Mathematicians in Industry — The First 75 Years

They have increased 12-fold each 25 years, and their relation to management is changing.

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In 1888 a local group called the New York Mathematical Society was organized at Columbia University. Two years later it took on national character and became the American Mathematical Society. From this event we may quite reasonably date the beginning of mathematical research on this side of the Atlantic; for, prior to this time, there were not only no important centers of discussion and no important mathematical journals but there were essentially no creative mathematicians.

In the decade or so preceding 1890, however, it became quite usual for young men from the more affluent families to complete their education abroad. Those who were interested in mathematics, or in the broad analytical aspects of physical sciences which were then called "natural philosophy," came under the influence of the lively groups of mathematicians in England and Germany and returned with a new vision of scholarship and a determination to emulate it at home. It was one of these men, T. S. Fiske, who was the prime mover in organizing the group at Columbia, and he is generally recognized as the father of the American Mathematical Society. This, as I said, was in 1888, and the Society is now passing its 75th anniversary.

It was in 1888, also, that a young German student of mathematics was to take his doctor's degree at the University of Breslau. His dissertation had already been accepted, and only certain minor formalities remained to be completed. He never got the degree. He was a socialist agitator whom the police just then decided to close in upon, and, having been mysteriously forewarned, he fled in the best story-book fashion, in the middle of the night, to Switzerland. The following year he came to America, where he secured employment in what is now the General Electric Company. He was, so far as I know, the first mathematician in the modern sense of the word to be employed in industry. So 1963 comes within a year of being also the 75th anniversary of the first employment of mathematicians in industry.

The man in question was Charles Proteus Steinmetz, and the title of his thesis was, "On Involutory Self-Reciprocal Correspondences in Space which are Defined by a Three-Dimensional Linear System of Surfaces of the *n*-th Order." It has, for its time, a remarkably modern sound.

Steinmetz met Fiske very soon after his arrival, and the two became close friends. He became a charter member of the American Mathematical Society and participated actively in its affairs during those early years. He presented several original papers, which are referred to in volumes 1 and 2 of the *Bulletin*.

We may also date the beginning of industrial research from this same period. (Not the beginning of the institution now known to us as the "industrial research laboratory." That is an organizational concept which came later.) Prior to this time, American industry—in fact, the industry of the world—had been flourishing through inventive genius of the purely Edisonian type. But the problem of transmission in telephony, and the problems of transmission and generation in the power industry, raised questions of a more subtle and analytical type and required a more scientific approach. I know of no single event which heralded the birth of such research, but it certainly began within a very few years of 1890.

With only negligible errors of approximation, therefore, we can say that in America the year 1888 marked the beginning of mathematical research, the beginning of industrial research, and the first employment of mathematicians in industry.

The growth in all three areas has been phenomenal, but to the hard-headed businessman of that day, today's use of mathematicians in industry would no doubt be the most surprising of all.

Growth

In a study published in 1940 (*1*) I made a serious attempt to estimate the number of professional mathematicians working in industry and came up with the figure 150.

This of course involved a matter of definition. In 1940, as today, many industrial physicists, chemists, and engineers had considerable mathematical training and ability and were using it in their work. It would have been foolish to count all these as mathematicians. I resolved the difficulty by counting the members of the American Mathematical Society who clearly had industrial or government employment. My thought in selecting this criterion was that those who had sufficient interest to belong to a society devoted exclusively to creative mathematical research could properly be defined as mathematicians.

This study was made in 1939, and the membership list on which it was based was probably that for 1938. There is a double coincidence in the fact that 1938 is just a quarter of a century ago and just a half century after 1888. So I thought it might be interesting to fill in the other quarter-century points.

The latest membership list is that for 1961–62, and from this, by a sampling process, I obtain a count of 1800. The closest list to 1913 available to me was that for 1915; from this, depending on whether or not some doubtful cases are included, I get 11 or 15.

