1 atm, 123.2×10^{-6} ohm-cm (*1*), and Bridgman's pressure-resistance data for bismuth (*8*). The inversion of the temperature coefficient is found at all pressures between 15 and 35 kb in bismuth I.

Figure 2 shows resistivity-pressure results obtained at 77.4°K between 15 and 60 kb. From 15 kb, the resistivity rises by a factor of 6 with increasing pressure. At the I-III transition at 42 kb the resistivity is 740×10^{-6} ohm-cm, much too high for a true metal. The drop in resistivity at higher pressures shows the presence of the metallic bismuth III phase.

In semiconductors, the resistance R follows the formula,

$$R = A \exp (E_g/2kT)$$

where E_g is the energy gap and A is the first term of a series.

The energy gaps resulting from the application of the simple formula are shown in Fig. 3, where they are plotted as a function of pressure. The series of points taken with bare anvils are from heating experiments only. For the two sets of experiments with the copper block method, heating and cooling values were averaged at each pressure. The gap values obtained by cooling are about 0.003 ev higher than those obtained by heating.

The energy gap in Fig. 3 is 0.006 ev at 15 kb and it rises to 0.018 ev at 35 kb. A linear extrapolation of the energy gap to 1 atm yields a value of -0.004 ev. This negative value, if it were correct, would represent the overlap energy of the valence and conduction bands. Other estimates of the overlap energy at 1 atm are much larger, ranging from -0.012 to -0.035 ev (*17*) at low temperatures. The semiconductor formula from which the energy gaps are derived is accurate only if the gap is greater than kT. Since kT is 0.007 ev at 77.4°K and several gap values are smaller, the formula does not apply accurately, and the gap values must be considered approximate. The smallness of the energy gap explains why work around room temperature fails to show the semiconducting properties of bismuth. At room temperature, the gap is less than kT and bismuth appears to be a metal. To observe this energy gap, the temperature must be reduced to the point where the gap is of the order of more than kT.

It is possible that the negative temperature coefficient of resistance in bismuth is due to a decrease in carrier mobility as temperature decreases at a given pressure. However, no such mechanism is known in metals. Moreover, no mechanism affecting mobility in metals has been found to yield a resistivity as large as 740×10^{-6} ohm-cm. The negative temperature coefficient is more likely due to the creation of an energy gap as the pressure increases. The decrease in carrier concentration with increasing pressure found in work at room temperature on Hall coefficients (*11*) supports this view. The decrease in overlap energy to 1 kb (*12*) offers further support.

It is interesting to note the wide range of behavior shown by the element bismuth upon compression to 30 kb. In phase I, it is poorly metallic to semiconducting, but not superconducting (*18*). Phases II and III, formed under pressure, are both purely metallic, with III and possibly II both being superconducting (*19*).

