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CURRENT PROBLEMS IN RESEARCH

Superfluids

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For quite a long time experiments at low temperatures were aimed at reaching the absolute zero. Absolute zero, as such, is a rather old concept. Two hundred and fifty years ago Amontons had already visualized some state of matter at which all motion would come to rest, and which thereby would constitute a lower limit to our temperature scale. The concept of the ultimate cessation of molecular motion was maintained in this form for quite some time, until 50 years ago, when Nernst, by enunciating the third law of thermodynamics, somewhat modified it by substituting for the zero point of energy the zero point of entropy.

Entropy is a concept which many people consider "difficult." Even scientists dislike it sometimes, but, after all, it is not such an abstract thing. It simply measures the degree of disorder in any physical system. Thus, with the advent of the third law of thermodynamics, a change occurred in the approach to absolute zero as the approach to complete order, and not, as it was thought formerly, to complete rest. At the same time, the third law of thermodynamics contained the provision that absolute zero could not be reached by any physical process, and thereby the emphasis of research has been shifted to the question of approach rather than the question of attainment of a final point in the temperature scale.

Recent low-temperature research has therefore been directed toward better understanding of the state of ultimate order rather than that of ultimate rest in matter. As such, it has revealed aspects which are unique. These strange phenomena, which have now been classed under the general term *superfluids,* have no counterpart in the observational world of everyday physics. They stand entirely on their own and, possibly for this reason, have so far resisted theoretical explanation.

It seems quite clear that these phenomena must be related in some way to the amount of energy remaining at absolute zero in any substance—the zero-point energy—or rather to the ratio between zero-point energy and thermal energy. We can expect on the basis of first principles that the region in which the zero-point energy is large compared with the thermal energy will be distinguished as the region in which energy quantization assumes an ever larger part in the aspect of the physical world. In fact, in the phenomena of superfluidity, quantization of energy, which at normal temperatures is restricted to particles of interatomic or perhaps intermolecular size, now transcends into large-scale phenomena, and this is a condition which is alien to our normal observational experience.

Superconductivity

The first of the superfluid phenomena, superconductivity, was discovered by H. Kamerlingh Onnes in Leiden in 1911, three years after the first liquefaction of helium. What Kamerlingh Onnes did was to investigate the dependence of the resistance of a metal on the absolute temperature. At that time there were two different ideas about what the resistance of a metal might do at absolute zero. One school assumed that the resistance should gradually drop to the value zero as zero temperature was approached, whereas the other opinion held that it should rise to infinity. Kamerlingh Onnes used mercury as the first metal to be investigated for the simple reason that he could get it very pure. What he found was rather astonishing. It was so astonishing, in fact, that he had to do a second experiment in order to verify properly the first results. The results showed that the resistance of mercury at first dropped constantly as the temperature was lowered until, at 4°K, it attained an unmeasurably small resistance. It was only in the second experiment, when he had made his method more sensitive, that he found the amazing fact that the resistance, at about 4 degrees, disappeared completely and discontinuously (Fig. 1). This was the first observation of superconductivity, a phenomenon which has since been found in a great number of metals and even in compounds.

If you take a closed ring of a superconductive metal such as lead and induce a current in it, then this current will run for seconds, minutes, days, weeks, months—as long as the ring is kept in liquid helium. Recently it has been shown that the current will run for years without any decrease in its strength. These so-called persistent currents are clearly something that do not occur in ordinary macroscopic physics. A striking way of demonstrating persistent currents is shown in Fig. 2. In a Dewar vessel containing liquid helium, such a current was induced in a set of rings, and a lead sphere was dropped into the system of rings. As the sphere approached the rings, the persistent currents in the rings induced a system of persistent currents on the surface of the sphere. These currents being mutually repellent, the sphere was slowed down in its fall until it came to rest at a position where the mutual repulsion of the currents just counterbalanced the weight of the sphere. There it hung suspended

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in space by this unusual and strange phenomenon. It does not require much imagination or detailed calculation to see that this is a phenomenon which has no counterpart in the other domains of physics.

Figure 3 shows the distribution of the superconductors in the periodic table. For a little while it looked as if they were grouped in two regions in the table, but now other superconductors have been discovered between these regions. One thing seems certain: the monovalent metals are not superconducting. Perhaps a more impressive and slightly more informative diagram can be made when we plot the atomic volume against the atomic number (Fig. 4). Then the superconductors are all contained in a belt which has intermediate atomic volumes. An interesting region of this diagram is the superconducting series, lead, thallium, and mercury. Bismuth is just a bit too large to be a superconductor, and gold is just a bit too small. But if an intermetallic compound of gold and bismuth (Au_2Bi) is made, then this compound will be a superconductor. The case is quite analogous to that of the Heusler alloys in ferromagnetism, where two nonferromagnetic elements will produce a ferromagnetic compound. Here you have two nonsuperconductive parent metals, gold and bismuth, producing a superconductor. This also indicates that superconductivity cannot be a property of the atom, but must be due to some rearrangement of the free electrons in the energy spectrum of the metal. Superconductivity is confined to rather low temperatures, which means that the energy difference is of the order of 10^{-4} electron volts, and it would be difficult to find energy transitions of this order in an individual atom. Figure 5 shows that the transition is connected with an anomaly in the specific heat which by its small absolute size clearly marks it out as an electronic phenomenon.