Using these figures, and recording a "1" for Steinmetz opposite 1888 (which is within 1 year of the correct date), I arrive at the data of Table 1. The figures in column 3 are interesting. The actual count for 1963 is exactly 12 times that for 1938; that is, there was

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a 12-fold increase in the last quarter century. If we extrapolate backward at the same exponential rate, we get a figure of 12 for 1913 and of 1 for 1888—results which are both remarkably close to the actual counts. In other words, this exponential rate of growth has proceeded with amazing consistency for three-quarters of a century. If anyone is daring enough to infer from this that the same rate will continue for the next half century, he will conclude that there will be 22,000 mathematicians with nonacademic employment in 1988 and 270,000 in the year 2013.

I do not offer this as a serious prophecy. On the contrary, both numbers, and particularly the one for A.D. 2013, impress me as fantastic (*2*). They do serve to indicate, however, the tremendous thrust of the social forces which have been injecting mathematicians in large numbers into industrial and government laboratories, an environment which only a few generations ago would have been judged inhospitable. I think they justify a closer examination of what these forces have been, and of the nature of the role mathematicians have played.

Science and Industry—1888–1913

If we examine the 25-year periods defined in Table 1, we can observe a very significant progression in the character of scientific thought as we pass from one to another. Not that there were abrupt transitions between them. The transitions were gradual and, to the men experiencing them, largely imperceptible. But the accumulated change over each quarter century is great enough to give character to the periods.

In industry the first period (1888–1913) can be characterized as one of Edisonian invention and handbook engineering. Looking back from the modern age, where science pervades almost every aspect of our lives, it is difficult for us to appreciate how primitive engineering was. Perhaps two anecdotes may bring this into focus.

The first concerns Steinmetz, and relates to the 1890's. One of the two principal things upon which his fame rests is his effort to awaken electrical engineers to the use of complex quantities in alternating-current theory. Mathematically, what he was teaching was not news. Mathematicians had been using the method for decades—perhaps for centuries. And in his *Theory of Sound,* which was published in

Table 1. Data on mathematicians in industry. X is the number of members of the American Mathematical Society with addresses in government or industry. X' is computed from the exponential growth function $X' = e^{(Y-1888)/10}$, which gives a 12-fold increase every 25 years.

Year (Y)	X	X'
1888	1	1
1913	11–15	12
1938	150	150
1963	1,800	1,800
1988		22,000
2013		270,000

1877, Lord Rayleigh had used it extensively in discussing the sinusoidal oscillations of mechanical systems. Nor was the method difficult. It depended basically on two simple facts: that $e^{ix} = \cos x + i \sin x$, and that the sum of two solutions of a linear differential equation is also a solution. But the vast majority of electrical engineers found it incomprehensible, and were completely mystified that the square root of minus 1 should have anything to do with electric currents.

By 1913, when this quarter century ended, the idea was beginning to catch on. But the mathematics used by engineers, even in the universities, was still primitive, as my second anecdote will show.

In 1913 I was an instructor in mathematics at the University of Wisconsin. My classes were composed entirely of engineers, and the course material was selected and presented with the engineering student in mind. Yet no semester went by, either in my classes or in those of my fellow instructors, without someone's asking, "What is all this good for?" It revealed a hostile and foolish attitude, but an understandable one. For in the College of Engineering—which was reputed to be one of the most progressive in the country—the deflection of beams was still being taught with no reference whatever to the calculus, and most of alternating-current theory was also.

Viewed against the contemporary state of natural science, this primitive state of the world of industry is understandable. For, prior to 1900, physical science was entirely Newtonian and chemistry was entirely empirical. Scientifically speaking, engineering had had little to feed upon, and it is perhaps a little surprising that in 1915 there were even a dozen members of the American Mathematical Society with industrial or government addresses. But, with the new century, things began to happen in the more esoteric fringes of physics which would revolutionize first physics, then chemistry, and in the end engineering and society as well. The vast growth in employment of mathematicians in industry is one aspect of this revolution.

Let us note a few of these events. The quantum hypothesis was formulated in 1901. The vacuum tube (*3*) was invented in 1907. The special theory of relativity was published in 1908, and the general theory, in 1914. Millikan's measurement of the charge on the electron, which gave the first solid proof of the existence of a class of identical electric particles, was in 1912. Bohr's paper on the hydrogen atom appeared in 1913, and Mosley's on atomic numbers, in 1914. I believe these can properly be regarded as the beginnings of modern chemistry. Biochemistry began at about the same time; Fischer's discovery of the protein building blocks was also in 1913.