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References and Notes

1. D. T. J. Hurle and S. Weintroub, *Proc. Phys. Soc. London* **76**, 163 (1960). This resistivity value, used here to represent a polycrystalline sample, is the average of readings parallel and perpendicular to the principal axes in a single crystal of bismuth.
2. H. Jones, *Proc. Roy. Soc.* (*London*) *Ser. A* **147**, 396 (1934).
3. For a partial list of references, see C. F. Gallo, B. S. Chandrasekhar, P. H. Sutter, *J. Appl. Phys.* **34**, 144 (1963).
4. G. K. White and S. B. Woods, *Phil. Mag.* (8th ser.) **3**, 342 (1958).
5. C. S. Barrett, *Acta Met.* **7**, 810 (1959).
6. A. L. Jain, *Phys. Rev.* **114**, 1518 (1959).
7. S. Tanuma, *J. Phys. Soc. Japan* **16**, 2349 (1961).
8. P. W. Bridgman, *Proc. Am. Acad. Arts Sci.* **72**, 157 (1938).
9. L. S. Kan and B. G. Lazarev, *Zh. Éksperim. i Teor. Fiz.* **34**, 258 (1958), translation in *Soviet Physics-JETP* **7**, 180 (1958).
10. J. Hatton, *Phys. Rev.* **100**, 681 (1955).
11. A. I. Likhter and L. F. Vereshchagin, *Zh. Éksperim. i Teor. Fiz.* **32**, 618 (1957), translation in *Soviet Physics-JETP* **5**, 511 (1957).
12. N. V. Brandt and V. A. Venttsel, *Zh. Éksperim. i Teor. Fitz.* **35**, 1083 (1958), translation in *Soviet Physics-JETP* **8**, 757 (1959).
13. Obtained from Central Research Laboratories, American Smelting and Refining Company, South Plainfield, N.J.
14. Performed by G. Shalimoff in this laboratory.
15. P. W. Bridgman, *Proc. Am. Acad. Arts Sci.* **81**, 165 (1952); P. W. Montgomery, H. Stromberg, G. H. Jura, G. Jura, in preparation.
16. P. C. Souers and G. Jura, *Science* **140**, 481 (1963).
17. B. Abeles and S. Meiboom, *Phys. Rev.* **101**, 544 (1956); G. A. Ivanov, *Fiz. Tverdogo Tela* **1**, 1600 (1959), translation in *Soviet Physics-Solid State* **1**, 1461 (1960); also references (*3*) and (*6*).
18. N. Kurti and F. Simon, *Proc. Roy. Soc.* (*London*) *Ser. A* **151**, 610 (1935).
19. P. F. Chester and G. O. Jones, *Phil. Mag.* (7th ser.) **44**, 1281 (1953); N. B. Brandt and N. I. Ginzburg, *Fiz. Tverdogo Tela* **3**, 3461 (1961), translation in *Soviet Physics-Solid State* **3**, 2510 (1962).
20. Supported in part by the U.S. Atomic Energy Commission.

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Phase Separation in Suspensions Flowing through Bifurcations: A Simplified Hemodynamic Model

Abstract. *In the laminar flow of a suspension of neutrally buoyant spheres through a bifurcation, intended as a simplified macroscopic model of the flow of blood, the concentration in the side branch is generally lower than in the main branch, and is affected primarily by the ratio of discharges in the two branches, the concentration upstream, and by the branch size.*

The occurrence of lower concentrations of red cells in the side branches of bifurcations in the smaller vessels of the circulatory system is a well-known phenomenon (*1*), which has also been reproduced in vitro (*2, 3*). The phenomenon is of significance both in physiology and in fluid mechanics, but there have been few systematic studies of the subject.

To assess the influence of some of the factors affecting the phenomenon, we have conducted experiments with suspensions of rigid, neutrally buoyant, plastic spheres in a mixture of 87 percent water and 13 percent glycerine (by volume), in steady laminar flow through bifurcations formed by a side branch leaving a straight main tube, as shown in Fig. 1*A*. In three bifurcations the diameter (D_2) of the side branch was the same as that of the main branch (D_1 = 6.32 mm) both upstream and downstream of the bifurcation, and the side branches formed angles of 45°, 60°, and 90°, respectively, with the downstream portion of the main branch; in the fourth bifurcation the side branch was at an angle of 45°, but had a narrower diameter ($D_2 = 1/2D_1$ = 3.16 mm). The two values of the ratio D_2/D_1 (1.0 and 0.5) cover the common range for branchings in the circulatory system.

The bifurcations were drilled through Lucite blocks and presented a sharp edge at the junction of the branches. The main branch was always vertical, and was attached to the downstream end of a vertical Lucite tube of the same diameter (6.32 mm) and 190 cm in length. The flow was supplied to the apparatus from a tank in which the liquid surface was maintained at a constant level; the water and glycerine mixture was returned to the tank by a pump. The flow rates in the two branches of the bifurcation were varied by means of resistances; the liquid and solid flow rates were measured volumetrically, and the possible error in the determination of the concentrations did not exceed 3 percent.

The particles were almost uniform in size, with an average diameter of 600 μ, corresponding to a tube-to-particle-size ratio of approximately 10, a value selected as being in the range where the plasma skimming phenomenon has been observed to occur with blood (*3*). The concentration upstream of the branches, C_T, was varied from 5 percent to 45 percent. In general, for a given concentration, a two- to threefold range of upstream flow rates Q_T was investigated, in the overall range of 2 to 12 cm^3/sec. For the 45°, $D_2 = D_1$ bifurcation, and for concentrations C_T up to 30 percent, some exploratory experiments were also performed at much higher flow rates (up to 43 cm^3/sec). Characterization of the flow in terms of a Reynolds number would be ambiguous, given the non-Newtonian properties of the suspensions (except at low concentrations); however, some indicative values based on an extrapolated viscosity of the suspensions at infinite shear rate are: 48 for $Q_T = 2$ cm^3/sec and for $C_T = 45$ percent; 1000 for $Q_T = 12$ cm^3/sec and for $C_T = 5$ percent; and 1560 for $Q_T = 43$ cm^3/sec and for $C_T = 30$ percent.