This very strange and unusual phenomenon of superconductivity has been known for 45 years, and quite a number of attempts have been made to explain it. On the whole, in the intervening years these attempts have averaged three in peacetime and two in wartime per year, but none of them have been very successful. This failure is not due to the fact that we know so little about superconductivity; actually, the trouble is that we know rather too much. Any theory now will have to contain a whole set of phenomena which must all find a consistent explanation. However, before we hazard any opinions on what kind of state the electrons in the metal pass into when the metal becomes superconducting, let us first have a look at the other superfluid, liquid helium itself.

Liquid Helium

Liquid helium was first produced in 1908 by Kamerlingh Onnes, but somehow the discovery of its unusual properties seemed to be dogged by misfortune, or perhaps rather by the fact that people did not quite believe their eyes. The first indication that there was a change in the liquid helium itself at a temperature well below its boiling point was obtained by Dana and Kamerlingh Onnes. They measured the heat of vaporization of helium and found that there was a sort of indentation in the curve for the heat of vaporization against temperature. They also carried out some preliminary specific heat measurements, which, if these had been pursued, would have shown that the difficulty experienced in reaching lower temperatures with helium was due to a large anomaly.

The next indication was found when the density of helium was investigated by Kamerlingh Onnes. The density, as for any ordinary liquid, first increases with falling temperature, but then, at about 2.2 degrees Kelvin, there is a change in the direction of the density curve, and the density decreases again. Helium thus has a density maximum at about 2.2 degrees Kelvin (Fig. 6). There are, of course, other substances, among them water, which have density maxima in the liquid phase, because of some secondary effect. But this sort of explanation became quite untenable when

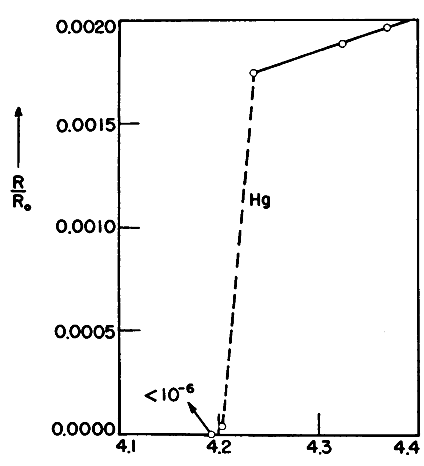

Fig. 1. Kamerlingh Onnes' discovery of superconductivity in mercury.

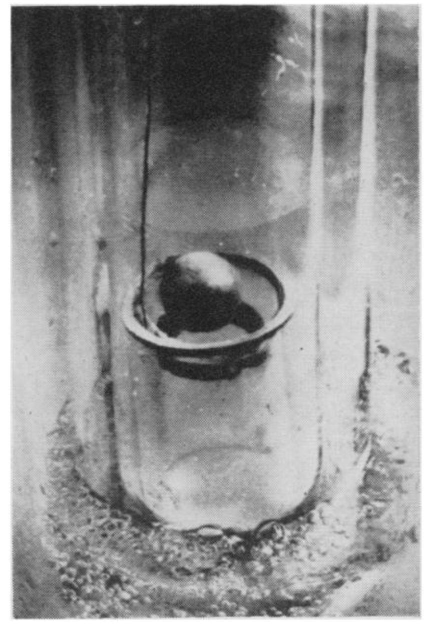

Fig. 2. Superconductive lead sphere suspended in space by the repulsion of persistent currents.

the specific heat was measured by Keesom and Clusius, whose results are shown by Fig. 7. These results led Keesom to the concept of the two states of liquid helium, which are separated from each other by this large anomaly known as the lambda-point. On the basis of these measurements, Keesom proposed to call the form of the liquid below this temperature, helium II, and the one above, helium I. One has to be careful in the term used because it would not be correct to use *phase* or *modification,* for with these terms, one always connects the separation of the forms in space. As is evident from the specific heat curve, there is never a coexistence of helium I and helium II, since there is no latent heat separating the two forms.

For a while the hope remained that there would be a conventional explanation—such as, perhaps, the appearance of liquid crystals—for helium II. Indeed, one has to remember that, owing to its high zero-point energy, helium remains a liquid down to absolute zero. As is clear from Fig. 8, external pressure is required to transform it into the solid phase. Therefore, for a little while the idea was entertained that the anomaly was after all the lost triple-point. Then the x-ray investigations made by Keesom and Taconis showed unmistakably that the form below the lambda-point, helium II, was a liquid showing none of the ordered structure of a solid.