This is an exceedingly impressive list of discoveries. I think we can properly say that by 1913 atomic physics and atomic chemistry had been born. It was still true, of course, that the geometry of chemical bonds was inaccessible, but the diffraction of x-ray crystals would be observed by the Braggs shortly before 1915, and thus the mechanism for studying chemical geometry would be provided.

Science and Industry—1913–1938

The period from 1913 to 1938 was equally exciting, though in a quite different way.

In physics, I think it can best be described as a period of consolidation of the non-Newtonian concepts which had been so recently born, and exploitation of the great possibilities of electronic measurement. It was the period when quantum mechanics and electronic physics were the center of excitement.

Going on at the same time, of course, were the experimental studies of cosmic rays and the rather advanced thinking about atomic nuclei which we now know to have been the beginnings of particle physics, but these were somewhat out of the mainstream.

It was chemistry, rather than physics, which moved explosively ahead during this period under the impetus of the clear-cut structural ideas which grew out of the work of Bohr, Mosley, and the Braggs.

This was also a period of tremendous change in industry, which discovered that profits could be derived from scientific research, as distinguished from engineering development. Research laboratories sprang up by the hundreds, many in industries in which management was ill-equipped to direct them or even to understand the nature of their activities. If we call the preceding period in industry the age of the engineer, we may, not too inaccurately, call this the age of the scientist. Not that the engineer no longer had a function to perform; such an idea would be quite false. His function was, in fact, enlarged because of industrial scientific research, and his productivity was increased. This was the age of scientific research only in the restricted, but tremendously important, sense that scientific research was now being consciously organized and exploited by industry.

I had the good fortune in 1916 to be employed by one of the earliest and best of the industrial research laboratories, when it was almost newborn. I was, moreover, hired not as an engineer or physicist but as a mathematician. Thus I was in a favorable position to observe how the opportunities for mathematicians were affected by this awakening to the industrial value of scientific research.

For one thing, the presence of other scientists made the environment less awkward for the mathematician. More important, however, was the contrast between the attitude of the engineers on the job and that of my student engineers only a few years before. As the scientific method replaced Edisonian cut-and-try, the engineer's methods of design became more and more analytical. The practical engineer got his mathematics where he could—often through self-education, sometimes by seeking the help of his long-haired colleagues. But he did not question its value. Instead, a curious reverse situation arose in which the engineers, conscious of their own limitations, tended to give a high rating to anyone with mathematical training and interests who was reasonably articulate, regardless of his true mathematical ability.

Indeed, if my observation is sound, the industrial mathematician has seldom, if ever, been without honor in his own country. I have seen a weak scholar and a strong one honored equally because their associates were incapable of appraising their work critically. And I have seen a good mathematician and his associates equally frustrated when their working relations had not been properly defined. But I can recall no instance where a talented mathematician who attempted to cooperate with his engineering associates was not rewarded with their respect and appreciation.

Here again an anecdote from my own experience may illustrate the point. Between 1931 and 1933, the depression years, the professional staff of Bell Telephone Laboratories was reduced by about one-third. Each department head was required to select, on a pro-rata basis, the individuals to be separated from his department; then a conference of department heads was held in which these selections were discussed and adjusted. The experience was a very grievous one for all concerned. But in the end, not a single member of the Mathematical Research Group was among those released; whenever the name of a mathematician was mentioned, the conference group decided that he could not be spared, and one of the other supervisors supplied a substitute.

The First Mathematical Research Department in Industry

The problem presented by the mathematician in industry was not then, and I do not think it ever has been, lack of appreciation. It was lack of understanding. Basically, it arose from the fact that the interests of mathematics and industry are almost antithetical. For the function of industry is to produce things and services and to make a profit in the process, while, in the whole spectrum of science, the discipline which is least concerned with things or profits and most dedicated to ideas for their own sake is mathematics.

Once this simple (one might say "obvious," except that it went for some years unnoticed) fact had been stated, certain consequences followed at once.

