Numerical Mathematics

Introduction to the Constructive Theory of Functions. John Todd. Academic Press, New York, 1963. 127 pp. Illus. $6.

This is the first volume in a new series of monographs, the *International Series of Numerical Mathematics*, which is printed by Birkhäuser in Basel, Switzerland. The printing is excellent, the book seems to be relatively free of errors, and its overall appearance is attractive.

The volume is primarily aimed at graduate students of mathematics. The author's goals are to present selected topics from the classical analysis of approximation, interpolation, and orthogonal polynomials and to provide mild propaganda for numerical analysis without actually doing anything really hard. Todd has given considerable attention to learning through doing, and to this end, he has provided about 100 problems. These, together with their solutions, take up all of 50 pages.

The book deals entirely with continuous real functions $f(x)$ defined on an interval $a \leq x \leq b$. In chapter 1 Todd reviews a few basic properties of roots of polynomials, continuity, and infinite series. In chapter 2 he presents the fundamental Weierstrauss theorem that each such function may be approximated uniformly by polynomials. Several proofs are sketched, and S. Bernstein's probabilistic proof is given in detail.

Chebyshev's theory of best polynomial approximations is presented in the third chapter. This and the short fourth chapter, on the Markoffs' theorems on bounds on the derivative, are most elegant. In the next chapter, Todd discusses the classical orthogonal polynomials of Chebyshev, Legendre, Laguerre, and Hermite; this chapter as well as chapter 6, on interpolation, and the final chapter, on approximate integration, should appeal to scientists. In chapter 7 the Bernoulli polynomials and the Euler summation formula are presented. Chapter 8, which is written in a different spirit, has the briefest possible introduction to the (modern) abstract space methods of analysis, with an application to Borel's main existence theorem in the Chebyshev theory discussed in chapter 3. There are many references in the text to induce the reader to go to the literature for further material.

In my opinion, the author has made a valuable contribution to the literature. The material is well chosen and exciting to read; one might only wish there were more of it. The problems that are provided will be invaluable to the serious student who wants to master the subject.

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Symmetry Principles

Representations of the Rotation and Lorentz Groups and Their Applications. I. M. Gel'fand, R. A. Minlos, and Z. Ya. Shapiro. Translated from the Russian edition (Moscow, 1958) by G. Cummins and T. Boddington. Pergamon, London; Macmillan, New York, 1963. xviii + 366 pp. Illus. $10.

It is hardly an exaggeration to say that the bulk of our knowledge of physical theories rests on the proper interpretation of symmetry principles of one sort or another. Although the group SU3 has lately come in to prominence for the study of nuclei and elementary particles, the rotation group has played a dominant role in the interpretation of atomic and molecular spectra, while the Lorentz group and the ideas of relativistic invariance have given their present forms not only to electromagnetic field theory but also to a large portion of high energy physics.

Although the applications of these two groups have been extensive, the publication of several articles by Gel'fand, Shapiro, and Yaglom, and the subsequent appearance of the English translation of this book that is based on those earlier articles, fills a badly felt need for a comprehensive and accurate account of these particular groups, an account which would follow the spirit of the contemporary mathematical theory of semisimple Lie groups, but which nevertheless would be sufficiently detailed with respect to practical matters. These latter include an analysis of the actual matrix representations with their generators and matrix elements, recursion relations, Clebsch-Gordan coefficients, and similar topics. It is a testimony to the extent of this subject that in more than 350 pages of concrete results concerning these groups, the authors can barely hint at the extensive mathematical lore underlying their treatment, nor do they venture very far into such aspects as the *n-j* coefficients. One judges that the application which most interests them is the determination of the possible differential equations which possess rotational or Lorentz invariance; in fact their treatment of the transformation properties of vector and tensor fields was one of the novel features of their original papers.

With regard to the mechanics of publication, the translation is quite readable and in keeping with English terminology, although on page 269 the "Klein-Gordon" equation became the "Clebsch-Gordan" equation. The bibliography lists "M. Rouse, Multipole Fields" (undoubtedly this should be M. Rose), and no publisher is mentioned. The overall quality of the printing is a bit poor, and this is bound to detract from the otherwise warm regard in which this book will certainly be held.

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Politics and Water Resources

The Politics of Water in Arizona. Dean E. Mann. University of Arizona Press, Tucson, 1963. xviii + 317 pp. Illus. $6.50.

Arizona is a state in which development has proceeded sufficiently rapidly relative to the available water supply that its water problems are as acute as those of nearly any other state in the Union. Owing to the fact that, in the past, the principal use of water was for irrigation, and that the areas where the water has been utilized were geographically separated from the mountain zones where the water originated, surface water resources were developed fairly early in the state's history.

As irrigation expanded in the fertile valley and desert portions of the state, and far more arable land became available than there was water to serve it, intensive ground water development ensued. This resulted in a total draft on the ground water reserve which was far in excess of recharge, a situation which still persists and one for which no completely satisfactory solution has appeared on the horizon.