Whereas the helium I, admittedly somewhat gaslike owing to its high zero-point energy, still retains the properties of a liquid, or at least those of a reasonably classical aggregate, helium II is quite different. A long time elapsed before this was realized. In retrospect, it seems strange that so many people should have seen and talked about the phenomena in helium below this temperature without having hit upon the correct explanation.

As a bath of liquid helium is cooled by withdrawal of the vapor from above the liquid, the liquid boils vigorously until the lambda-point is reached, when the boiling suddenly stops abruptly. From then on, although one knows from the flow meter that liquid is evaporating, the meniscus stands sharp against the glass wall like the edge of a knife. This phenomenon was quite well known and, while the explanation of it is obvious, it must have appeared very far-fetched. Later on, when Keesom and Miss Keesom made accurate determinations of the heat conductivity, they found indeed that the heat conduction of this dielectric liquid was increasing at this temperature by a factor of about 1 million. This result explained the cessation of boiling immediately, but who would have postulated that a dielectric liquid should suddenly become a substance with the best heat conductivity known, more than a hundred times better than copper or silver. Moreover, it was soon discovered that the heat conductivity of helium II depended on the temperature gradient, on the size of the capillary in which it was measured, on the shape of the vessel, and on other factors. In fact, while there exists high heat flow in liquid helium, the concept of heat *conduction* seems to have lost its meaning.

The same kind of behavior was discovered when the viscosity was investigated. It turned out that the viscosity of liquid helium II also depends on the type of measurement which is used (Fig. 9). A value of about 10^{-5} poise is obtained when an oscillating disk is used as the measuring instrument. If, on the other hand, one uses a Poiseuille method of outflow through narrow channels, the viscosity is less than 10^{-11} poise. Here again is a clear factor of a million between the measured values of the viscosity. Moreover, it was found in a great number of experiments that helium flows through quite narrow channels at a very high rate and largely independent of pressure. This was the discovery of superfluidity. In addition to these phenomena, there exist peculiar thermal effects.

Thermal Effects

If two vessels are connected by a capillary (Fig. 10), and the level of liquid is higher in one vessel than in the other, helium will flow through the capillary toward the lower level, but in doing so the helium below the dropping level will heat up, and the helium below the rising level will cool down. This effect has its exact opposite in that, when one starts with equal levels and supplies heat to one of the reservoirs, the level in the heated vessel will rise, and that in the other vessel will fall. We therefore have a flow of helium from the lower to the higher temperature. This effect can be demonstrated very strikingly, as shown in Fig. 11, by substituting for the capillary a little vessel which

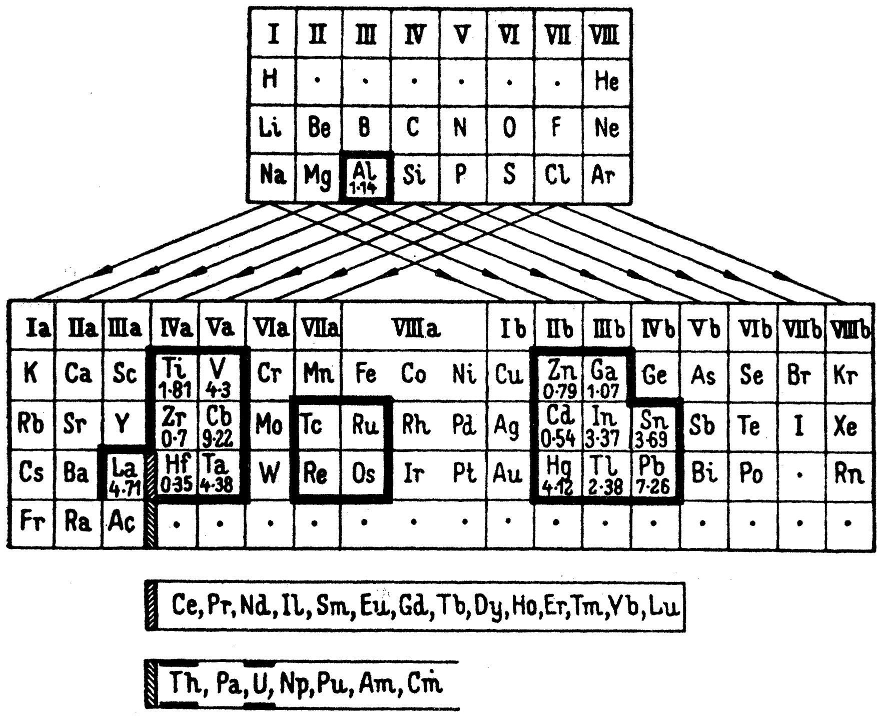

Fig. 3. Superconductors in the periodic table.

