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Geodesy by Satellite

Satellite data have added to knowledge of the earth's size and shape, gravity field, and internal processes.

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Geodesy is a field of study with three main purposes: (i) to determine the size and shape (that is, the figure) of the earth; (ii) to locate points upon its surface; and (iii) to describe its gravity field at every point upon its surface. A satellite is a new tool in this ancient study, whose recorded history goes back more than two thousand years.

Belief that the earth is a sphere was common among philosophers by the time of Aristotle, who gives 400,000 stades as its circumference (*1*). Eratosthenes is frequently credited with being the first man to measure the earth, but he cannot have been, since he lived a century later than Aristotle. However, his is the oldest recorded method (*2*). He was told that the sun at noon on Midsummer Day shone straight down a well at Syene, the modern Aswan, and he measured the direction of the sun at Alexandria at the time of this event to be 1/50 of a circle from the vertical. Since the distance from Syene to Alexandria is 50 days' journey by camel caravan, the circumference of the earth, according to Eratosthenes, is 2500 camel-days.

Eratosthenes' "arc-measuring" method (with better-calibrated instruments) is still one of the basic methods of "geometrical geodesy." Physical geodesy, the other great branch of observational geodesy, is not so old. Physical geodesy deals with measurements of gravity and could not have come into existence until man had conceived of gravity as a quantitative force. The first measurement in physical geodesy was probably that of Richer (*3*), who found in 1672 that a pendulum clock which ran correctly in Paris lost about 2½ minutes per day near the equator. Soon afterward, Newton (*4*) computed that the earth's rotation should cause its figure to be an oblate spheroid, and he used Richer's measurements in testing his theory. We can consider this the beginning of theoretical geodesy.

The question raised by physical geodesy about the departure from sphericity was settled through recourse to geometrical geodesy. If the earth is not a sphere, the ground length of a degree should vary with the latitude. The first measurements of variation in the length of a degree indicated that the earth is a prolate spheroid, but the second measurements showed that it is oblate, and these were taken as definitive (*5*).

A satellite can be used as a tool of physical geodesy, or it can be used in a purely geometric fashion. To use a satellite for physical geodesy, we observe its motion from widely separated points and at various times, and deduce the forces acting on the satellite by analysis of the motion. This gives us information about gravity, and also tells us the locations of the observing stations. In contrast to this procedure, we can observe the satellite simultaneously from points close together and determine the relative positions of the satellite and the observing stations, without knowing anything about the motion of the satellite. In my opinion, the opportunities for making measurements of this sort are so limited, and the cumulation of errors is so great, that the first procedure is more effective.

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Representation of Gravity

Mathematical description of the gravity field is usually given in terms of the potential V. The potential varies from point to point, and thus specifying a point is a necessary part of specifying the potential. I shall specify a point by giving its radius r from the center of mass of the earth, its latitude θ (*6*), and its longitude λ. By the potential at a point, written $V(r, \theta, \lambda)$, we mean the work required to carry a unit mass from an infinite distance to the point. All that matters in this definition is the point; we can carry the mass by any path we choose, such as along a radius. For a spherical earth, the potential would be $V(r, \theta, \varphi) = -K/r$. K is the product of the universal gravitational constant and the total mass of the earth, and is about 3.986×10^{14} m^3/sec^2. A geophysicist or astronomer would probably omit the minus sign in expressing the potential.

There are surfaces called level surfaces, on which V does not change. For a spherical earth (as opposed to the actual earth, which is oblate and bumpy), there would obviously be surfaces of constant r—that is, spheres concentric with the earth. If we move a mass from one point to any other on a level surface, we do no work, so the force exerted is perpendicular to the surface at every point. But this force is just the force to counteract the acceleration due to gravity, so the direction of gravity is also normal to the level surface.

