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The "Clock Paradox" and Space Travel

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In the pages of *Nature* there have appeared recently exchanges of correspondence between McCrea (*1*), Dingle (*2*), and Crawford (*3*). Dingle maintains that there will be no difference in age between a returned space traveler and his twin brother who stays home, while the other authors maintain that such a difference will exist, in just the amount computed by a straightforward application of the Lorentz transformation of special relativity. Dingle's argument is based on an old difficulty known as the "clock paradox," which stems from an apparent ambiguity in the answer to the question: If all motion is relative, how does one decide who traveled and who stayed home?

McCrea and Crawford have in my opinion clearly won the argument, and perhaps further remarks are superfluous. However, because of the considerable interest aroused, it may be worth while to restate the situation in new words and all in one place; hence this article. It has three sections. In the first, the "paradox" is stated and resolved, using only inertial coordinate systems; in the second, a treatment of accelerated coordinate systems based on the principles of special relativity is given; in the third, the possibility of practical implications for space travel is examined.

The "Paradox" and Its Resolution

The apparent difficulty in the "paradox" can be stated very simply. The twin brothers B and B' are in relative motion with the velocity v. If B considers himself at rest, he concludes from the usual formula for the time dilatation that the watch carried by B' will register a time interval

$$\sqrt{1-(v^2/c^2)}\,\Delta t$$

while his own watch registers a time interval Δt. If B' makes a journey at velocity v and returns home at the same velocity, his elapsed time (as measured by his watch or by his own physiological aging) will be smaller by the factor

$$\sqrt{1-(v^2/c^2)}$$

than the elapsed time experienced by B, who stayed home. However, if "all motion is relative," B can just as well say that he went on the trip while B' stayed at rest, and he can state that he is the one that should be younger. If both statements were correctly derived from the principles of special relativity, there would indeed be a paradox and one would conclude that the theory is not self-consistent and must be rejected. What we shall show is that the first statement is correct while the second is wrong; there is no true paradox, and the result that travelers live longer than stay-at-homes, while sometimes called "paradoxical," is really in the "strange but true" category.

To examine this matter more closely, we must set up the situation in greater detail. Suppose that B is at rest at the origin of an intertial coordinate system S, while B' has the same relation to an inertial coordinate system S', and that the two coordinate systems are in relative motion with a velocity v. Suppose also that the watches of the two brothers are set so that they each read zero when the two origins coincide, which defines the starting point of the trip. The coordinates and times in the two systems are then related by the Lorentz transformation

$$x'=\gamma(x-vt) \qquad (1)$$

$$t'=\gamma\left(t-\frac{v}{c^2}x\right) \qquad (2)$$

$$\gamma=1/\sqrt{1-(v^2/c^2)}$$

From these equations, and from the fact that the brothers are located at their respective origins, we find for the relation between times observed at the location of B (where $x=0$)

$$t'=\gamma t \qquad (3)$$

while at the location of B' (where $x'=0$), we find

$$t=\gamma t' \qquad (4)$$

The quantities in these equations are clock readings t at coordinates x in S, and t' and x' similarly defined in S', while the equations give the relations between these quantities at localized events which are perceived in both systems. We may picture each system as carrying a long line of measuring rods and clocks, with observers to note down the clock reading and distance coordinate whenever an event occurs. These of course do not actually exist, but we are allowed to use them in discussion since the Lorentz transformation is constructed in such a way that all relations between events (such as the sending and reception of light signals) perceived where there are actual observers are consistent with observations that might be made elsewhere.

Now imagine a very simple event in which B' merely looks at his watch, notes that it has the reading τ, and also observes the reading of the particular clock in S that coincided with his location at that instant, obtaining for that the value $\gamma\tau$, as given by Eq. 4. This event alone is not sufficient to determine the relative elapsed time as seen by B and B'; it must be correlated with another event, which is a watch reading made "simultaneously" by B.

Here we run into trouble in the interpretation of the word *simultaneous*. First consider the situation from the standpoint of B. One of his corps of observers reports (at any time after the event) that he saw B' read his watch when he went by, and reports his readings of t and t', which are the same as those noted by B'

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at the same time and place. B can then say: "When B' looked at his watch, he saw the reading τ; my own observer read the time as $\gamma\tau$ on his local clock; that clock is synchronized with my own watch; therefore, if I had looked at my watch when it indicated the time $\gamma\tau$, that event would have been simultaneous with the event consisting of B' looking at his watch." Second, consider the situation from the standpoint of B'. He says to his observers: "One of you was next to B when your clock read τ, which is the time when I read my watch. What did you see on the watch of B?" The result, from Eq. 3, would be τ/γ. Therefore, if B had looked at his watch when it read τ/γ, B' would have considered that event to be simultaneous with his own watch-looking event.