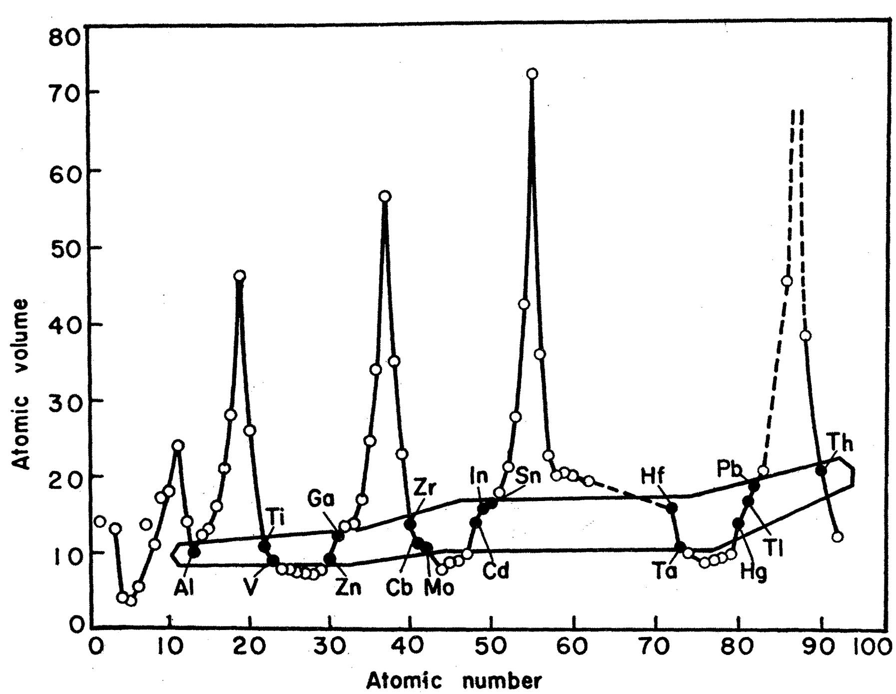

Fig. 4. All superconductors have similar atomic volume.

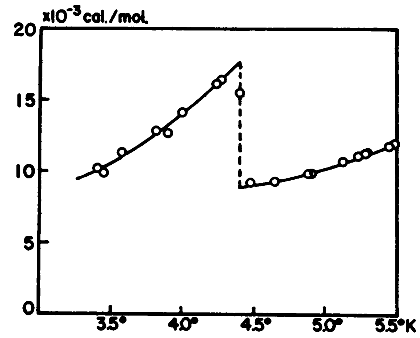

Fig. 5. The change in the specific heat of tantalum at the occurrence of superconductivity.

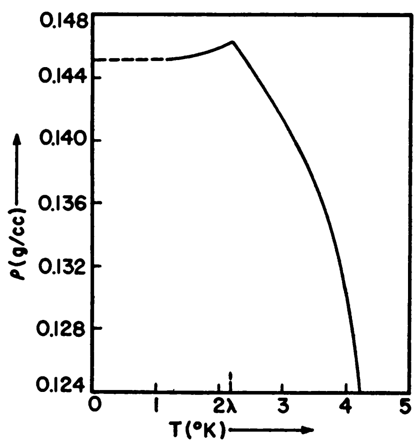

Fig. 6. The density maximum of liquid helium.

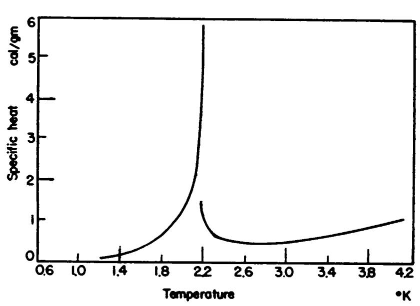

Fig. 7. The lambda-point anomaly in the specific heat of liquid helium.

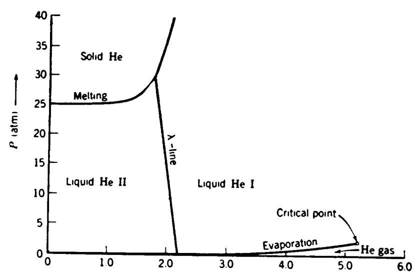

Fig. 8. The phase diagram of helium showing the absence of a triple point.

is open to the helium bath and is filled with powder and which is heated by shining a light on it. The pressure generated by the liquid flow toward the heat will be so high that helium will squirt out in the form of a little fountain. This has become known as the "fountain effect," but it is less confusing to talk about the two thermal effects as the mechanocaloric effect and the thermomechanical effect.