Let us consider two neighboring level surfaces a distance ds apart along the normal—that is, along the direction of gravity. If dV is the difference between the values of V on the surfaces, it is therefore the work done in moving along ds while exerting the force $-g$, where g is the acceleration due to gravity. Hence, $dV = -g\ ds$, or $g = -dV/ds$, and if we know the function $V(r, \theta, \varphi)$, we can calculate the direction and magnitude of g at every point. Thus, $V(r, \theta, \varphi)$ contains all of the information about gravity, and it is easier to manipulate mathematically than the acceleration g itself. For the spherical earth, gravity is directed along the radius, so $ds = dr$, and

$$g = -d(-K/r)/dr = -K/r^2.$$

For the actual earth, V is not simply $-K/r$. To describe V in the actual case, we resort to an analogy with a taut string. Suppose we could strike a string with a hammer and then suddenly freeze it. The shape would look quite complicated, but it could always be resolved into simple sine waves called harmonics. One of these would have one full loop (half a wavelength) in the full length of the string, others would have 2, 3, . . . , loops. Each would have its own amplitude, or maximum displacement from equilibrium. If we figure out the shape of each wave, and add these shapes up, we get the shape of the actual wave. Each of the individual waves, or harmonics, is identified by an integer, which is the number of half-waves in the string.

Now suppose we have a spherical earth uniformly covered with water and throw a large meteorite into it. After waiting a short time to let the waves spread over the surface, let us freeze the surface. Again we will have a complicated shape, but one that is also resolvable into individual harmonics, each with its individual amplitude. Each harmonic is identified by two integers, n and m. Integer n is the total number of full waves (not half-waves as in the case of the string) encountered in going completely around the earth, along the great circle that will cross the maximum number of waves for the particular harmonic in question. Integer m is the number of undulations encountered in going around the equator. Obviously, m can be any integer from 0 through n, but it cannot exceed n. Let S_n^m denote the mathematical function that describes the form of one of these harmonics.

In addition to its amplitude, we must tell one other thing about a harmonic. The wave can have any orientation about the earth's axis. We pick some arbitrary longitude—say, the longitude of a point on the equator at which the displacement is maximum—and call it λ_n^m. Then the function S_n^m, the longitude λ_n^m, and an amplitude factor yet to be introduced suffice to describe any individual wave (7). If $m = 0$, the harmonic is called zonal; since there are no oscillations in a wave going around the equator, the zonal harmonics are symmetrical about the earth's axis. For these, there is no characteristic longitude λ_n^m, and only the amplitude is needed.

So far we have used these harmonics to describe the shape of an undulating surface. The surface of most interest in geodesy is the level surface that coincides with mean sea level over the oceans; it is called the geoid. If the geoid is not spherical, and thus contains harmonics, other level surfaces for the actual earth are not spherical either. Thus, if we move around on a spherical surface, we are continually changing from one level surface to another; in other words, V changes from point to point on a spherical surface. But if V is not constant on a spherical surface, it must contain undulations, or harmonics. The harmonics in the gravity potential are simply related to those in the geoid; in fact, they differ only in the numbers used to give the amplitudes. I shall use J_n^m to denote the amplitudes of the harmonics in the gravity potential V. Further, I shall scale the amplitudes on the basis of the value for K/r at the equator, and I shall scale r on the basis of the earth's mean equatorial radius. Thus, the size of J_n^m relative to unity tells us approximately the importance of the corresponding harmonic.

If we like, we can call $-K/r$ a harmonic, and in mathematical formalism this is an advantage. The amplitude would be denoted by J_0^0, and in the system used here, $J_0^0 = -1$. Here it is simpler not to consider $-K/r$ one of the harmonics, and I shall not do so.

The foregoing discussion of the geoid would be correct if the earth were not rotating. For any element of the earth, including any particle that rotates with the earth, there is another force besides gravity, and that is the centrifugal force. The simplest way to handle the centrifugal force is to add the quantity $-\omega^2\rho^2/2$ to the potential, where ω is the angular velocity of the earth and ρ is distance from the axis of rotation. The geoid is then more properly described as the surface on which the sum of the gravitational potential V and the centrifugal potential $-\omega^2\rho^2/2$ is constant. The earth's rotation is the largest cause of its departure from sphericity.

A satellite does not rotate with the earth, and is not subject to this added potential.

Measurements in physical geodesy are made with instruments that rotate with the earth. Therefore these instruments do not measure the acceleration arising from gravitational forces alone, but measure the acceleration resulting from gravitational plus centrifugal forces. When a physical geodesist uses the symbol g, or refers to the acceleration due to gravity, he generally means the acceleration resulting from both forces, whereas in the foregoing discussion I used the symbol g and the term *gravity* to mean the acceleration due solely to the gravitational force.

The value for J_2^0, the amplitude of the harmonic which results from the polar flattening, is about 10^{-3}. All other amplitudes are of the order of 10^{-6} or less. That is, in looking for the other harmonics, we are working to an accuracy approaching 1 part per million.