1) It became apparent that when a mathematician was practicing his trade —that is, so long as he was dealing only in ideas—he was working outside the mainstream of the industry's activities. A mathematician should not logically be responsible for any stage of the development process. Instead, he should function as a consultant to those who are.

2) Some men are by temperament interested exclusively in ideas and some in things, but there are many who are deeply interested in both. Hence a good mathematician may also be a good engineer. In an industrial environment there is a strong tendency to assign such a man the duties and responsibilities of an engineer, and when this is done his availability as a mathematician is reduced. This process, if allowed to continue, drains off the mathematicians who are interested in things as well as in mathematics and leaves behind as consultants only those who are not, and who for that reason may be the least effective consultants.

With these ideas in mind, a Mathematical Research Department was created at Bell Telephone Laboratories in the late 1920's. I have described its organization and functions in some detail elsewhere (*1*). For the present discussion I need only say that it was explicitly understood that no project responsibility would be assigned to the men in this group, but that they would be consultants to the project engineers.

This concept was a wise one for its time, and though the department was not large, it was highly respected and performed a valuable service. Indeed, it was this group which, as I have said, the project supervisors preserved intact throughout the depression at the penalty of losing good men from their own staffs.

I am proud of the achievements of the men who were in it. Among these were John Carson and Sergei Schelkunoff, two outstanding experts in electromagnetic propagation and antenna theory; George Stibitz, whose early (1937) ideas regarding modern automatic computers have never been adequately recognized; Hendrik Bode, who contributed so much to the mathematical theory of feedback control, and who is now a vice president of the Laboratories; Claude Shannon, who originated information theory; and Walter Shewhart, the father of quality control.

Science and Industry—1938–1963

It is a little harder to tag the science of the final period, from 1938 to the present, because we are too close to it. We lack what my inimitable friend W. O. Baker calls "the exquisite acuity of hindsight." But some things of a very fundamental sort can be distinguished.

For one thing, I think it is safe to call it the era of particle physics. There have, of course, been important advances in other areas, notably solid-state physics, but none have the social impact of controlled and uncontrolled nuclear power.

Maybe we should also call it the biochemical age, for the progress through chemistry toward an understanding of life processes and heredity has already been spectacular, and one has the feeling that tomorrow will be even more exciting.

There is also information theory, which in effect quantizes all intelligible thought and may lead to consequences not now foreseeable.

And, finally, there is something else which I find difficult to name. The electronic computer is the most ubiquitous example, but a somewhat special one. I refer to our emerging ability to control systems of all kinds, from the simplest machine to the most involved spacecraft, not through rigid procedures but through flexible processes akin to thought, where the only invariant is the underlying system of logic. Whatever this ability may be called, it is something new and important, and because of it the world will never be the same again. It may well be that 50 years from now particle physics, biochemistry, and this thing to which I have not given a name will stand out as the great scientific achievements of the period.

When we turn our attention to industrial research, the situation is not so confusing. Here I believe the most important evolution has been the team. Even today it is quite clear that without the team approach we could not have effectively exploited the better materials and better understanding which science has given us. With these materials we now make systems whose complexity exceeds that of the recent past by several orders of magnitude, but designing them often requires more skill and knowledge than a single man can give, and much more time than he is given. The team transcends these limitations by linking several or many brains into a single interacting agency—an agency which is as necessary for the final accomplishment as are the materials or the scientific theories.

The industrial research team has introduced problems of management from without, and of communication within, which are quite as revolutionary as were the problems accompanying the initial introduction of industrial research laboratories. I will not discuss these in detail. It is, however, important to note that, simultaneously with the emergence of the team and to some extent because of it, the place of the mathematician in industry has become more complex and sometimes more central. To understand why this is so, it will be helpful to digress for a moment and consider the nature of mathematics itself.

The Parts of Mathematics

The word *mathematics* does not connote a simple entity with a single facet. It relates to a useful art with at least four separate and important aspects. More specifically, mathematics is an art, a language, a tool, and a reckoning.

