# Book Reviews

**Numerical Analysis.** With emphasis on the application of numerical techniques to problems of infinitestimal calculus in single variable. Zdeněk Kopal. Wiley, New York, 1955. 556 pp. Illus. $12.

Numerical analysis is the science upon which the art of computing is based. Details of the art will vary with the computing device being employed—for example, abacus, slide rule, or Univac. However, the glamor of the electronic digital computers and their explosive propagation focus the attention on digital computation and lead to a restricted usage of the terms *computing* and *numerical analysis*. At any rate, most recent books whose titles include the words have little if anything to say about analog computers.

A digital device is designed to represent numbers as sums of powers of some base, in practice 2 or 10. Only a limited class of numbers can be represented exactly (for example, those expressible by ten decimal digits); others are only approximated. The device is further designed to perform only the four arithmetic operations, and the result can be exact in general only when it, as well as the operands, are all members of the limited class of numbers that are exactly representable. The output of a digital device is a finite set of numbers of the limited class. Thus a function, as such, is not a possible output, although a finite set of its functional values, or approximate values, would be.

Many problems of great practical importance, however (differential equations, integral equations), have a function, or several functions, among the unknowns. Such a problem must be somehow reduced so that a finite set of numbers will suffice. These numbers may be values of the function or functions at selected points. A somewhat more general approach is to set up a limited class of functions (polynomials, perhaps), which may or may not include a required function among its members, but from which one may hope to select a member that in some sense approximates the required function. The aforementioned reduction then amounts to finding a system of equations satisfied by those parameters which distinguish the approximating function (for example, the coefficients of an approximating polynomial) from among the others of its class.

Thus the problems for numerical analysis tend to stratify at two fairly distinct levels. At the higher, or secondary, level are the problems of reduction of functional equations to finite equations. At the primary level lie, however, the solution of these finite equations, along with the location of the extremes of functions, and the approximation of functions of rather general classes by functions of some fairly restricted class.

The areas concerned with equations, extremes, and approximations are not wholly distinct. In fact, the approximating function may be designated by parameters that satisfy a given set of equations, or minimize a given function, or both. Thus a separation is rather artificial, and reasonable justice can hardly be done to any one topic without some discussion of the other. Certainly a treatment of one of these topics to the exclusion of the others can scarcely justify being called "numerical analysis" without some qualification.

Nevertheless, Kopal's book, in spite of the general title, is in fact so restricted. It is true that the longish subtitle, which few will remember, gives warning that the contents are less general than the title. But even this, while excluding matrix theory and systems of equations, leaves open the possibility for including a consideration of the zeros and poles of functions of a single variable.

The book deals exclusively with polynomial interpolation, numerical differentiation and integration, and orthogonal polynomials, at the primary level; and, at the secondary level, with ordinary differential equations, including both initial-value problems and boundary-value problems, and with integral equations. An appendix discusses the use of Chebyshev polynomials for optimum-interval interpolation, but Chebyshev approximation (minimal departure) is not discussed.

Within the limited area there is, indeed, a great deal of material in the book. A brief history of numerical analysis provides an interesting introduction. Thereafter, two chapters on polynomial interpolation and numerical differentiation are followed by three on differential equations; and a chapter on mechanical quadrature is followed by one on integral equations. Noteworthy features are the sections on error in Chapters II and IV, the treatment of Runge-Kutta methods, the discussion of mechanical quadrature, the appendix on Chebyshev polynomials, the numerous problems, and the bibliography and notes.

The treatment is certainly not that of a mathematician; it is often prolix, sometimes confusing, and generally uninhibited by undue concern for rigor. As so often in books on the subject, the emphasis is on recipes and not principles. The phraseology is occasionally bizarre. Thus we are told that numerical analysis does not know of irrationals.

Problems in an area not treated are sometimes shrugged off as trivial. Thus, in the treatment of boundary-value problems, the author is careful to keep the matrices of low order, and we are told that (p. 284) once the characteristic values are known "the corresponding characteristic functions can be constructed without difficulty." The author then proceeds to "demonstrate the fact" by an example!

In brief, this is an excellent reference covering a limited area and might make a good textbook within that area, provided that it is consulted with circumspection or presented by an instructor who will fill the gaps.

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**Combustion Processes.** vol. II, **High Speed Aerodynamics and Jet Propulsion.** B. Lewis, R. N. Pease, and H. S. Taylor, Eds. Princeton University Press, Princeton, N.J., 1956. 662 pp. Illus. + plates. $12.50.

The purpose of this series and, in particular, of this volume is perhaps best expressed by the following quotations from the series editor and the volume editor. "Rapid advances made during the past decade on problems associated with high speed flight have brought into even sharper focus the need for a comprehensive and competent treatment of the fundamental aspects of the aerodynamic and propulsion problems of high speed flight." [This volume] "deals with rate processes in chemical reactions, the propagation of chemical reaction by the mechanism of combustion waves and detonation waves, with the effect of turbulence on combustion waves, with processes of simultaneous mixing and combustion of fuels and oxidants and with chemical equilibria."