duction, but with a terrestrial K/U ratio this corresponds to a uranium concentration of 3.3×10^{-8} g/g. If the integrated heat production for a chondritic earth is a limit in earth heat balance models, then a mantle and crust with a uranium concentration of 5.2×10^{-8} g/g and a terrestrial K/U ratio is permissible. This range in uranium concentration of from 3.3×10^{-8} g/g to 5.2×10^{-8} g/g is consistent with the values estimated by Hoyle and Fowler (*1*) for solar system nucleosynthetic material. This agreement of course cannot be taken as direct evidence that the earth is composed of unfractionated solar system material.

The relative contribution of K^{40} to the heat production is drastically different for chondritic material and for material with a terrestrial K/U ratio. At present, K^{40} produces 59 percent of the heat generated in chondrites while for material with the terrestrial K/U ratio, K^{40} produces only 15.8 percent of the heat. A material having a uranium concentration of 2.255×10^{-8} g/g and a terrestrial K/U ratio of 10^4 would have the same present production rate as chondrites, but over the past 4.5 aeons would produce only 63 percent as much heat as chondrites. This lower heat production is due to the relatively short half-life of K^{40} (1.3 aeons) as compared with the longer half-lives of U^{238} (4.5 aeons) and Th^{232} (13.7 aeons). The fundamental difference between models in which potassium is the principal heat source and ones in which uranium plus thorium dominate is that in the former case the potassium releases almost all its energy in the first 2 aeons (*20*). This early release of heat provides a longer time scale for escape, whereas in uranium- and thorium-dominated models, the heat production is distributed more uniformly over earth history. The ratio of the rates of heat production 4.5 aeons ago to the present rate for chondrites is 8.2, while for the terrestrial model the ratio is 4.5. In the past 3 aeons the production rates for the terrestrial ratio model have changed only by a factor of 2.2. Because of the more uniform heat production, the terrestrial ratio model may require early differentiation so that the heat sources are near the earth's surface (outer few hundred kilometers) during most of earth history. Such a near surface concentration of heat sources would permit a close approach to matching the rate of heat production with loss; this circumstance is of course not permitted in a chondritic model where the largest part of the potassium is buried in the lower mantle, unless, of course, the conductivity is extremely high.

The detailed development of the model for the terrestrial ratio requires a consideration of the differentiation of the earth, the mechanism of heat transport, and the conditions prevailing during the early stages of development of the earth. Preliminary calculations have been carried out by MacDonald and will be published elsewhere.

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Negative Temperature Coefficient of Resistance in Bismuth I

Abstract. *Measurements of the electrical resistance have been made on bismuth I between 15 and 35 kilobars at temperatures between 77.4° and 120°K. Above about 150°K, the temperature coefficient of resistance is positive, as in a metal; below 150°K, the coefficient becomes negative, as is characteristic of semiconductors. On the basis that bismuth is a semiconductor, the energy gap, calculated by the exponential resistance formula, is 0.006 ev at 15 kb with a steady rise to 0.018 ev at 35 kb. At higher pressures, bismuth I is transformed into a metallic modification with the normal temperature dependence of the resistance. The energy gap in bismuth I is not visible at room temperature because thermal excitation populates the conduction band and metallic behavior is the result. From available evidence the observed behavior is due to an energy gap rather than to a decrease in carrier mobility.*

Bismuth I, the form found under normal conditions, has long been considered a semi-metal. The resistivity, 123.2×10^{-6} ohm-cm (*1*), is very high for a metal. From a study of bismuth-tin alloys, Jones (*2*) suggested that all five of bismuth's valence electrons lie in a single energy zone (valence band). A slight overlap into a higher zone (conduction band) gives the metallic conduction found in pure bismuth. Since Jones introduced this hypothesis, many experiments—the deHaas-van Alphen effect, cyclotron resonance, galvanometric effects, and the anomalous skin effect (*3*)—have verified the

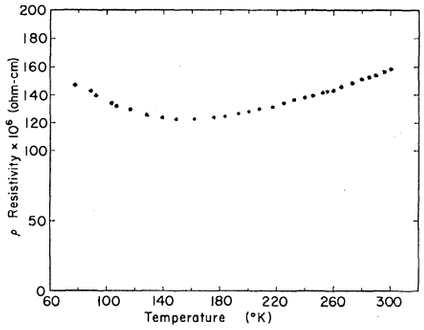

Fig. 1. Resistivity-temperature curve for bismuth at 15 kb.

low number of conduction electrons, somewhere between 10^{-2} and 10^{-5} per atom. However, bismuth at 1 atm shows the positive temperature coefficient of resistance characteristic of a metal (*4*). Bismuth also shows no crystallographic transition upon cooling to 4.2°K (*5*).