Thus, to one event at B', there correspond two different "simultaneous" events at B, depending on whose idea of simultaneity is accepted. This result is well known, and is not considered troublesome so long as B and B' are in relative motion at a distance apart; because of the symmetry of the situation, there is no way to decide who is "right," and there is no meaning in such a decision.

The usual next step is to have one of the observers reverse his motion, so that ages can be compared directly when the two come together again. However, the argument is just as cogent if we have them come to a state of relative rest, where age comparisons are meaningful even at a distance, and this approach is used because of its simplicity. The end of the trip will be defined by having one of the observers (say B') come to rest relative to the other.

Now an element of dissymmetry is introduced, since B' is the one who has to push the firing button of his decelerating rocket and whose accelerometer will deflect, while nothing will happen to B. This fact is used in a way which does not involve any arguments from general relativity (except insofar as it may give meaning to the concept of an inertial system) or any discussion of accelerated coordinate systems. The important thing is that, once B' has stopped, he has dissociated himself from S' and attached himself to S (with a shift in the zero point of his time scale, of course); the system S' remains unchanged. (We may note here that a reversal, rather than a stopping, of B' can be treated by having him transfer to a third inertial system S'' moving in the opposite direction; this more complicated procedure leads to a result for the round trip consistent with what we get for the outward part.) It is also postulated, as is usually done and as seems physically reasonable, that acceleration as such does not change his clock reading. His readings of τ on his own watch and $\gamma\tau$ on the clock in S at which he stopped are, one may say, transferred bodily into S, and he and B can then compare notes with no ambiguity since they are at relative rest and since both use the same definition of simultaneity. Both B and B' agree that the ratio of elapsed times during the trip is that given by Eq. 4, and there is no "paradox."

Looking at the situation from another standpoint, one might inquire about the interpretation of a signal sent out by B' at such a time that its arrival at B is simultaneous (in S') with the stopping of B'. Such a signal, seen by B and interpreted as indicating "the end of the trip," would lead to a wrong result (to the usual "paradox" in fact). However, if B is more careful, he will realize the discrepancy in concepts of simultaneity and correct for it. From Eq. 2 we find that the difference in t between two events occurring at the same value of t' and at locations a distance Δx apart (in S) is equal to $v\Delta x/c^2$. The distance Δx in the case being considered is $v\gamma\tau$, the length of the journey in S; the events are the arrival of the signal at B and the stopping of B'. The time correction $v\Delta x/c^2$ has the value $(v^2/c^2)\ \gamma\tau$, or $[\gamma-(1/\gamma)]\tau$. When this is added to τ/γ, the observed value of t at the arrival of the signal, the value $t=\gamma\tau$ for the time of stopping of B' is obtained, in agreement with what was seen directly by observers in S. What will B' think of this? He sent the signal with the expectation that it would reach B when he reached the end of his journey, but when he gets into communication with the observers in S after the trip is over, they tell him that the signal arrived before he stopped. He resolves this apparent dilemma by realizing that when he jumped from S' into S he had to replace the concept of simultaneity in S' by that of simultaneity in S, the difference introduced thereby being equal to the "time correction" already defined.

At the cost of some repetition, we return to the question: How does one distinguish the stopping of B' from the starting of B, when the change in relative velocity is the same in both cases? Physically, the acceleration is felt by B' and not by B; mathematically, the acceleration is treated as a transfer from one inertial system to another, and here again the distinction is just as apparent. The essential symmetry of the treatment is clear if one notes that the concept of simultaneity is used in that inertial system in which *both* of the brothers are initially and finally at rest.

By a simple but interesting transformation, it is possible to relate the age difference of the brothers to their *changes* in velocity. For symmetry, allow both brothers to move, with velocities corresponding to γ_1 and γ_2. Then the difference between their elapsed times t_1' and t_2' is given by

$$t_1'-t_2'=\int[(1/\gamma_1)-(1/\gamma_2)]dt=$$
$$[(1/\gamma_1)-(1/\gamma_2)]t-\int t\,d[(1/\gamma_1)-(1/\gamma_2)]$$

Since we have postulated that $\gamma_1=\gamma_2=1$ (that is, both brothers are at rest in the system in which t is measured) at the start and finish, the first term on the right vanishes. If the velocity changes are thought of as discontinuous (being expressed as changes in $1/\gamma$), the second term becomes a sum, giving:

$$t_1'-t_2'=\Sigma t\Delta(1/\gamma_2)-\Sigma t\Delta(1/\gamma_1)$$

This illustrates both the symmetry of the treatment and the importance of changes in velocity. Note that only the magnitudes of the velocities enter, and that the motion need not be confined to the x-axis.