Other factors being equal, we have found that within the range of conditions investigated, the following statements hold true.

1) The ratio C_2/C_1 of the concentration in the side branch to that in the main branch downstream of the bifurcation is considerably less than unity for low values of the corresponding ratio Q_2/Q_1 of the flow rates, but increases with Q_2/Q_1. Only in exceptions, when Q_2/Q_1 is considerably greater than unity, is $C_2/C_1 > 1$. Typical results are shown in Figs. 1 and 2.

2) The ratio C_2/C_1 is generally higher for the narrower branch at 45° ($D_2/D_1 = 0.5$) than for the larger one ($D_2/D_1 = 1$) (Fig. 1*A*), the difference becoming, however, less marked at the higher values for C_T.

3) For branches of equal size ($D_2/D_1 = 1$), C_2/C_1 is not greatly affected by bifurcation angle (Fig. 1*B*).

4) The ratio C_2/C_1 is only slightly affected by changes in the upstream flow rate Q_T (as exemplified by Fig. 2 where results at the extremes of a tenfold range in Q_T values are compared).

5) The ratio C_2/C_1 decreases, reaches a minimum, and then increases as C_T increases from zero toward the maximum value investigated. In Fig. 3, a typical curve shows the variation of C_2/C_1 with C_T and Q_2/Q_1 for the 45° bifurcation with narrower side branch. Similar results are obtained with the other bifurcations.

These results are consistent with a qualitative description of the phenomenon as being the resultant, basically, of two effects: the tendency for the layers of lower momentum (that is, of lower velocity, which are closer to the boundary) to be more easily deflected into the branch, and the presence of concentration gradients in the flow upstream.

On account of both friction and geometric factors, the layers deflected into the side branch, rather than being circular cylinders concentric with the axis of the main branch, will have a more complex configuration favoring the region closer to the branch. This is essentially in agreement with the observations of Barnett and Cochrane on the deflection of a dye filament into a bifurcation (*4*) (which, however, do not provide complete information as to the contour of the layers deflected, and were performed with a Newtonian fluid).

Although radial drift of neutrally buoyant, rigid, spherical particles has been observed at sufficiently high Reynolds numbers, still little is known of the distribution of radial concentrations, except for very dilute concentrations (*5*). In our tests, the presence of a particle-deficient (but by no means uniform) layer near the boundary could be clearly discerned at the higher concentrations; at the lower concentrations the layer became more difficult to define, and at all concentrations the particle distribution did not appear to be greatly affected by changes in flow rate.

In any event, a nonuniform concentration profile in the flow upstream, of which the particle-deficient layer may be but one manifestation, will lead, in general, to a concentration of the fluid deflected into the side branch different from that of the fluid in the straight branch. The increase in C_2/C_1 and Q_2/Q_1 can accordingly be explained with the average concentration of the deflected layers becoming increasingly close to that of the rest of the flow as a progressively larger portion of the upstream flow is deflected into the side branch. Similarly, the trend toward larger C_2/C_1 ratios at larger values of C_T can be explained with the nearly uniform distribution of concentrations that must prevail upstream at the larger concentrations.

The larger C_2/C_1 values in the 45° bifurcation when the side branch is narrower would indicate the deflection into the narrower branch of layers having relatively greater concentration. Barnett and Cochrane have shown that with a highly viscous Newtonian fluid (glycerine), the contributions to a 45° side branch usually come from farther across the main branch when the side branch is narrower. A similar pattern for the suspensions used in our experiments, coupled with a monotonic increase in concentration from the wall to the centerline, would then immediately explain the higher C_2/C_1 values for the smaller branch; however, the lack of complete information on the deflection pattern and the distribution of radial concentrations could make this an over-simplification.