Nature of Orbit Perturbations

If there were no gravity harmonics for the earth—that is, if the potential were $-K/r$—the orbit of a satellite would be an ellipse (8). Any departure from an exactly elliptical orbit, or the force that produces this departure, is called a perturbation. The context usually makes it clear whether the term *perturbation* refers to the motion or to the producing force. Every gravity harmonic produces a perturbation in the orbit of the satellite, and the nature of the perturbations depends upon the harmonic.

Consider, first, how the perturbation depends upon the value of m, the number of complete waves encountered in going around the equator. Since the earth rotates once per (sidereal) day under the satellite orbit, which is a curve almost stationary in space, the potential due to any harmonic S_n^m repeats itself, at any point on the orbit, every $(24/m)$ hours. That is, the perturbation due to the harmonic S_n^m is periodic, with period $(24/m)$ hours, regardless of the value of n. Thus, it is easy to separate the harmonics with different m by the periods of the perturbations they introduce into the orbit.

For $m = 0$, the simple formula for the period yields infinity; that is, the perturbation always continues in the same direction. Such a perturbation is called secular. For $m = 0$ and even values of n, the perturbation is indeed secular. For $m = 0$ and odd values of n, small effects make the perturbation periodic rather than secular, but the period is long, of the order of a thousand revolutions of the satellite.

Now consider the dependence of the perturbation upon n. Let us take two points with the same r but lying in exactly opposite directions from the center of the earth. If n is even, the harmonic S_n^m is of the same magnitude and sign at these two points. If n is odd, the potential is of the same magnitude but of opposite sign at these points. This symmetry property is independent of the value of m.

Because of this difference in symmetry, which depends upon the evenness or oddness of n, there is a profound difference in the effects of the respective harmonics on the satellite orbit. Harmonics with even values of n primarily affect the orientation of the orbit in space, and the phase of the motion (that is, the time at which the satellite will arrive at a particular point on the orbit). Harmonics with odd values of n primarily affect the eccentricity (that is, the departure of the orbit from a circle).

Consider sequences of harmonics with the same m and with n differing by multiples of 2—for example (*9*), the sequences S_1^m, S_3^m, S_5^m, . . . , or S_2^m, S_4^m, S_6^m, . . . (the sequences start with the first value of n that is not less than m; possible first harmonics in a sequence are S_1^0, S_1^1, S_2^1, S_2^2, S_3^2, S_4^3, . . .). Each member of one of these sequences gives perturbations, for a particular orbit, that are almost indistinguishable from the perturbations given by any other member of the same sequence but that are readily distinguishable from the perturbations given by any member of another sequence.

To distinguish harmonics of the same sequence, we take advantage of the fact that perturbations produced by different harmonics in a sequence depend in different ways upon the orbit parameters. Thus, each orbit studied yields an independent function of the harmonics in a given sequence, so that we can solve for as many harmonics in each sequence as we have orbits. By analyzing perturbations that are less significant than those described, it is possible, in principle, to solve for more harmonics, but usually with less reliability. A safer procedure is to use more orbits than harmonics, if possible.

Another goal in geodesy, besides that of studying the gravity field, is the goal of finding the geocentric coordinates of points on the earth's surface—that is, the coordinates referred to the center of mass as origin and to the spin axis as a preferred direction. Determining geocentric coordinates of the tracking stations is a necessary part of determining the gravity field in satellite geodesy. To see why, let us imagine, as a simple example, that a station is tracking a satellite in an orbit that passes over the poles, and that the station is actually 1 kilometer north of where it is believed to be. When the satellite is traveling north, it appears to arrive over the station late, and when it is traveling south, it appears to arrive early. Thus, there seems to be a perturbation in the phase of the satellite motion qualitatively similar to the perturbation produced by some harmonics, in this case, by S_n^2 where n is even. Fortunately, the effects of position errors and of the harmonics change in different ways with different orbits, and thus can be separated through the use of different orbits. Also, we are aided by the fact that the coordinate errors are fairly random from station to station, while the gravity perturbations are not. In principle, then, if we have a well-distributed set of stations, we can determine the number of gravity harmonics and simultaneously determine station coordinates, but the extra uncertainties introduced by errors for station coordinates make the use of a redundant number of orbits all the more desirable.