Film Flow

This is not all. All solid substances in contact with helium II are covered with a film of about 10^{-6} centimeter thick, and this film also acts as a vehicle for superflow. Figure 12 shows what happens in a very simple type of experiment. An empty glass beaker has been dipped partly into liquid helium. Helium is then seen to appear in the bottom of this beaker, and it slowly rises until the same level is attained on the inside as on the outside. If the beaker is now withdrawn from the bath, the level will drop until it has reached the level of the bath. If the beaker is pulled out completely, the level will still drop, and one can see little drops of helium forming at the bottom of the beaker and falling back into the bath (Fig. 13). This is the sort of thing that makes one look twice and rub his eyes and wonder whether it is quite true. I remember well the night when we first observed this film transfer. It was well after dinner, and we looked round the building and finally found two nuclear physicists still at work. When they, too, saw the drops, we were happier. Incidentally, one can also see in Fig. 13 how still the liquid is and how completely free of bubbles. This demonstrates the high heat conduction of liquid helium II.

Figure 14 shows the really striking aspect of the film flow—namely, that if one withdraws the filled beaker and now plots the drop of level against time, the outflow will be quite constant. As the level in the beaker falls, a number of conditions change. First of all, the difference in height, and with it, the pressure, varies. Second, the length of the path over which the helium has to travel changes. Third, the height of the barrier over which the film passes changes. However, all these variations leave the rate of outflow quite unaltered. If we look around for physical phenomena which have any similarity to this steady outflow, the best thing we can find is a

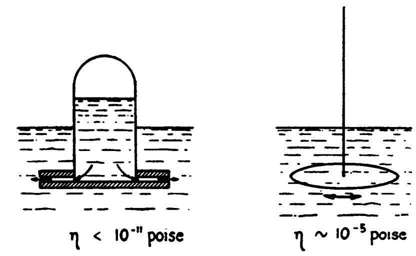

Fig. 9. Different methods of measurement yield different values for the viscosity of liquid helium II: (left) flow through narrow channels; (right) damping of an oscillating disk.

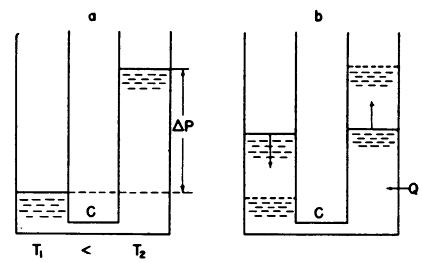

Fig. 10. The (*a*) mechanocaloric and the (*b*) thermomechanical effects.

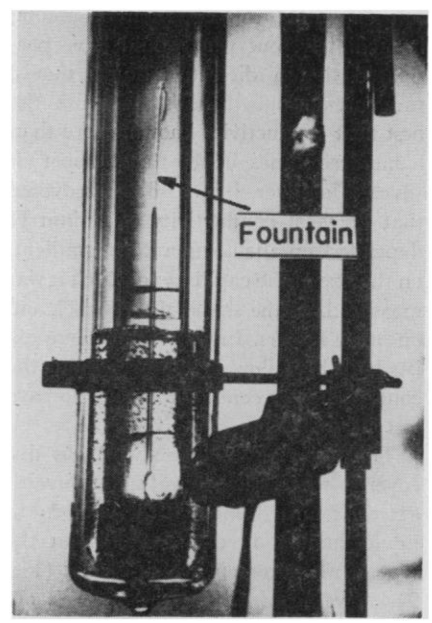

Fig. 11. Helium fountain produced by the thermomechanical pressure.

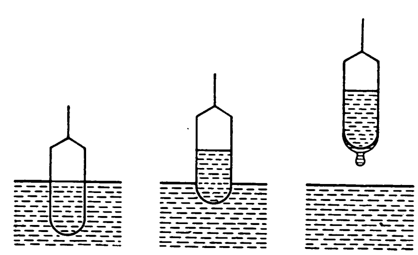

Fig. 12. Film flow of helium II in and out of a glass beaker.

lump of radium in which, uninfluenced by external conditions, the nuclei break up at a given rate, disgorging alpha particles. One is greatly tempted to look at superfluidity in the same way and to regard the outflow simply as the quantum statistical probability of finding a helium atom outside rather than inside the beaker. Whereas the uncertainty principle in the case of the radium atoms does not operate over very large distances, here, as a result of the disproportion between zero-point energy and thermal energy, it will transgress into these large-scale dimensions.

In the case of an individual nucleus of radium, one cannot of course say whether it will explode in the next millionth of a second or will not explode for a hundred million years. All one knows is that, of the large number of atoms in the lump, half will have decayed in 1600 years. I think the same consideration must be applied to the phenomena in helium. One should not try to look at one helium atom as a billiard ball running up one side of the wall and down the other. The helium atoms are wave packets which leak out over the beaker.

In a slight modification of this experiment, superfluidity can be even more clearly demonstrated. Figure 15 shows an arrangement which consists of two beakers placed concentrically into each other. To start with, the level of the helium is the same in both the inner and the outer beaker and in the bath. When the apparatus is lifted out, liquid helium will run from the inner into the outer beaker and from the outer beaker into the bath. What is significant is that there is *no difference in level* between the inner and the outer beakers. The wall of the outer beaker acts as a restricting perimeter to the outflow, and zero potential difference exists between the inner and outer levels. This means that the helium in the film flows completely without friction.