The small effect of the angle of bifurcation on C_2/C_1, when $D_2 = D_1$, also is consistent with Barnett and Cochrane's observation that the angle has little influence on the deflection pattern, but the effect of the angle in other configurations, for example, in bifurcations where $D_2 \neq D_1$, and in Y-shaped bifurcations, remains to be investigated.

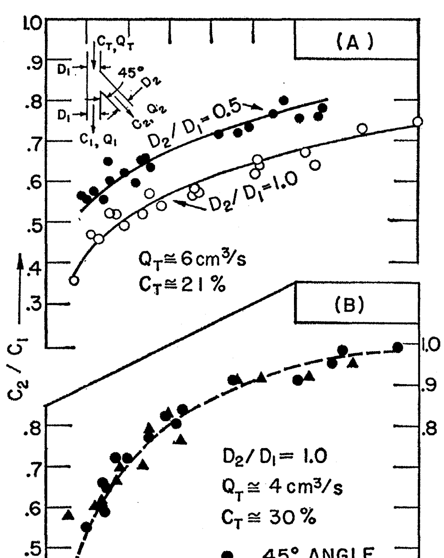

Fig. 1. Examples of variation with relative flow rate, Q_2/Q_1, of the ratio C_2/C_1 of the concentration in the side and main branch of bifurcations. (*A*) Effect of the ratio D_2/D_1 of the diameters of the side and straight branches. (*B*) Comparison of two bifurcations with different side-branch angles.

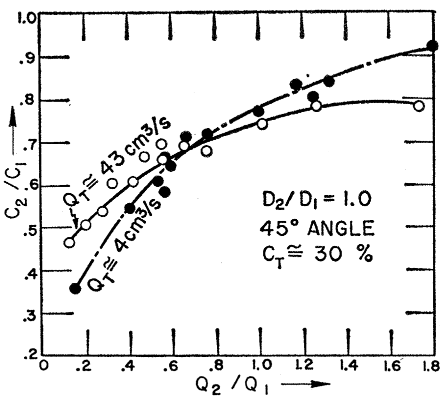

Fig. 2. Effect on C_2/C_1 of a tenfold change in the rate of flow upstream, Q_T.

Finally, the limited influence of the upstream flow rate Q_T on C_2/C_1 can be explained as due to a limited influence of Q_T both on the deflection pattern, a fact observed by Barnett and Cochrane, and on the concentration distribution, a fact as yet hypothetical but consistent with our qualitative observations of the limited variation of the particle distribution with flow rate, and also consistent with quantitative observations of the peripheral layer in the flow of blood in vitro (*6*). At values of Q_T below the range investigated, C_2/C_1 must be expected to be affected by Q_T, because, as shown by Goldsmith and Mason (*7*), a rigid particle would not exhibit a tendency to drift away from the boundary [but a small layer of lower concentration would still be present near the boundary due to purely geometric effects (*8*)].

Application of these findings to the circulatory system requires caution and more understanding of the comparative behavior of suspensions of rigid versus highly deformable particles (such as the erythrocytes) in a shear field.

With deformable particles, drift away from the boundary occurs at very low rates of flow (*7*), whereas, with rigid particles, rates of flow higher by several orders of magnitude are necessary, other factors being equal. Thus, to achieve a system in which the particles behave as in the circulatory system, a model in which rigid particles are used must first be made to operate at sufficiently high Reynolds numbers, as did that in our experiments, so as to insure the occurrence of phase-separation effects. The distribution of the cross-sectional concentrations must also be similar, likewise the variations in concentration with flow rate. Lack of information on both these phenomena, for suspensions of rigid particles as well as for blood, prevents an assessment of the extent to which our model fulfilled these requirements. In first approximation, however, the concentration in the side branch would be governed primarily by the peripheral layer, and the rigid-particle model should, therefore, provide at least a qualitative understanding of the phase-separation phenomena at branchings of similar configuration in the circulatory system. In particular, our experiments concur with the existing observations of the plasma skimming phenomenon, in showing that the hematocrit ratio in the side branches of the smaller vessels should be appreciably lower than in the straight branch downstream of the bifurcation, since, in general, the ratio Q_2/Q_1 will be smaller than unity—that is, in the range where phase separation effects are the most pronounced. With branchings in series, the process could thus lead rapidly to very low hematocrit ratios. Within the range of validity of the results obtained with the model, the phenomenon should be affected only to a minor extent by the angle of the branching and by changes in the circulation rate, but would be sensitive to the relative diameter of the side and straight branches.