Satellites and Measurement Methods

Most of the tracking data used in satellite geodesy have come from three measurement methods: (i) radio interferometry, (ii) a method based on optical angles, and (iii) a method based on Doppler shift (*10*). Two methods of measuring the range from a station to a satellite have been developed (*11*), but these have not been much used, so far. Radar tracking, though extensive, has been mostly intended for simply keeping up with the enormous number of artificial objects in orbit (*12*), and the data have not been used much for geodesy.

The optical method has a precision of around 1 second of arc, or about 10 meters at average ranges. Radio methods are not as precise, being limited at present to a precision of 30 meters or more. However, the radio methods have a compensating advantage in that the measurements can be made at any time of day in any weather. Optical measurements can be made only when it is night at the station and when the sky is clear. Thus, the optical data always have a biased distribution.

The satellite 1962 $\beta\mu 1$, nicknamed Anna, was described in the press as the first geodetic satellite. This is correct only in a limited, though important, sense. The first satellite that yielded data useful in geodesy was the second satellite to be launched, 1957 β, and the first satellite launched primarily for geodetic purposes was 1961 $\alpha\eta 1$.

The important point about 1962 $\beta\mu 1$ is that it incorporated equipment to allow tracking by three independent means—optical angles, radio Doppler shifts, and radio ranging. Use of independent methods has two advantages: It increases the volume of data, and, more importantly, it allows simultaneous measurement by independent methods. Such simultaneous measurement is necessary if we are to have valid estimates of measurement accuracy. (The estimates given earlier in this section are my personal estimates, based upon estimates of the foreseen sources of error and upon internal consistency.)

The 1962 $\beta\mu 1$ also carries equipment to allow measurement of another important quantity, the time. Since the position of a satellite is continuously changing, time is an essential part of any data point, and the accuracy required in the measurement for time is quite stringent. If the accuracy of measuring position is dr and the satellite speed is v, the accuracy dt needed in measuring time in order not to distort the finding for position is dr/v or less. Since dr is 10 meters (or possibly less) and v is about 7000 meters per second, $dt \leqslant 10/7000$. Accuracy of 1 millisecond or better is a reasonable requirement.

This exceeds the accuracy with which time can be measured on a world-wide basis by the use of standard time signals, such as those from station WWV. To meet the need for accurate timing, the satellite carries a clock whose drift rate is less than 0.1 millisecond per day. This clock controls time signals of two types. It controls the times at which a flashing light, the one that is used in photography to yield optical data, is flashed. It also controls modulation pulses that are imposed on the radio transmissions from the satellite. By observing either of these types of time signal it is possible to measure time synchronously at any tracking sta-