The experiment is the flow counterpart to a well-known electrical phenomenon, namely that of a super current. Figure 16 shows the circuit which is usually employed to find out whether a substance is a superconductor. There is a battery in the circuit with an ammeter, a superconductor, and a limiting resistance which will keep the current below the critical value. Then a voltmeter connected across the superconductor will show zero potential difference. In Fig. 15 the difference in levels between the bath and the inner level

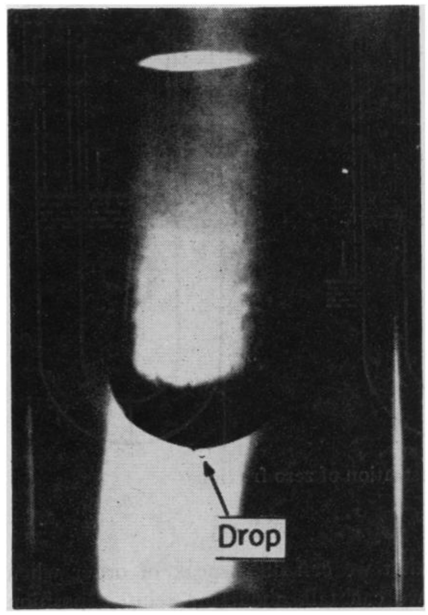

Fig. 13. Liquid drop formed by film transfer.

corresponds to the battery, the outflow rate is the ammeter, the wall of the outer beaker represents the limiting resistance, and the wall of the inner beaker is the superconductor, across which there is zero potential difference.

Entropy

Coming back to our starting point—namely, the postulate that as absolute zero is approached the order in the system increases—we can now ask whether the order is increased when these superfluid phenomena occur. The question can easily be answered by simply measuring the specific heats and deriving the entropy diagrams. As Fig. 17 shows, the two entropy diagrams for helium and for a superconductor are remarkably similar. There is a linear drop toward absolute zero in the normal state —in the metal before it becomes a superconductor, or in the helium I. This is followed by a much more rapid decrease in entropy as superfluidity sets in. It is tempting to connect this drop in entropy with the phenomenon of superfluidity; this can be done because superfluids seem to be mixtures of a curious kind. The so-called two-fluid model for superfluids has been used extensively, and in spite of certain shortcomings it is useful in explaining the properties. In this model the superfluid is considered as being made up of the same particles—that is, electrons diluted in electrons in the superconductor, and helium atoms diluted in helium atoms in the lambda phenomenon. As, on cooling, the transition temperature is passed, some of these particles pass into a state of lower energy, and finally, at 0 degrees Kelvin, the whole assembly is made up of these lower energy particles. So, as the temperature is decreased, the superfluid constituent in both cases increases all the time at the expense of the normal one. Such a mixture model will have exactly all the requisite properties. For instance, if one has an oscillating disk in helium, it will be impeded by the normal part of the fluid, but if one has a capillary, then the superfluid part will run through the capillary and give the impression of a much smaller viscosity.

As long as we just talk about the normal and the superfluid part without going into any atomistic explanation, the model is comparatively innocuous and we can use it as a working hypothesis. For instance, we can make experiments on the flow of the superfluid constituent, measuring its entropy separately. In a persistent current only the superconducting electrons take part in the motion, and in flow through a capillary only the superfluid part. By making a suitable type of measurement, we can determine the entropy of that part of the two-fluid model which moves without friction. It turns out that this entropy is zero in both cases. The drop in entropy (Fig. 17) is thus due to the growth of the superfluid

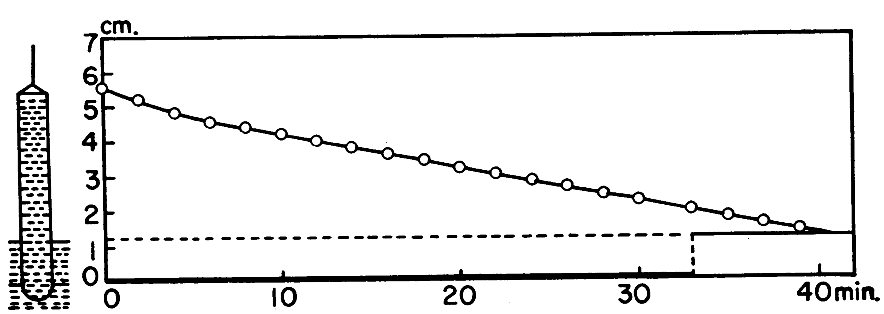

Fig. 14. Superflow takes place at a steady rate.