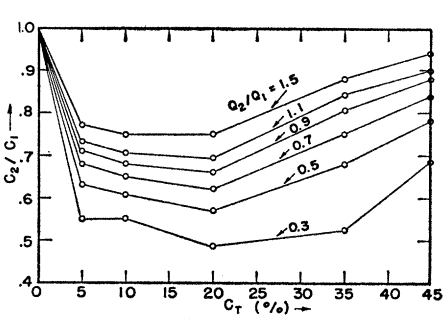

Fig. 3. Variation of C_2/C_1 with upstream concentration C_T, for the narrower side branch ($D_2/D_1 = 1/2$). The values shown are averages over a range of Q_T values.

In vivo, the pulsating nature of the flow, and the degree of sharpness of the bifurcation edge must also be considered. However, since pulsations are less pronounced in the smaller vessels, where plasma skimming is of importance, and since in our experiments the angle of bifurcation was shown to have limited influence, neither of these factors should alter the basic phenomenon.

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References and Notes

1. L. C. Bayliss, in *Deformation and Flow in Biological Systems*, A. Frey-Wyssling, Ed. (North-Holland, Amsterdam, 1952), p. 393; H. P. Wright, in *Textbook of Physiology*, J. F. Fulton, Ed. (Saunders, Philadelphia, 1956), p. 509; J. R. Pappenheimer and W. B. Kinter, *Am. J. Physiol.* **185**, 377 (1956); E. W. Emery, A. H. Gowenlock, A. H. Riddel, D. A. K. Black, *Lancet* **I**, 298 (1958); J. Fourman and D. B. Moffat, *J. Physiol.* **158**, 374 (1961).
2. L. E. Gelin, *Acta Chir. Scand.* **122**, 287 (1961).
3. L. E. Gelin, *Biorheology* **1**, 119 (1963).
4. C. H. Barnett and W. Cochrane, *Nature* **177**, 740 (1956).
5. G. Segrè and A. Silberberg, *J. Fluid Mech.* **14**, 115, 136 (1962).
6. G. Bugliarello and J. W. Hayden, *Trans. Soc. Rheol.* **7**, 209 (1963).
7. H. L. Goldsmith and S. G. Mason, *Nature* **190**, 1095 (1961).
8. J. Vand, *J. Phys. Chem.* **52**, 300 (1948); A. D. Maude and R. L. Whitmore, *Brit. J. Appl. Phys.* **7**, 98 (1956).
9. Supported in part by grant HE-05557 of the National Heart Institute, U.S. Public Health Service. The assistance of A. Brandt, A. Toro, B. E. Imhoff, D. A. DeVos, and D. O. Seiple in performing the experiments is gratefully acknowledged.

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Volatile Growth Inhibitors Produced by Aromatic Shrubs

Abstract. *Root growth of* Cucumis *and* Avena *seedlings is inhibited by volatile materials produced by leaves of* Salvia leucophylla, S. apiana, *and* Artemisia californica. *The toxic substance may be deposited when dew condenses on affected seedlings in the field.*

The role of metabolic products in various forms of growth inhibition has been reviewed extensively since 1950 (*1*). We have in progress an analysis of inhibition of annual herbs by *Salvia leucophylla, S. apiana, S. mellifera, Artemisia californica*, and other aromatic shrubs. The localization of the toxic principles is a first step in their identifications and in the determination of ecological relationships.

The spacing and patterning of annual grassland species in and about colonies of *Salvia leucophylla* and *Artemisia californica* in the Santa Inez Valley, Santa Barbara County, California, suggested this study. Numerous isolated patches of both shrubs occur surrounded by grassland. Annual grasses and forbs are usually absent from the interiors of such patches and there is frequently a zone of bare soil extending 60 to 90 cm beyond the canopy of the shrub branches. Beyond this, a zone of differential inhibition may extend 2 to 6 or even 9 m. In the proximal part of this differential zone an almost pure