Some alloy work indicates that bismuth might become a semiconductor upon a reduction of volume. Jain (*6*) found that the temperature coefficient of resistance became negative for bismuth alloys containing more than 5 percent antimony. Using the exponential resistance formula for semiconductors, he found an energy gap which rose to 0.012 ev at 12 percent antimony and then dropped at higher concentrations, bismuth becoming a metal again at 40 percent antimony. Tanuma (*7*) measured both conductivity and Hall coefficients on the full range of bismuth-antimony alloys and found that the lowest net carrier concentration at 15 percent antimony with carrier mobilities decreased slightly from that of pure bismuth. The lowest carrier mobilities

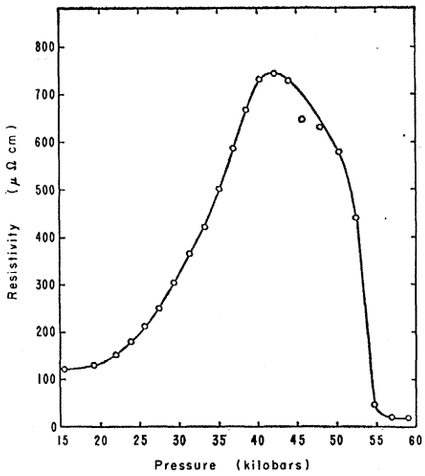

Fig. 2. Resistivity-pressure curve for bismuth at 77.4°K.

were at 88 percent antimony and the highest carrier concentration was in pure antimony. Since the minimum mobility occurs in an alloy with metallic properties, the carrier concentration determines the electrical properties. The minimum carrier concentration at 15 percent is equivalent to creating an energy gap, and the alloy is semiconducting. Both Jain and Tanuma measured the lattice parameters and found that they decreased with increasing antimony. Thus addition of antimony to bismuth crudely approximates the compression of the lattice by pressure. Semiconducting behavior might therefore occur in both cases.

The results from experiments on the direct compression of bismuth confirm that the application of pressure apparently decreases the metallic properties. At room temperature, the resistivity increases 50 percent with pressure to 25.5 kb, where the crystallographic transition to the metallic bismuth II occurs (*8*). Experiments on bismuth to 1.7 kb at 77.4°K (*9*) and to 5 kb at 4.2°K (*10*) show the same increase of resistance with pressure observed at room temperature. Work on the Hall effect at room temperature (*11*) shows a steady decrease in carrier concentration as the pressure is raised from 1 atm to the I-II transition. Magnetic susceptibility work to 1 kb (*12*) shows a decrease with pressure of the Fermi energy, and consequently of the overlap of valence and conduction bands. This decrease of the metallic properties of bismuth has led to the prediction by Kan and Lazarev that, somewhere in the bismuth I phase, there will be semiconducting behavior (*9*). It is not clear that Kan and Lazarev actually found such semiconducting behavior. By cooling and compressing bismuth, we have found that the temperature coefficient of resistance changes from positive to negative owing to the probable presence of an energy gap.

The bismuth (*13*) was supposedly 99.999 percent pure. Spectral analysis (*14*) showed five parts per million each of copper, magnesium, and silicon. The magnesium may have come from the spectrometer arc.

The bismuth was extruded into wire 0.007 cm in diameter. The Bridgman anvils and the method (*15*) of mounting the samples and the pressure measurement and calibration, as well as the resistance and temperature measurement, have been described (*16*).

The electrical resistance of bismuth was studied at pressures between 15

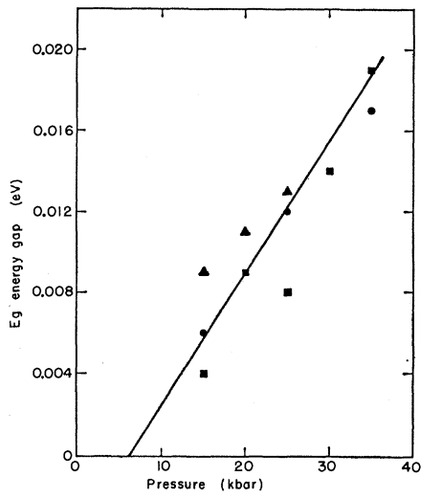

Fig. 3. Energy gap of bismuth as a function of pressure. Circles refer to experiments without copper blocks; squares and triangles refer to experiments with copper blocks.

and 35 kb over the temperature range 77.4° to 120°K. In one series of experiments the exposed anvils were cooled to the temperature of liquid nitrogen at three different pressures. As the liquid nitrogen evaporated, there was a temperature difference of as much as 20°K between the top and bottom anvils. Measurements during cooling cannot be taken since the liquid nitrogen lowers the temperature of the sample too abruptly.

To better equilibrate the system, heavy copper blocks were fitted snugly around the anvils and the blocks backing up each anvil. All cracks were plugged with Duxseal. With this method the liquid nitrogen cannot reach the anvils as it cools the massive blocks. Both heating and cooling experiments can be made with this system, and the temperature difference between the top and bottom anvils is thus decreased to 2° to 6°K. Two series of runs at various pressures were made by this method. With either method, the temperature of the bismuth is assumed to be the mean between the top and bottom anvil temperatures.