Accelerated Coordinate Systems

It is of some interest, although not necessary for this problem, to inquire what happens in an accelerated coordinate system. In the treatment given in the preceding section, the system S' was not accelerated, but the traveler simply left it, like a man jumping off a train. Now we suppose that the train slows down with the man on it. The coordinate system (that is, the train, or a line of space ships if you prefer) carries measuring rods and clocks by which times and distances are measured. It is accelerated by applying power to the wheels or firing rockets at times considered simultaneous in itself. We start again with Eqs. 1 and 2 and suppose that the acceleration occurs suddenly at $t'=0$. We now stand in S and watch the process. We note that the acceleration does not now seem to be simultaneous; Eq. 2 shows that, at each t, it occurs at an x given by $x=(c^2/v)t$. Thus the acceleration looks like a wave traveling with the velocity c^2/v. (The fact that this velocity is greater than c is not objectionable since the wave does not convey a signal.)

An acceleration wave in the form of a step of finite amplitude will not retain its shape since the wave velocity varies with v; therefore we deal at first with a step wave of infinitesimal amplitude dv. The new Lorentz transformation after the wave has passed is

$$x'=(\gamma+d\gamma)[x-(v+dv)t] \qquad (5)$$

$$t'=(\gamma+d\gamma)\left[t-\frac{v+dv}{c^2}x\right] \qquad (6)$$

where t' retains its meaning as a time considered simultaneous in the new S'. However, it no longer corresponds to the clock readings in the new S' except at

$x' = 0$. To see this, we note that a clock at a given x' does not change its reading suddenly when the acceleration occurs, while t' does make a sudden change. The latter is computed by subtracting Eq. 2 from Eq. 6, evaluated at the appropriate place and time in S—that is, at $x = (c^2/v)t$, with the result that t' at a given point changes suddenly by the amount

$$dt' = -\gamma x dv/c^2 = -\gamma^2 x' dv/c^2$$

The clock reading at x', which we shall call T', does not suffer a corresponding instantaneous change, so that a difference between T' and t' appears:

$$d(T' - t') = \gamma^2 \frac{x' dv}{c^2} \qquad (7)$$

A similar treatment applied to Eqs. 1 and 5 shows that the change in x' at a given point (say a given scale division on an actual measuring rod in S') is equal to zero. This is not trivial; x' is not defined as a scale reading but as a coordinate in terms of which the Lorentz transformation is valid; scale readings, like clock readings, cannot change suddenly, but their interpretation as coordinates can. For example, suppose that an acceleration were made to take place as a wave traveling with a velocity (in S) other than c^2/v. Then there would be a change in x' at a fixed scale reading in S'. The observer in S' would interpret this as an actual physical expansion or compression of his system arising from the fact that the acceleration no longer occurs simultaneously at all points in his system. In other words, the specification that the acceleration be simultaneous according to t' is necessary if we require that no "strains" be introduced into S'.

The concept of an acceleration wave with velocity c^2/v in S gives a simple pictorial representation of the generation of a Lorentz contraction with no discontinuous coordinate changes. Consider a measuring rod in S' with an apparent length l in S. The time interval in S for the wave to pass the length of the rod is $l/[(c^2/v) - v]$. During this interval, one end is moving faster than the other by the amount dv, and the apparent final change in length is

$$-l dv/[(c^2/v) - v] = -\gamma^2 l v dv/c^2$$

According to the Lorentz contraction $l = l'/\gamma$. Differentiating this, we find that

$$dl = -\gamma l' v dv/c^2 = -\gamma^2 l v dv/c^2$$

in agreement with the result obtained from the wave picture.

It is now of interest to find out whether Eq. 7 is consistent with general relativity, which says that two clocks, at a distance x' apart in a system suffering acceleration a', will appear to differ in rate by the amount $a'x'/c^2$, and therefore will acquire a difference in reading $x' du/c^2$ if the acceleration continues just long enough to produce a velocity change du'. The velocity change du' is a small increment of velocity from rest, as seen by the observer in the accelerated system; to find the corresponding change in the velocity v as seen by an outside observer, we differentiate the relativistic formula for the composition of velocities, with the result that $dv = (1/\gamma^2)\, du'$. The change in clock readings, expressed in terms of dv, is therefore in agreement with Eq. 7, and we have derived the change in clock rates in an accelerated system using special relativity alone. The generalization to a gravitational field requires, of course, the use of the equivalence principle of general relativity.