As an art, it deals with postulates and their logical consequences, the system of logic being a part of the postulates. In this aspect it is creative and has no necessary connection either with the physical world or with other parts of mathematics. As a simple example, two of the postulates of matrix algebra are

$$C = A \cdot B \quad \cdot\supset\cdot \quad c_{ij} = \Sigma\, a_{ik}\, b_{kj},$$
$$C = A + B \quad \cdot\supset\cdot \quad c_{ij} = a_{ij} + b_{ij}.$$

A mathematical artist contemplating these postulates might wonder what the consequences would be if they were replaced by two other postulates,

$$C = A \cdot B \quad \cdot\supset\cdot \quad c_{ij} = a_{ij}\, b_{ij},$$
$$C = A + B \quad \cdot\supset\cdot \quad c_{ij} = \Sigma\, (a_{ik} + b_{kj}),$$

which, in effect, interchange the rules for forming the elements of the sum and the product. The new mathematics thus created might or might not turn out to be interesting; as an entirely separate matter, it also might or might not turn out to be useful. But whether or not anything of permanent value results, the process is the *art* of mathematics.

One cannot, of course, practice the art of mathematics without using its *language*. The converse, however, is not true. Consider, for example, the pair of equations

$$l\frac{\partial i}{\partial t} + ri = \frac{\partial e}{\partial x},$$

$$k\frac{\partial e}{\partial t} + ge = \frac{\partial i}{\partial x}.$$

They express certain physical laws which define the propagation of electromagnetic disturbances in a one-dimensional medium. What they say is not mathematics; it is physics or engineering. They are either the speech of a physicist using the language of mathematics or the speech of a mathematician who is talking about physics.

When physical laws have been expressed in mathematical language, it becomes possible to make use of known mathematical facts and arrive at the physical consequences of the laws. To state it more simply, we could solve the foregoing differential equations and thereby derive formulas for the current and potential. In doing so, we would be using mathematics as a *tool*.

Finally, those procedures by which accounts and inventories are kept are also a part of mathematics, which I have called *reckoning*. It is a very important part, because, for one thing, without it no monetary system of trade could exist, and only primitive barter would be possible.

The order in which I have stated these aspects of mathematics is significant. From the point of view of the professional mathematician, they proceed from the most sophisticated, and therefore the most important, to the least. From the standpoint of industry, the order of importance would be reversed, for the art has no necessary connection with things and is therefore of little immediate value, whereas the language and the tool clearly have value, and, without reckoning, trade as we know it could not exist.

Mathematics may also be characterized by what we speak of as its method, and here there are three principal attributes. It is precise, concise, and rigorous—precise, because the discipline of mathematics requires that all terms be well defined; concise, because redundancy is recognized and avoided; rigorous, because logical principles are part of its clearly stated postulates, and are adhered to.

In industrial research, of course, the principal interest centers in the language and the tool. These are indispensable to science in general, and to industrial research in particular, precisely because they are precise, concise, and rigorous. This is why so many of today's scientists and engineers acquire such a high degree of skill in mathematics.

The Mathematician's Role in Industry Today

What role, then, does the mathematician play in today's industrial research, and how does it differ from his role in the past?

To begin with, his former role of consultant has not been eliminated because higher mathematical skills are now prevalent among project scientists. On the contrary, project scientists are now able to use his assistance more effectively. Organizationally this implies that mathematical research departments, such as the one I described earlier, are as logical now as they were some decades back.

In addition, two new functions have evolved from the scientific and organizational changes I have discussed.

The electronic computer has brought with it a greater need for experts on numerical analysis than existed a quarter- or a half-century ago. This is a rather specialized role, but nevertheless an important one.

Even more significant is the role which is growing out of the team concept. Clearly, such a team cannot function effectively without free and unambiguous communication between the experts of which it is composed. These may be from many disciplines, each with its own special language and special mode of thought. But today all, or almost all, have a fair training in mathematics, and many are highly skilled. Mathematics therefore provides the precise and unambiguous common language by means of which members of the team can communicate with each other and in terms of which they can formulate the problem with which they are concerned.

