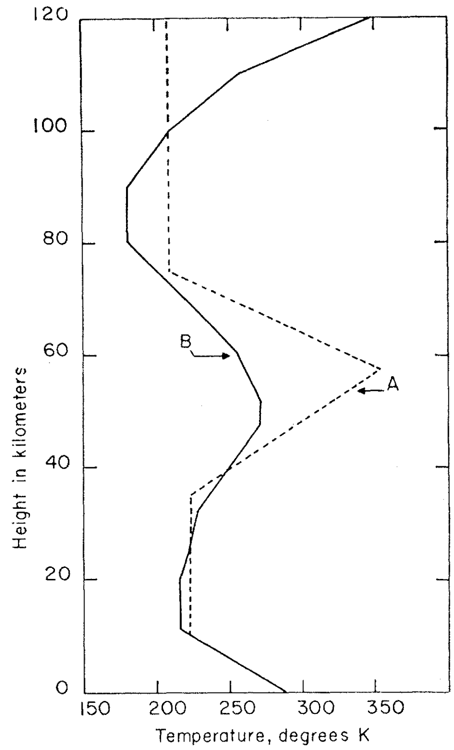


Fig. 8. Vertical temperature distribution in the atmosphere. (*A*) Temperature distribution assumed by Pekeris (*11*); (*B*) U.S. Standard Atmosphere curve, 1962 [U.S. Government Printing Office, Washington, D.C., 1962].

work has been done since Kelvin's time to determine the free oscillations of the terrestrial atmosphere and the response of the atmosphere to both gravitational and thermal types of excitation. From these investigations it follows that the free oscillations of the atmosphere are the same as the free oscillations of an incompressible and homogeneous ocean of "equivalent depth" h. This quantity h depends on the vertical temperature distribution of the atmosphere. G. I. Taylor (*10*) has shown that there is in general an infinite number of equivalent depths for a given atmospheric temperature distribution, and that h is related to the velocity V of long atmospheric waves by the formula $V^2 = gh$, where g is the acceleration of gravity. Such waves were observed during the eruption of the volcano Krakatao in 1883 and gave $h = 10$ km, approximately. On the other hand, according to theory the equivalent depth $h = 7.8$ km is required for a free oscillation with period of 12 solar hours.

In order to discuss what values the equivalent depth h of the real atmosphere may assume, let us consider Fig. 8, which represents two vertical atmospheric temperature distributions. Before rocket data became available, the information about the temperature distribution above 30 kilometers, then the ceiling of meteorological balloon ascents, came mainly from observations of the anomalous propagation of sound. These observations indicated, for altitudes around 50 kilometers, temperatures well above those found at the ground, and Pekeris (*11*) showed that with the temperature profile A of Fig. 8, which is characterized by a temperature maximum of 350°K at 60 kilometers, two values are obtained for h—namely, 10 kilometers, as suggested by the Krakatao waves, and 7.8 kilometers, very close to the value required for strong resonance magnification. Since that time, however, rocket ascents have given a much more reliable picture of the temperature distribution of the atmosphere; this distribution is represented by profile B of Fig. 8, which shows temperatures considerably lower than 350°K at altitudes of 50 to 60 kilometers. With these lower temperatures the value $h = 10$ km is retained, but not the value $h = 7.8$ km. Thus, the resonance magnification for the solar semidiurnal oscillation is not strong (not $\times$ 60 or more, as was previously thought) but quite weak (on the order of $\times$ 3 or $\times$ 4).

For these more realistic models with little magnification of S_2, the lunar semidiurnal oscillation would be magnified to about the same extent as the solar semidiurnal oscillation, in agreement with the observed magnitude of the lunar tide.

Thermal Excitation

With this low resonance magnification the gravitational excitation cannot contribute noticeably to the observed magnitude of the solar semidiurnal pressure oscillations. It becomes necessary to examine the thermal excitation in detail. Only a small amount of the incoming solar-radiation energy is absorbed in the atmosphere before reaching the ground. Thus, the heating of the atmosphere, at least in the lower troposphere, proceeds mainly from the ground upward, by turbulence and by long-wave radiation. These processes are effective only through a very limited height range. Hence, the resulting temperature oscillation is reduced to insignificance at a few hundred meters' altitude and affects only a small part of the total mass of the atmosphere, one-tenth or less. So long as only this part of the total heating and cooling effect of the sun was considered, it seemed necessary to assume large magnification, and the contribution of the thermal excitation appeared to be only of about the same magnitude as the gravitational excitation (*12*), insufficient to explain the observed semidiurnal pressure oscillation without very appreciable magnification.

As a way out of this difficulty, Sen and White (*13*) and Siebert (*13*) pointed out that the amount of incoming solar energy absorbed directly in the atmosphere, while small, must give rise to a daily temperature variation in the atmosphere which makes a very significant contribution to the atmospheric oscillation, since it affects the whole atmosphere.

Moreover, at higher layers in the atmosphere, ozone becomes important in the atmospheric heat budget. Because of its high absorptive power in certain parts of the ultraviolet region the ozone that is present heats the upper atmosphere between 30 and 50 kilometers very considerably and is in fact responsible for the relatively high temperatures at about 50 kilometers shown in Fig. 8. The ozone also produces pronounced daily temperature variations in this whole layer, which must contribute to the diurnal oscillations and their higher harmonics. Butler and Small (*14*), in fact, conclude that by far the largest part of the semidiurnal pressure oscillation is due to the temperature oscillation in the ozone layer. If the temperature wave producing S_2 occurs mainly in a higher atmospheric layer, a node for S_2 should exist at about 30 kilometers, and the phase should here change by 180 degrees (*14, 15*). The geomagnetic variations seem to indicate a phase reversal between the surface layers of the atmosphere and the ionosphere. But this phase difference can also be accounted for by the gradual phase changes of S_2, observed in the tidal wind oscillations at meteor heights, between 80 and 100 kilometers. Data obtained in observations over the Azores (*7*) at elevations around 30 kilometers do not show the node and sudden phase change which should exist if the cause of S_2 is to be found in the ozone layer, but the elevations were not sufficiently high to rule it out conclusively. Thus, it is at present impossible to decide on observational grounds whether S_2 is caused by the heating of the whole atmosphere or by the heating of the ozone layer alone.