Figure 1 shows the resistivity of bismuth as a function of temperature at a pressure of 15 kb. From room temperature to 155°K, the resistivity drops with decreasing temperature, as is expected of a metal. But from 155°K down to 77.4°K, the resistivity rises with decreasing temperature, as is characteristic of a semiconductor. The measured resistance of bismuth has been converted to resistivity by using the value at room temperature and

1 atm, 123.2 × 10^{-6} ohm-cm (*1*), and Bridgman's pressure-resistance data for bismuth (*8*). The inversion of the temperature coefficient is found at all pressures between 15 and 35 kb in bismuth I.

Figure 2 shows resistivity-pressure results obtained at 77.4°K between 15 and 60 kb. From 15 kb, the resistivity rises by a factor of 6 with increasing pressure. At the I-III transition at 42 kb the resistivity is 740 × 10^{-6} ohm-cm, much too high for a true metal. The drop in resistivity at higher pressures shows the presence of the metallic bismuth III phase.

In semiconductors, the resistance R follows the formula,

$$R = A \exp (E_g/2kT)$$

where E_g is the energy gap and A is the first term of a series.

The energy gaps resulting from the application of the simple formula are shown in Fig. 3, where they are plotted as a function of pressure. The series of points taken with bare anvils are from heating experiments only. For the two sets of experiments with the copper block method, heating and cooling values were averaged at each pressure. The gap values obtained by cooling are about 0.003 ev higher than those obtained by heating.

The energy gap in Fig. 3 is 0.006 ev at 15 kb and it rises to 0.018 ev at 35 kb. A linear extrapolation of the energy gap to 1 atm yields a value of −0.004 ev. This negative value, if it were correct, would represent the overlap energy of the valence and conduction bands. Other estimates of the overlap energy at 1 atm are much larger, ranging from −0.012 to −0.035 ev (*17*) at low temperatures. The semiconductor formula from which the energy gaps are derived is accurate only if the gap is greater than kT. Since kT is 0.007 ev at 77.4°K and several gap values are smaller, the formula does not apply accurately, and the gap values must be considered approximate. The smallness of the energy gap explains why work around room temperature fails to show the semiconducting properties of bismuth. At room temperature, the gap is less than kT and bismuth appears to be a metal. To observe this energy gap, the temperature must be reduced to the point where the gap is of the order of more than kT.

It is possible that the negative temperature coefficient of resistance in bismuth is due to a decrease in carrier mobility as temperature decreases at a given pressure. However, no such mechanism is known in metals. Moreover, no mechanism affecting mobility in metals has been found to yield a resistivity as large as 740 × 10^{-6} ohm-cm. The negative temperature coefficient is more likely due to the creation of an energy gap as the pressure increases. The decrease in carrier concentration with increasing pressure found in work at room temperature on Hall coefficients (*11*) supports this view. The decrease in overlap energy to 1 kb (*12*) offers further support.

It is interesting to note the wide range of behavior shown by the element bismuth upon compression to 30 kb. In phase I, it is poorly metallic to semiconducting, but not superconducting (*18*). Phases II and III, formed under pressure, are both purely metallic, with III and possibly II both being superconducting (*19*).

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Phase Separation in Suspensions Flowing through Bifurcations: A Simplified Hemodynamic Model

Abstract. *In the laminar flow of a suspension of neutrally buoyant spheres through a bifurcation, intended as a simplified macroscopic model of the flow of blood, the concentration in the side branch is generally lower than in the main branch, and is affected primarily by the ratio of discharges in the two branches, the concentration upstream, and by the branch size.*

The occurrence of lower concentrations of red cells in the side branches of bifurcations in the smaller vessels of the circulatory system is a well-known phenomenon (*1*), which has also been reproduced in vitro (*2, 3*). The phenomenon is of significance both in physiology and in fluid mechanics, but there have been few systematic studies of the subject.

To assess the influence of some of the factors affecting the phenomenon, we have conducted experiments with suspensions of rigid, neutrally buoyant, plastic spheres in a mixture of 87 percent water and 13 percent glycerine (by volume), in steady laminar flow through bifurcations formed by a side branch leaving a straight main tube, as shown in Fig. 1*A*. In three bifurcations the diameter (D_2) of the side branch was the same as that of the main branch (D_1 = 6.32 mm) both upstream and downstream of the bifurcation, and the side branches formed angles of 45°, 60°, and 90°, respectively, with the downstream portion of the main branch; in the fourth bifurcation the side branch was at an angle of 45°, but had a narrower diameter ($D_2 = 1/2D_1 = 3.16$ mm). The two values of the ratio D_2/D_1 (1.0 and 0.5) cover the common range for branchings in the circulatory system.

The bifurcations were drilled through Lucite blocks and presented a sharp edge at the junction of the branches. The main branch was always vertical, and was attached to the downstream end of a vertical Lucite tube of the same diameter (6.32 mm) and 190 cm in length. The flow was supplied to the apparatus from a tank in which the liquid surface was maintained at a constant level; the water and glycerine mixture was returned to the tank by a pump. The flow rates in the two branches of the bifurcation were varied by means of resistances; the liquid and solid flow rates were measured volu-