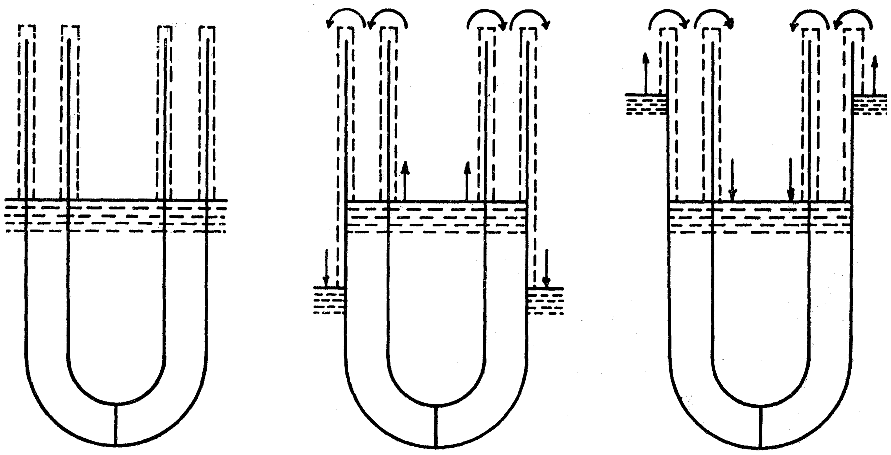

Fig. 15. Experimental demonstration of zero friction.

phase at the expense of the other phase, and to the fact that the superfluid phase has zero entropy. This constitutes no violation of the third law of thermodynamics for the simple reason that we cannot separate these two phases without a change in temperature.

Whereas we can determine the amount by which order is increased, the concept of entropy is unfortunately so general that we cannot say anything about the way in which this order is increased. In order to get some idea, we have to start thinking afresh about the basic principles of defining order. In both these cases—that of the free electrons and that of the helium atoms—we deal with fluids and not with solids. There is, therefore, no crystallization, and all the aspects which we normally connect with the concept of order are missing. Therefore we have to ask ourselves: What is actually the concept of order?

For instance, the chairs in a lecture room are arranged in a definite predetermined pattern, which we consider as order. It seems to us quite natural that order involves position. We feel that we can only speak of order when we can talk about the relative position of one particle to another. But that is not quite true, because, as Gibbs showed long ago, in a description of a set of particles we do not deal only with the three dimensions of position-space; we must take into account all the six dimensions of phase-space—the three coordinates of position and the three coordinates of momentum.

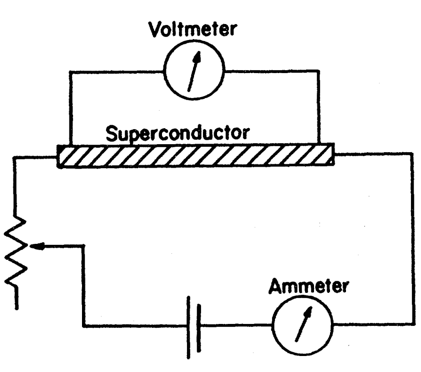

Fig. 16. Superconductive analogy of the double beaker experiment.

Momentum and Position

The great strength of physics as a method is that it allows comparatively sure predictions. Modern technology is based on the combination of well-known principles, and these principles allow a fairly clear prediction of what will happen in the end. In order to make such a prediction, it is not sufficient to know the position of particles at one given time. We must also know their positions in the next instant. In other words, the momenta of the particles are as important as their positions. This complementarity is made clear by the fact that the uncertainty relation, the basic principle of quantum mechanics, does contain these two quantities, the position and the momentum, as complete equivalents. They are not similar; they are as different as lock and key. However, lock or key alone would not make sense; both must be considered together.

The idea of taking into account also the momentum coordinates immediately offers an entirely new aspect of the definition of order. Since the coordinates of momentum are quite equivalent to those of position, we must be able to conceive of order with respect to the momenta in the same way as we are accustomed to regard the chess-board pattern of position as a state of order. And it is perhaps significant that in the superfluids the striking features are provided by the transport phenomena. This means that the relevant phenomena are those which are connected with momentum, with the properties of motion, rather than with any static properties. The only difficulty appears to be that somehow our brains are so constructed that we cannot imagine happenings in momentum-space in the same way as we visualize happenings in position-space. On the other hand, if what we observe in the superfluids are momentum condensations, they do not occur in dimensions which are inaccessible to us. On the contrary, these are very striking phenomena which we can see. Perhaps we can make a model of velocity condensations conceivable even with our brains which are accustomed to position order.