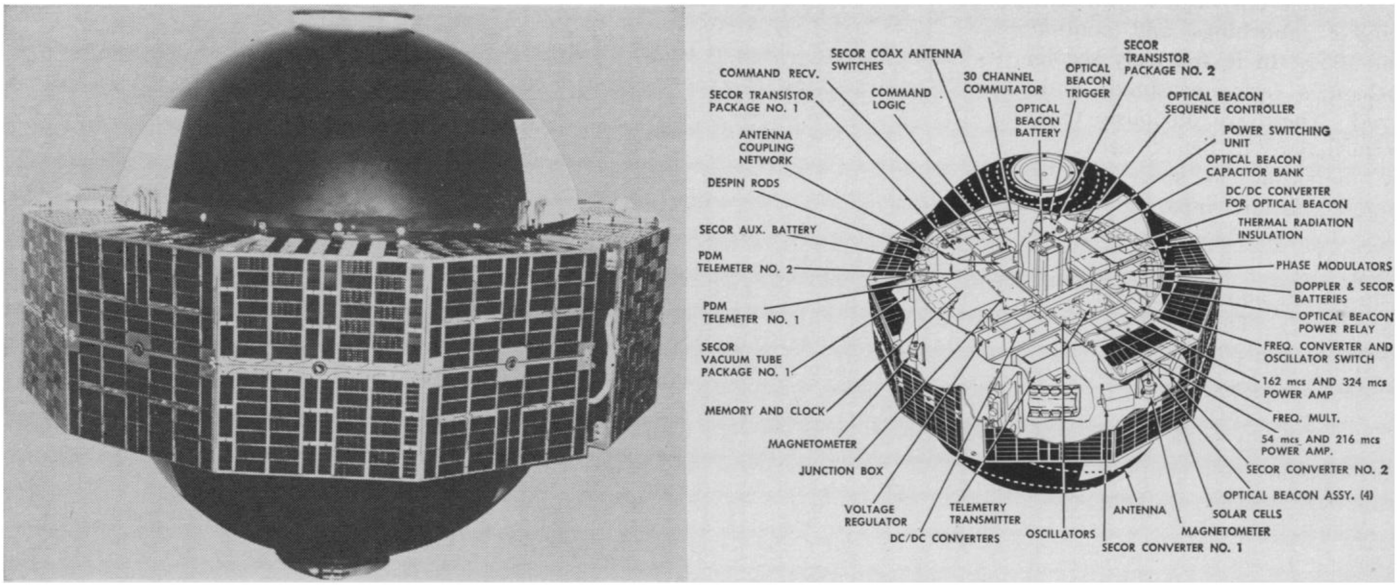

Fig. 1. Satellite 1962 $\beta\mu 1$ (Anna 1B). (Left) Exterior view, showing the solar cells and the paints used for thermal control. (Right) Cutaway view, showing the main components.

tion with an accuracy of the order of 0.1 to 0.2 millisecond.

Figure 1 shows a photograph of 1962 $\beta\mu 1$ before launching, together with a cutaway drawing showing its main components. Figure 2 is a photograph of the same satellite after launching. Optical angle measurements are obtained from this and similar photographs by interpolating the positions of the satellite between the positions of the stars.

Unfortunately, failure of a component shortly after launching prevented use of the radio-ranging method. The optical system and the Doppler system are still (on 5 November 1963) functioning well, though on an ever-decreasing duty cycle, caused by radiation damage to the solar-cell power supply.

Data Analysis

The method of analyzing tracking data to obtain geodetic results is familiar to anyone who has ever had to infer physical parameters from experimental data. The basic method applies to any type of tracking data or to any combination of types.

Tracking data consist of measurements of some quantity u at a set of times t_i. Let us suppose that u has been measured at N different times, so that i is an index running from 1 to N, and let us denote the body of tracking data by $u(t_i)$. In addition to the measurements, we must have a theory that gives u as a function of time and of the parameters of the situation. In the geodetic situation the set of parameters includes, as a minimum, the following: (i) the parameters that would fix the orbit if we knew the forces (there are six of these for each segment of an orbit used); (ii) the amplitudes of the gravity harmonics (there are as many of these as we feel justified in including; to work at an accuracy of 10 meters will probably require about 50); (iii) coordinates of the tracking stations (there are three for each station). There may also be other types of parameters needed, depending upon the situation. As an example, if a satellite is appreciably perturbed by the pressure of solar radiation, this perturbation depends upon the reflectance of the satellite, which is usually unknown, and hence must be included in the list of unknown parameters.

Let O denote the total set of orbit parameters needed, S the set of station coordinates, and G the set of gravity parameters. Let $f(t_i, O, S, G)$ denote the function given by theory for calculating the measured quantity at time t_i. We subtract the measurement $u(t_i)$ from the theoretical value $f(t_i, O, S, G)$ for each time t_i, square the result, and find the average value of the square. This yields a function

$$F = \frac{1}{N} \sum_{i=1}^{N} [f(t_i, O, S, G) - u(t_i)]^2,$$

which is no longer a function of the times but is a function of all of the parameters. The best estimate of the parameters is usually taken to be that set of values which gives F its minimum value.

If we like, we can multiply each term inside the summation by a weighting factor W_i. The value for W_i should depend upon our a priori estimate of the accuracy of each data point. If we expect all the data to be equally accurate, we take all of the W_i values to be unity; if we expect some to be more accurate than others, we give W_i for the "more accurate" data points a larger value. If the W_i values are not unity, we replace the N in the denominator in front of the summation by $\sum W_i$, summed over all i.

Function F is such a complicated function that it is not possible to find an explicit formula for the best estimate—that is, the values which give F its minimum value. Instead, it is necessary to proceed by iterations. We start with some estimated values of the parameters and compute F and all of its derivatives for the estimated values. From these calculations we make a new estimate of the parameters, and we repeat this process until no further significant improvement can be made in the value of F.

Nature of the Results

Although satellite geodesy is only 6 years old, it is old enough to have a history that can be divided into three eras. These eras, which are not sharply divided, are characterized by levels of difficulty in finding and analyzing orbit perturbations.

