Book Reviews

Approximations for Digital Computers. Cecil Hastings, Jr., assisted by Jeanne T. Hayward and James P. Wong, Jr. Princeton Univ. Press, Princeton, N.J., 1955. viii + 201 pp. $4.

The advent of automatic computing machinery has raised a host of problems and necessitated a thoroughgoing reexamination of numerical techniques. A striking example of this is to be found in the treatment of special functions. The computer whose only mechanical aid is a desk calculator must lean heavily upon tables. If a differential equation is to be solved, the first endeavor will be to express the solution in terms of tabulated functions. Even an expansion into an infinite series of such functions may be worth while, provided that the convergence is fast enough that relatively few terms contribute significantly to the result.

The use of punched-card machines did not substantially affect this approach. Numerous tables were set up on punched cards at various computing centers and often made available for duplication as needed elsewhere. Apart from the ad hoc methods of circulation, the major novelty lies in the tendency toward higher-order interpolation to permit reducing the number of tabular entries. The Harvard MARKS, all slow by current standards, and the ENIAC, with very limited general purpose storage, each had a few tables built in.

With most of the machines that have begun operating during the current decade, the built-in table has been abandoned, and for several reasons. An obvious factor is that the built-in table ties up a great deal of hardware that might better be used more flexibly, although this was probably as true of the ENIAC, which had tables, as of the UNIVACS which have none. More to the point is the fact that at current speeds it is generally more economical to compute a function, from a power series or continued fraction, or even by solving numerically a differential equation, than it is to refer to a table and do the necessary interpolation. An exponential, for example, can be computed in a few milliseconds whenever needed, under the direction of a program prepared once and for all and occupying only a very small amount of memory space. This very fact implies that there is less need for the function itself than would otherwise be the case. Instead of attempting to transform a differential equation into a form whose solutions are tabulated, one will attempt to transform it into a form more readily amenable to direct numerical solution.

Nevertheless, the need for evaluating special functions has certainly not disappeared, and, furthermore, there is a higher synthesis of the superficially disparate points of view. Clearly in its actual utilization a power-series expansion is always truncated, and it therefore provides only a polynomial approximation to the function. But a table itself ordinarily provides also, by way of a suitable algorithm, a polynomial approximation, or rather many such, according to the interval and the interpolatory technique. But if, in either case, the function is being represented by a polynomial, it seems in order to make a direct attack and ask explicitly for the best polynomial.

The problem of obtaining an optimal representation of an arbitrary, but fixed, continuous function by means of a polynomial of given degree was investigated extensively by the Russian mathematician Chebyshev about a century ago and has been under study ever since. *Optimal* means, here, that the maximal departure of the representation from the function being represented is minimized. Chebyshev proved the basic theorems. If one allows the polynomial to be of degree n, then there will be $n+2$ points at least where the maximum departure is actually reached. It is understood that the representation is required over only a given finite segment of the x-axis. The best approximating function then exists and is even unique.

For machine purposes one can just as well use a quotient of two polynomials instead of a single one. The theory is very much the same, and the flexibility is much greater. In either event the machine has only to store the coefficients (usually six or seven) and a short program to direct the evaluation of the polynomial or fraction.

Unfortunately the proof of the existence of the best approximation is not a constructive proof, and although the best approximations are known for a few simple functions, these are very few indeed. Every case must be handled laboriously on its own. During the late 1940's, Cecil Hastings of the Rand Corporation began constructing Chebyshev approximations (or approximately (!) Chebyshev approximations) for various functions arising in the course of the computing at Rand. The "Hastings approximations" came to be well known and rather extensively used in this country and, doubtless, abroad. The approximations themselves, with error plots, were circulated, but without explanation, and the results gave no hint as to how one might go about constructing other approximations or improving upon those available.

The present little volume is designed to let the computing world in on the preparation and to collect together some of the results. Almost as laconic as the original memoranda, the volume is made up of a series of sketches, each with a brief legend. These illustrate the basic Chebyshev theorem and proceed step by step to show how one can start with an initial crude approximation and subsequently improve it. There are no mathematical demonstrations, but the presentation is admirably intuitive. The computing world is greatly indebted to Hastings for this tour of his workshop.

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Speech: Code, Meaning, and Communication. John W. Black and Wilbur E. Moore. McGraw-Hill, New York–London, 1955. vii + 430 pp. Illus. $4.50.

This book is distinctive in its field by virtue of the information it contains about the neurophysiological processes of abstracting and projecting that are necessarily involved in speaking. These basic organic activities are either disregarded or markedly subordinated to other matters in most textbooks concerned with speech. By comparison, therefore, this textbook by John W. Black, of Ohio State University, and Wilbur E. Moore, of Central Michigan College of Education, is distinguished by its vital and meaningful treatment of vocabulary, meaning, evaluative reactions, logic, probability, semantics, and the related facets of communicative behavior. Any old grad who gets as far as Chapter 6, "The speaker's meanings: speech and evaluation," is likely to wedge a forefinger in the book at that point and go looking for his old speech teacher to show him how he could have made "Freshman Speech" a whale of an interesting and significant course.

Other aspects of speech, dealt with in