In fact, a certain type of momentum condensation has been invented by all civilized governments—that is, those whose subjects own motors cars—by the introduction of a speed limit. Clearly if you have cars moving at all velocities from 5 to 100 miles per hour over the road and you then impose (and enforce) a speed limit of 30 miles per hour, then all the higher velocities will be compressed to 30 miles an hour. If you were to make a movie of this from a great

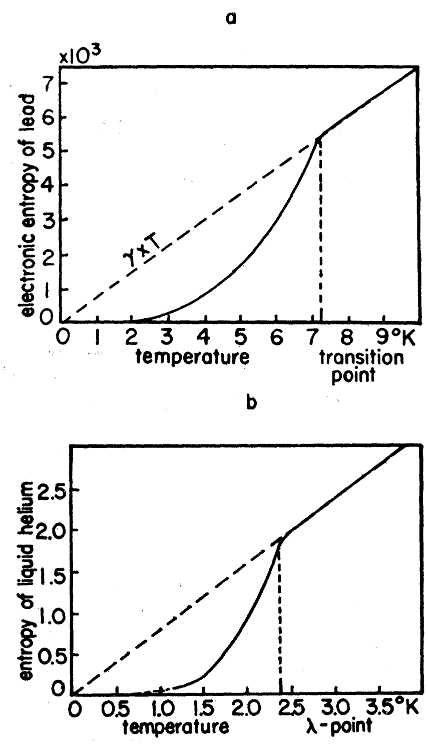

Fig. 17. Entropy diagrams of the electrons in a superconductive metal (*a*) and of liquid helium (*b*)

height, then, although you would only observe positions at intervals, you would nevertheless see a pattern of order appear. Take a section of road somewhere where there are a bus, two cars, another bus, and a truck, and later look at the same string at quite a different part of the road. You will then note a degree of order if the relative positions of and distances between the vehicles have been maintained. This is clearly a condensation in momentum space. I would like to make it clear straight away that this model has no other merit than to show that we can understand, conceive, and recognize momentum condensations in our imagination, but this is certainly not the model which can be applied to liquid helium and superconductors. In our example, recognition of the pattern depends on our ability to make a distinction between the different vehicles, and of course one must not make a distinction between atoms. This is a classical model, and it shows nothing more than that momentum condensations can be created and might therefore possibly exist in nature.

Aggregation of Momenta

If we go that far and assume that momentum condensations are the basis of the phenomena of superfluidity, then we must immediately go one further step in our speculations. That is to assume that this order pattern must be of a very fundamental nature, because in the two cases which we know, electrons in superconductors and helium atoms in liquid helium, we deal with particles in very different settings and of a very different nature. In one case they are charged light particles in a metal matrix, which obey Fermi-Dirac statistics, and in the other case they are uncharged atoms which by themselves form a liquid and obey Bose-Einstein statistics. If the superfluid state occupies in momentum-space the same position as a crystal in coordinate space, then we can expect that, quite independently of the originating process, the model may be one of great generality. In the cohesion of a solid we may have exchange forces such as in a metal, or van der Waal's forces as in solid argon, or ionic forces as in sodium chloride. These are quite different processes, but they all result in a pattern with very much the same general properties—that is, they result in a crystal.

Last of all, one might ask whether helium and superconductivity are the only representatives of this new state of aggregation. We must call it a state of aggregation in the same way as we call a solid a state of aggregation, only here the aggregation is in momenta and not in positions. Probably on the crust of the earth, which is after all a very unrepresentative bit of the universe, these states may occur only in the laboratory. If, on the other hand, one considers the interior of astronomical bodies of not too high a temperature, it is quite conceivable that under those conditions of very high zero-point energy, the state of aggregation of this frictionless pattern may be more favored than that of position. In fact, we know that here position-order is quite impossible. It thus may turn out that the frictionless state of aggregation is the common form of aggregate matter in the universe, and that the solid state is a rather strange oddity reserved for our own odd corner of the universe.

That is one possibility, but there is another one. We can, of course, become bolder and, knowing the position condensates in the form of crystals, having postulated and almost believed in momentum condensates in the form of superfluids, we can look around in Gibbs space for further possibilities. There are regions in it where we must find other condensates, mixed condensates of position and momenta. We do not know what these mixed condensates can be, but we can say beforehand what properties they must have. They must have very low entropy, because you have here the chance of ordering to a very great extent. Second, they must have the properties of position-order—that is they must be tangible things with a given volume and shape. Third, they must have the characteristic properties of a condensation in momentum. One of the significant properties is that some essential feature is spread over the whole volume of the momentum condensate. Take, for instance, a superconducting ring. If there were no current in the ring and you cut it somewhere, not much harm would be done. There would be an array of atoms not much different from the atoms before cutting. It would not be generally noticed that the ring had been cut. But once a persistent current is flowing and you cut the ring, the whole current disappears from it. This means that the condensation or ordering in respect to the momentum vector, which you have destroyed locally, is now destroyed over the whole of the ring. Therefore, the properties of these velocity condensates are characteristic of the whole structure, and if you cut it in two, it has lost its meaning. If one tries to think where we might find structures like this, one sees that they are very improbable in the first place. They must have low entropy, they must be tangible structures, and they will lose their significance if they are cut in two. There are indeed structures which fulfill these conditions—the structures of living matter. However, this is just an amusing speculation.