I am speaking particularly of the early phase in the evolution of a complex system which is often called "systems research." Here the exact definition of terms, and the rigorous formulation of questions and of logical answers to them, are necessary before the nature of the problem can be clearly understood and the requirements for its solution adequately formulated. The language of mathematics helps greatly in doing these things. Here also the mathematician, with his more severe schooling in the manipulation of abstract ideas, can be of very real service. Later, when the problem is thoroughly understood and requirements are set, and when the reduction of these requirements to "hardware" begins, his services are likely to be less needed. It is in the earlier phase, when general principles—sometimes unfamiliar ones —must be examined critically and without semantic or logical ambiguity, that he will be in greatest demand.

In this role the mathematician is no longer a consultant. He is a working member of the team, and if the problem is sufficiently analytical he may have a very central part indeed.

This is not a speculative suggestion. It has been repeatedly borne out by recent experience. For example, one of the leaders of the team which studied the ICBM interception problem and set the requirements for the Nike missile was H. W. Bode—one of the early members of the Mathematical Research Department at Bell Telephone Laboratories, who succeeded me as its director. And a leading member of the team which set the development requirements for nuclear warheads was Brockway MacMillan, a mathematician trained at M.I.T., who is now Undersecretary for Air.

To play this role well, however, it is not sufficient that the mathematician think straight and know the language of mathematics; for language alone does not suffice for intelligent conversation. Many English-speaking people cannot carry on an intelligent discussion of economic theory, though they know all the important words, and many mathematicians could not understand a discussion of atomic radiation, though they may be familiar with matrix algebra.

To be an effective member of the team, the mathematician must also understand the basic principles of the various disciplines which he is expected to discuss. He should be, in other words, the sort of man who a century ago was known as a natural philosopher —a man who had a keen analytical mind, adequate mathematical training, and a broad and sympathetic interest in a wide range of natural phenomena. There is already a clear need for such men, and, in my opinion, this may well become the most important role the industrial mathematician of the next generation will play (*4*).

Educational Requirements

If this judgment is correct, we may well ask where these men are to come from.

Those I have known have often been physics or engineering undergraduates who developed a love for mathematics and majored in it for their doctor's degrees. This was true, for instance of Bode, MacMillan, Schelkunoff, and Shannon, among the men whose names I mentioned earlier. This is not hard to understand, since such men have interest both in ideas for their own sake and in things.

But while this is an effective pattern of education, the reverse—an undergraduate major in mathematics followed by a Ph.D. in science—does not have equivalent value. The reason is that the ingredient which the mathematician adds to the team is his greater emphasis on precise definition of terms and rigorous logical analysis, an emphasis seldom obtained outside the graduate mathematics curriculum.

There is, then, a legitimate need for graduate mathematical training which is both sound mathematically and sympathetic to the phenomena of the real world. Whether we call it applied mathematics or something else makes little difference. Its object is to train men who can be—in the sense I have explained—natural philosophers. This requirement runs exactly counter to the oft-stated view that "mathematics is concerned solely with symbols and the logical relations between them, and has no concern for their significance in the world of phenomena." That statement is true of the *art* of mathematics, but not of its other three parts. And it becomes both false and very dangerous when, as is sometimes done, the statement is made, not of "mathematics," but of "a mathematician."

We need, I think, in the universities and the Mathematical Society as well, a broader concept of the social value of mathematics. Not a de-emphasis of the art, for that would be a tragedy, but a greater pride in the full scope of the discipline and a stronger interest in its social values. Such a concept would greatly facilitate the training of the "natural philosophers" which industry will increasingly need in the foreseeable years ahead.

References and Notes

1. T. C. Fry, "Industrial Mathematics," a report for the National Resources Planning Board published as part of "Research—A National Resource," vol. 2, a House of Representatives document, 77th Congress.

2. I must add, however, that in 1940 I would have found a prophecy of 1800 for today almost equally fantastic. This is obvious from the grossly inadequate estimate of future growth which I included in the study.
3. The inclusion of a gadget, the vacuum tube, in this very impressive array of scientific advances may perhaps appear incongruous. But the vacuum tube is not only a valve and an oscillator, a modulator and a power amplifier; it is also a measuring instrument which profoundly broadened the scope of scientific experiments. I believe the social impact of the millions of tubes which have been used in scientific measurement greatly outweighs that of the billions which have been used in the communications and other industries.
4. Mathematicians of this kind will, of course, not be the most numerous.