Many of the problems developed because the institutional and legal framework for governing water development was inadequate to prevent the serious problems that now confront the state. In addition, basic data and hydrologic investigations were, and are, insufficient in scope and quantity to forecast, in any satisfactory way, the consequences of various alternative actions that might have been taken.

Mann's book is concerned primarily with the history and status of these administrative and legal results as they are affected by the desires and the consequent pressures of various parts of the total community. But, in addition, the author explains the programs, philosophies, and plans of the various state and federal bureaus whose work impinges on water questions. The politics of the situation are explained in discussing the history that has led to the present situation; but in some respects the title is a little misleading, for the present political attitudes and pressures are not explained with the same force and clarity used in describing the sequence of events in the past. This difference between the explanation of past events and the present situation is understandable, but some of the more interesting intricacies of the present political framework are perforce side-stepped by the author. This can also be seen in the discussion of the attitudes and policies of the state and federal bureaus, the description of which comprises nearly half of the volume.

On the whole, however, Mann is amazingly forthright. In dealing with a situation as complicated as the one that exists in Arizona, no author can be completely forthright and still maintain objectivity; many interpretations of present attitudes and policies could be made, but they depend on the point of view of the observer.

In view of the fact that this is the first book to deal with the politics of water, it is an extremely informative and a highly commendable venture, one that other States should emulate, for such books are much needed to educate the public about water problems. The author conveys to the reader a considerable insight into these complicated matters but nevertheless leaves him with a feeling that this is an objective analysis which attempts to present various points of view in as fair a light as possible. The book is highly recommended to all persons interested in water problems, both administrative personnel and scientific hydrologists, as well as to community groups interested in the use and development of resources.

It is obvious that a book of this kind cannot be completely up to date, for events proceed apace even while it is being written. However, it is quite distracting to find that portions of the book are up-to-date with respect to the Supreme Court decision on the California-Arizona suit [373 U.S. **546** (1963) Ariz. vs. Calif.], which was concerned with the use of Colorado River water, but that other parts of the book, which deal with the same subject, are not. It would have been better to present the situation as it was before the Supreme Court decision was rendered, or to rewrite the book uniformly so that all portions of the volume reflect the Court decision.

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Mathematical Surveys, No. 9

Linear Approximation. Arthur Sard. American Mathematical Society, Providence, R.I., 1963. xii + 544 pp. Illus. $16.80.

The theory of approximation discusses the problem of finding, for a given complicated function $f(x)$, a simple function $P(x)$, which can replace $f(x)$ with a small error. In the simplest case, $f(x)$ is a continuous function for $a \leq x \leq b$, while $P(x)$ is an algebraic or a trigometric polynominal of degree n. Originally approximation was identical with interpolation, but it became an independent discipline after Chebyshev (1958) and Weierstrass (1885) found their fundamental results about approximation by polynomials. At first only approximation of individual functions was studied, but the introduction of electronic computers was in part responsible for a change in attitudes. In present-day computation, one often has to deal with a great number of functions at the same time. It is then not possible or profitable to pay attention to the individual features that some of them may possess. Thus, mathematicians were led to the study of approximation questions for large classes of functions. For this purpose, Kolmogorov successfully introduced the notions of width and of entropy of sets of functions.

Where does the Sard's book fit into this development? In his monograph, Sard presents in great detail the results of his research. They lie on the borderline between theoretical and practical approximation. One of Sard's purposes is to find the "best" approximation formulas. His main idea is to come back to interpolation, but with fewer restraints than in the classical formulas. The error is a linear functional which contains some free parameters. Sard minimizes its norm in a certain Hilbert space. This gives the parameters and the approximation itself. The result is best for a class, not for individual functions. Many formulas are thus obtained; their connection with spline interpolation (de Boor, Schoenberg) was discovered only recently and, thus, could not be included.

The second part of the book deals with functions of several variables. Sard is able to avoid some of the difficulties inherent in the problem by adopting spaces of functions for which the different data (the partial derivatives involved) are in a certain sense independent. A further major part of the book presents the probabilistic theory of Wiener-Kolmogorov, augmented by Sard's own results. The practical usefulness and theoretical interest of Sard's formulas are clear.

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Mathematics

Homology. Saunders MacLane. Springer, Berlin; Academic Press, New York, 1963. x + 422 pp. Illus. $15.50.

Here is a masterly, comprehensive treatment of an exciting area in contemporary mathematics. From the most primitive, intuitive ideas of "boundary," there has evolved in the past 70 years a sophisticated mathematical machinery called "homology." Like the differential calculus, homology arose as a collection of techniques, later strengthened and enlarged by an intensive study of the underlying theory. The present book offers the reader a working knowledge of homology—in theory, in practice, and in relation to many other branches of mathematics.

The author covers this wide range of material thoroughly, bringing out the local color of the various fields he traverses. Algebraic structures are his raw materials; categories and functors