The first era begins with the early satellite launchings and continues to mid-1959. In it, the truly secular perturbations are measured and analyzed. These are obviously the easiest perturbations to detect, because, steadily increasing in size, they become larger than any apparent perturbations likely to result from errors of measurement. They arise from the harmonics with n even and $m = 0$ (the even zonal harmonics). The first significant result (*13*) was the discovery that the earth's flattening (proportional to J_2^0) differs by about ½ percent from the previously accepted value. Work on these perturbations still continues; the most elaborate analysis of them is probably a recent one by King-Hele, Cook, and Rees (*14*), who analyzed this sequence of harmonics through J_{12}^0 (*15*).

The second era begins with the announcement by O'Keefe, Eckels, and Squires (*16*) of the "pear-shaped" term (J_3^0). It is characterized by emphasis on the harmonics with n odd and $m = 0$ (odd zonal harmonics). After the even zonal harmonics, these are the easiest to find, because their perturbations, although oscillatory, have amplitudes of several kilometers. This sequence of harmonics has been analyzed through J_9^0 by Kozai (*17*) and, more recently, by Guier and me (*18*).

The third era saw the introduction of nonzonal harmonics ($m \neq 0$) and the simultaneous refinement of station coordinates. The first such harmonic used was J_2^2 (ellipticity of the equator), first estimated from satellite data by Iszak (*19*) and shortly afterward, by me (*20*). Estimation of these harmonics has been carried through all those with $n = 4$ by several groups of workers (*21*), along with determination of station coordinates. The average change in station coordinates required by satellite data is of the order of 500 meters. Kaula (*22*) has also estimated the harmonics through $n = 8$ for all m, by combining satellite data with surface measurements.

Analysis of all these parameters is continuing. Different groups obtain different answers, because they work with different satellites, different station configurations, different types of tracking data, and different distributions of data. In spite of all these differences, it is encouraging to see the results converging. The level of disagreement is about 1 to 2×10^{-7} for the zonal harmonics and and about 5×10^{-7} for the nonzonal ones.

Only the specialist is interested in tables of the harmonics, and I shall not give such a table. It is more interesting, and more significant, to describe the shape of the geoid as computed from the satellite results.

The dominant departure from sphericity is the flattening, resulting from J_2^0. The flattening is about 1 part in 298.2—that is, the difference between the polar and the mean equatorial radius is 6378.2 (the mean equatorial radius) divided by 298.2, or about 21.39 kilometers. It is convenient to describe the remaining features of the geoid by reference to an ellipsoid of revolution having this flattening.

Because only harmonics with $n \leqslant 4$ have yet been determined, the geoid computed from satellite data does not have a complicated structure. Kaula (*23*) has computed a geoid for each of the principal gravity determinations from satellite data and finds that the geoids so computed have certain important features in common. Relative to the flattened ellipsoid, the geoid seems to be characterized by having isolated hills and bowls, rather than, say, long ridges and valleys.

There are four hills, of average height about 40 meters. These are found in the western Mediterranean or in northwest Africa; near New Guinea; near or to the west of the "waist"