It remains for me to explain why the diurnal pressure oscillation at the ground is only half S_2, even though the corresponding diurnal temperature oscillation is about 2.5 times the semidiurnal temperature wave. The explanation of this discrepancy is to be found partly in the resonance magnification of S_2, even though this magnification is small, and partly in the fact that the diurnal pressure oscillation is actually reduced in the atmosphere, rather than magnified. This reduction is due to the small equivalent depth required for S_1—less than 700 meters, a value which differs from the actual equivalent depth of the atmosphere (10 km) even more than the value required for S_2 (7.8 km) does. A simple calculation based on Siebert's work shows that a temperature oscillation of a given amplitude would produce a 12-hourly pressure oscillation of amplitude almost 4 times that of the 24-hourly oscillation. Thus, the discrepancy between (i) the amplitudes of $S_2(p_o)$ and $S_1(p_o)$ and (ii) the amplitudes of the corresponding temperature oscillations is greatly reduced.

In the discussion of the tidal theory given here I refer only to the lower layers of the atmosphere, up to approximately 100 kilometers. At greater elevations, where the tidal motions become as large as the mean motion, the basic assumption of the tidal theory—that the motion is sufficiently small that the equations can be linearized—is no longer valid. Furthermore, because of the increasing ionization of the atmosphere at these heights, hydromagnetic effects can no longer be neglected. The study of these problems has hardly begun.

Summary

The semidiurnal lunar tide in the earth's atmosphere is about 15 times smaller than the semidiurnal solar tide. Since the gravitational tidal force of the moon is 2.2 times that of the sun, the semidiurnal solar tide must be largely produced by the thermal action of the sun. The daily variation of the atmospheric temperature has, in fact, not only a 24-hourly but also a 12-hourly harmonic and other, higher harmonics, because of the asymmetry of the daily temperature curve. The amplitude of the 24-hourly harmonic of the temperature curve is considerably larger than that of the 12-hourly harmonic. To explain why the 12-hourly pressure oscillation is larger than the 24-hourly oscillation, the resonance theory postulates that the atmosphere has a free oscillation of period close to 12 hours, so that the 12-hourly pressure oscillation is greatly magnified. But such a free oscillation requires, at elevations around 50 kilometers, temperatures higher than those at the ground, and rocket observations do not show such high temperatures. That the semidiurnal pressure oscillation is nevertheless larger than the diurnal oscillation can be accounted for by the fact that the atmosphere does magnify the 12-hourly oscillation slightly, and that it has a tendency to suppress the 24-hourly oscillation.

While the atmospheric tidal motions are small at the bottom of the atmosphere, as compared to the mean values for the wind velocity, they increase with altitude and are greater by about two orders of magnitude at elevations between 80 and 100 kilometers than they are at the earth's surface. Thus, strong periodic motions occur at these high levels.

References and Notes

1. The millibar (mb) is the pressure unit now generally used in meteorology. It is 1000 times the pressure unit (dyne/cm^2) in the centimeter-gram-second system.
2. W. Thomson (later Lord Kelvin), *Proc. Roy. Soc. Edinburgh* **11**, 396 (1882).
3. See, for instance, S. Chapman and J. C. P. Miller, *Monthly Notices Roy. Astron. Soc., Geophys. Suppl.* **4**, 649 (1940).
4. See S. Chapman and K. C. Westfold, *J. Atmospheric Terrest. Phys.* **8**, 1 (1956), and earlier references given in that article.
5. B. Haurwitz, *Meteorol. Papers* **2**, No. 5 (1956).
6. J. S. Greenhow and E. L. Neufeld, *Quart. J. Roy. Meteorol. Soc.* **87**, 472 (1961); W. G. Elford, *Planetary Space Sci.* **1**, 94 (1959).
7. M. F. Harris, F. G. Finger, S. Teweles, *J. Atmospheric Sci.* **19**, 136 (1962).
8. S. Matsushita, in *Handbuch der Physik*, J. Bartels, Ed. (Springer, Berlin, 1964), vol. 49, p. 1.
9. B. Haurwitz, *Arch. Meteorol. Geophys. Bioklimatol.* **A12**, 426 (1962).
10. G. I. Taylor, *Proc. Roy. Soc. London* **A156**, 318 (1936).
11. C. L. Pekeris, *ibid.* **A158**, 650 (1937).
12. S. Chapman, *Quart. J. Roy. Meteorol. Soc.* **50**, 165 (1924).
13. H. K. Sen and M. L. White, *J. Geophys. Res.* **60**, 483 (1955); M. Siebert, thesis, Göttingen (1955); ———, *Advan. Geophys.* **7**, 105 (1961).
14. S. T. Butler and K. A. Small, *Proc. Roy. Soc. London* **A274**, 91 (1963).
15. R. Sawada, *J. Meteorol. Soc. Japan* **40**, 303 (1962); A. Matsushima and R. Sawada, *ibid.*, p. 309.
16. Figures 4 and 5 are based on a spherical harmonic analysis of the distribution of $S_2(p_o)$.