Now we wish to consider the effect of a finite change in v, which can be thought of as taking place by means of a set of superimposed infinitesimal stepwise acceleration waves. The resulting composite wave can be stepwise at only one point because the wave velocity varies with v. In the case of a stepwise increase in v (assumed to be initially positive) at $x = 0$ and $t = 0$, the wave will spread out in the forward direction as t increases, and cannot be defined for $x < 0$, $t < 0$. In the case of a decrease in v at the same place and time, the wave is spread out behind at earlier times, and cannot be defined for $x > 0$, $t > 0$. In order to generate such a wave, the firing of the rockets in the line of space ships defining S' must be scheduled differently at different points; if the acceleration is instantaneous at one point, it must be spread out, according to local clock readings, and the rate of acceleration must be correspondingly reduced, at other points. This corresponds to the fact that each velocity increment must occur simultaneously in t', while the clocks do not continue to indicate t'.

After the complete acceleration wave has passed, the total change in clock readings is obtained by integrating Eq. 7. If the relative velocity changes, for instance, from 0 to v, we find

$$T' - t' = \int_0^v \gamma^2 \frac{x'}{c^2}\, dv = \frac{x'}{c} \tanh^{-1}\left(\frac{v}{c}\right) \qquad (8)$$

This is not to be confused with the "time correction" used earlier, which has the same value only when $v \ll c$. The "time correction" is the result of a discrepancy in the estimation of simultaneity in *two different coordinate systems* with relative motion; Eq. 8 represents a desynchronization of clocks *in the same coordinate system.*

Finally, we inquire what relation this has to the "paradox." When B' stops, suppose that he brings his whole coordinate system to rest with him, and then compares the clock in his system at the location of B with the watch of B. This clock is out of synchronization with his own by the amount shown by Eq. 8, but he is presumed to be clever enough to know this (or to find it out by measurement after he has stopped), so he corrects for it, and the actual value of Eq. 8 does not enter into the result. He might just as well have used the already existing system S, in which he is at rest after stopping, and simply taken the reading of the clock opposite which he stopped to be elapsed time as estimated by B, which leads to the same result and is equivalent to the choice of Eq. 4 for the relation between the elapsed times.

Implications for Space Travel

The great recent interest in the "clock paradox" is based on its possible consequences for travelers in space; this transcends in popular appeal the more basic matters of principle involved. Therefore let us look at it from that standpoint. First, could an acceleration tolerable to human beings, acting for a reasonable time, produce velocities great enough to give an appreciable time dilatation? To answer this, we must find the distance x (in the rest system of the starting point) traveled in time t' (in the traveler's rest system) while the traveler feels the constant acceleration a'. We first find the velocity v by integrating the relation $dv = (1/\gamma^2)du'$ (introduced in the preceding section), noting that $du'/dt' = a'$, with the result

$$v/c = \tanh \frac{a'}{c} t'$$

Then

$$x = \int_0^t v dt = \int_0^{t'} \gamma v dt' = \frac{c^2}{a'}\left(\cosh \frac{a'}{c} t' - 1\right)$$

If x is measured in light years, t' in years, and a' in units equal to the normal gravitational acceleration at the surface of the earth, then

$$x = \frac{0.97}{a'}\left(\cosh \frac{a'}{0.97} t' - 1\right)$$

This is a remarkable result. For example, a man traveling for 21 years under a constant acceleration of 1 g would go a distance of 1.2×10^9 light years! If he then reversed his acceleration, he would finally come to rest at a point 2.4×10^9 light years distant from his starting place, having spent 42 years of his life on the trip. The results are less dramatic for shorter trips; even for rather high accelerations (on a human scale), the relativistic time modifications are negligible for travel within the solar system. For example, a man going to Neptune and stopping there, at an acceleration of 10 *g*, would spend 5 days on the trip but would gain only 1.5 minutes of time.