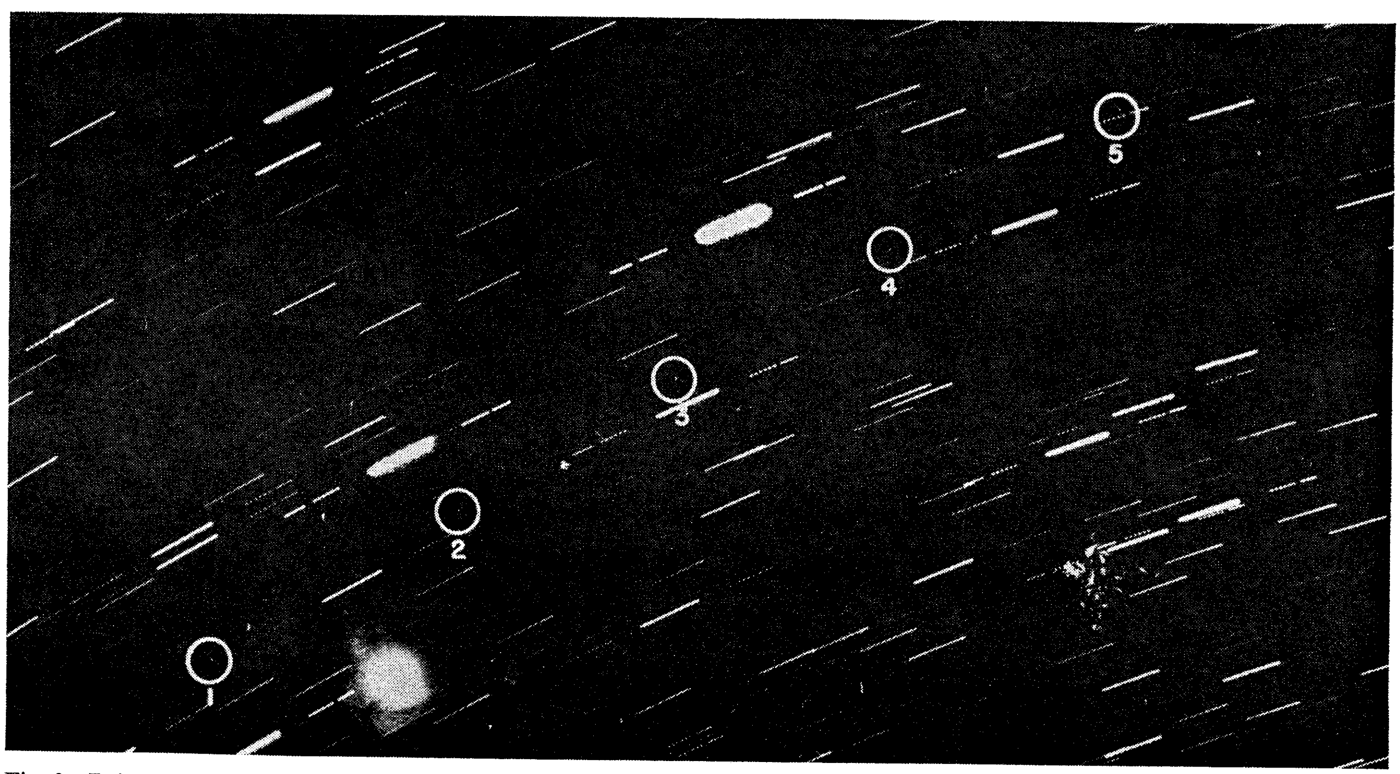

Fig. 2. Enlargement of photographic plate made during the strobe light test on 2 November 1962 to check the Air Force Cambridge Research Laboratories' optical system on Anna. The satellite was photographed at 0350 hours E.S.T., as it crossed the Boston area, by a BC-4, 300-mm FL camera located at Aberdeen, Maryland. The images on the camera plate averaged 50 to 55 microns in diameter. [Photograph by Coast and Geodetic Survey team; reproduced courtesy of Air Force Cambridge Research Laboratories, Bedford, Massachusetts]

of South America; and between southern Africa and Antarctica. Likewise, there are four bowls, of the same average depth. These are found near the tip of India, near Bermuda, between Hawaii and Japan, and near the Ross Sea off Antarctica.

Significance of the Results

A satellite is a new measuring tool in geodesy. Its greatest significance is that it gives us results which earlier tools had not given.

The type of gravity result that satellites can give most easily has already been described—namely, the harmonics with small values of n. These are equivalent to features in the geoid with continental dimensions. To study such features through surface measurement is difficult, because this requires accurate correlation of measurements over great distances. For harmonics with large values of n the situation is exactly the reverse. These harmonics give rise to fluctuations in gravity as large as those from harmonics with small n, but the fluctuations are of short range. These can be easily measured on the ground, but they hardly affect a satellite, which is necessarily a great distance away. Thus, surface and satellite geodesy neatly complement each other.

Satellites can also give accurate relative positions between points separated by oceans. Over small distances (less than the width of a continent) they do not give relative positions as accurately as surface measurements can, when the terrain is favorable for making such measurements. Over very difficult terrain, as in Antarctica, relative positioning by satellite may well be more accurate.

Accurate description of the geoid and of the earth's gravity field is important not only for accurate charting of the earth but also for learning something about the interior of the earth. Perhaps the most startling finding from satellite data is that the earth is not in hydrostatic equilibrium and departs from equilibrium by an amount that apparently exceeds the strength of its materials.

One theory that attempts to explain this departure from equilbruim is the theory that there are large convection currents in the mantle, the layer of the earth that extends from about the midpoint of the earth's radius to just beneath the surface. This theory predicts that harmonics with $n = 5$ should be the largest. There are not enough satellite data yet to make a definite test of this prediction, but there should be within a few years.

An interesting extension of the convection theory is the continental-drift (*24*) hypothesis, which attributes the distribution of the continents to the forces arising from convection. According to the hypothesis, the geoid should have long ridges and valleys. Moreover, the continents should be over the low places (valleys) of the geoid and the oceans over the high. The satellite results described earlier contradict these conclusions of the drift hypothesis. While the picture of the geoid will change considerably as additional results are obtained, I do not believe that it will change enough to come into agreement with drift hypothesis. Thus, the satellite results are an apparent obstacle to accepting the drift hypothesis, at least in its present form.

In summary, geodesy by satellite has already contributed greatly to our knowledge of the earth's figure, of its gravity field, and of processes that go on inside it. The future should see an increase in the application of satellite results in these fields.

References and Notes

1. Aristotle, in *Theories of the Universe,* M. K. Munitz, Ed. (Free Press, Glencoe, Ill., 1957), pp. 89–100. Aristotle does not give his source, but merely attributes the value he cites to "mathematicians who try to calculate the circumference."
2. For a description of Eratosthenes' method, see A. D. Ritchie, *Studies in the History and Methods of the Sciences* (Edinburgh Univ. Press, Edinburgh, 1958), p. 64. Eratosthenes goes on to estimate the circumference in stades, using 100 stades as a day's journey for a camel. Since there were several varieties of the stade in the ancient world, ranging from about 150 to 220 meters, and we do not know which one Eratosthenes used, there is ample room for argument about the accuracy of his answer. No one seems to have attacked the problem directly by measuring the average speed of a camel.
3. I. Newton, in his *Mathematical Principles of Natural Philosophy,* ed. 3 (originally published in 1726) [vol. 34 of *Great Books* (Encyclopaedia Britannica, Chicago, 1952), pp. 292–294], gives a historical summary of Richer's work and other early measurements.
4. Newton obtained a value, for flattening, of 1 part in 230, as compared with the currently accepted value of 1 part in 298.
5. W. A. Heiskanen, in *Encyclopaedia Britannica* (Encyclopaedia Britannica, Chicago, 1958), vol. 10, p. 127. The second set of measurements was the famous pair of surveys by (i) Bouguer and de la Condamine in Peru (1735) and (ii) de Maupertuis in Lapland (1736). Modern review of these measurements shows that they were not as accurate as was believed at the time, and in fact were not accurate enough to settle the question.
6. There are several types of latitude—geodetic, geocentric, and so on—which differ slightly from each other. The distinctions are not important here. The interested reader may consult, for example, G. Bomford, *Geodesy* (Oxford Univ. Press, Oxford, 1952), appendix I.
7. Analytic properties of the functions S_n^m can be found in many places; for example, in W. E. Byerly, *Fourier's Series and Spherical Harmonics* (Ginn, Boston, 1893), chaps. 5 and 6.
8. There are sources of perturbation other than the gravity harmonics. Examples are atmospheric drag, and forces due to the sun and moon. These must be included in any accurate study of orbits, but it is not necessary to describe them here.
9. The coefficients of S_1^0 and S_1^1 are zero if the origin of coordinates is taken as the center of mass of the earth. However, we will always have errors in the coordinates of the tracking stations, and some of these errors are qualitatively similar in their effects to coefficients of these harmonics. Hence we may include S_1^0 and S_1^1 in this qualitative discussion.
10. For discussion of the interferometry method, see *Minitrack System Training Manual,* issued by NASA (Government Printing Office, Washington, D.C.); on the Doppler method, see G. C. Weiffenbach, *Proc. I.R.E. (Inst. Radio Engrs.)* **48**, 750 (1960); on the optical method, see G. Veis, in *Proceedings of the First International Symposium on the Use of Artificial Satellites for Geodesy,* G. Veis, Ed. (North-Holland, Amsterdam, 1963), pp. 141–145.
11. R. C. Huntington, J. E. Kirch, K. Porter, *Motorola Military Electronics Div. Rept. W2719-2-1* (23 Nov. 1962); "Geodetic Secor," Cubic Corp. proposal No. P-59073A (21 Dec. 1959).
12. R. L. Easton and J. J. Fleming, *Proc. I.R.E. (Inst. Radio Engrs.)* **48**, 663 (1960).
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