Then there is the question of the energy involved. The man who travels for 21 years at 1 *g* reaches a value of γ equal to 1.2×10^9, at which point his kinetic energy is utterly fantastic. If his vehicle weighs (at rest) 1 ton, then its energy content is equal, in round numbers, to the energy released in the annihilation of 10^9 tons of matter, or in the fission of 10^{12} tons of uranium; it would be sufficient to melt the entire crust of the earth to a depth of about 30 miles. The man who makes the more modest trip to Neptune at 10 *g* reaches $\gamma = 1.0025$, and the kinetic energy of his 1-ton ship (2×10^{17} joules) corresponds to that released in the fission of about 2 tons of uranium; because of the limited efficiency of rocket propulsion, the actual energy needed would be much greater. The use of such energy quantities in a rocket ship is so far beyond any foreseeable practical limits, and the time gain in that case is so small, that it is hard to picture a practical case of space travel in which the time dilatation can be considered important. This conclusion, of course, does not detract from the interest of the fundamental principles involved in the "clock paradox" (*4*).

References and Notes

1. W. H. McCrea, *Nature* 167, 680 (1951); 177, 784 (1956); 178, 681 (1956); 179, 909 (1957).
2. H. Dingle, *Nature*, 177, 782, 785 (1956); 178, 680 (1956); 179, 865, 1242 (1957).
3. F. S. Crawford, Jr., *Nature* 179, 35, 1071 (1957).
4. I would like to express my appreciation to Frank S. Crawford, Jr., David L. Judd, W. K. H. Panofsky, Henry P. Stapp, and Edward Teller for many useful discussions. Since I have not read widely in the literature of this subject, I apologize to any authors who may have already published any of the material given.

Image of the Scientist among High-School Students

A Pilot Study

Margaret Mead and Rhoda Métraux

This study is based on an analysis of a nation-wide sample of essays written by high-school students in response to uncompleted questions. The following explanation was read to all students by each administrator. "The American Association for the Advancement of Science (*1*), a national organization of scientists having over 50,000 members, is interested in finding out confidentially what you think about science and scientists. Therefore, you are asked to write in your own words a statement which tells what you think. What you write is confidential. You are not to sign your name to it. When you have written your statement you are to seal it in an envelope and write the name of school on the envelope. This is not a test in which any one of you will be compared with any other student, either at this school, or at another school. Students at more than 120 schools in the United States are also completing the statement and your answer and theirs will be considered together to really find out what all high-school students think as a group of people."

In general, the study shows that, while an official image of the scientist—that is, an image that is the correct answer to give when the student is asked to speak without personal career involvement—has been built up which is very positive, this is not so when the student's personal choices are involved. Science in general is represented as a good thing: without science we would still be living in caves; science is responsible for progress, is necessary for the defense of the country, is responsible for preserving more lives and for improving the health and comfort of the population. However, when the question becomes one of personal contact with science, as a career choice or involving the choice of a husband, the image is overwhelmingly negative.

This is not a study of what proportion of high-school students are choosing, or will eventually choose, a scientific career. It is a study of the state of mind of the students among whom the occasional future scientist must go to school and of the atmosphere within which the science teacher must teach. It gives us a basis for reexamining the way in which science and the life of the scientist are being presented in the United States today.

Objectives

Our specific objectives in this study were to learn the following.

1) When American secondary-school students are asked to discuss scientists in general, without specific reference to their own career choices or, among girls, to the career choices of their future husbands, what comes to their minds and how are their ideas expressed in images?

2) When American secondary-school students are asked to think of themselves as becoming scientists (boys and girls) or as married to a scientist (girls), what comes to their minds and how are their ideas expressed in images?

3) When the scientist is considered as a general figure and/or as someone the respondent (that is, the student writer) might like to be (or to marry), or, alternatively, might not like to be (or to marry), how do (i) the postive responses (that is, items or phrases, not answers) cluster, and (ii) the negative responses (that is, items or phrases) cluster?

4) When clusters of positive responses and clusters of negative responses are compared and analyzed, in what respects are the two types of clusters of responses (i) clearly distinguishable, and (ii) overlapping?

5) Is a generally positive attitude to the idea of science, an attitude which we

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There is a great disparity between the large amount of effort and money being devoted to interesting young people in careers as scientists or engineers and the small amount of information we have on the attitudes those young people hold toward science and scientists. The Board of Directors of the AAAS has on several occasions discussed this disparity and the desirability of learning more about what high-school students actually think of science and scientists. This paper is one result of those discussions. Hilary Deason, director of the association's Traveling High-School Science Library Program, made all of the arrangements with the high schools and supervised the collection of the students' essays. The analysis of those essays and the preparation of this report were the responsibility of the two authors, Margaret Mead and Rhoda Métraux. Dr. Mead is associate curator of anthropology, American Museum of Natural History, New York, and Dr. Métraux is a research fellow at Cornell Medical College, New York.