5. Addis, T., *et al. J. Exptl. Med.*, **89**, 131 (1949).
6. Masson, G., Corcoran, A., and Page, I. *Am. J. Physiol.*, **162**, 379 (1950).
7. Sellers, A., *et al. Ibid.*, **166**, 619 (1951).
8. Barnafi, L. Personal communication.
9. Burn, H. J. *Quart. J. Pharm. Pharmacol.*, **4**, 517 (1931).
10. Dexter, L., Haynes, F. W., and Bridges, W. C. *J. Clin. Invest.*, **24**, 62 (1943).
11. Braun-Menéndez, E., *et al. Hipertensión Renal.* Buenos Aires (1943).
12. Fasciolo, J. C. *Acta Physiol. Latin Am.*, **1**, 7 (1951).
13. Croxatto, H., Rojas, G., and Barnafi, L. *Ibid.*, 179.

Manuscript received May 9, 1952.

Mapping Functions in Tetrad and Recombinant Analysis

S. Spiegelman

Department of Bacteriology, University of Illinois, Urbana

In the ascomycetes and basidiomycetes, all four products of a single meiosis can be isolated and characterized. Data so obtained possess obvious advantages in precision and statistical efficiency over those provided by the usual genetic material, which involves random sampling of single strands. In particular, the analysis of tetrads offers a sensitive method for examining the details of the meiotic process.

Various methods have been proposed for employing tetrad data in the mapping of loci. Whitehouse (*1*) and Mather and Beale (*2*) have provided a careful and exhaustive analysis of the information available in those cases (e.g., *Neurospora crassa, Bombardia lunata*) in which the linear order of the segregants is known.

The problem of mapping in the more general and widespread situation of unordered spore arrays has been considered by Lindegren (*3*), who developed a graphical method.

A less cumbersome and inherently more accurate approach was made possible by an analytic solution to the problem. To aid in the subsequent discussion we shall adopt the following symbolism: P_{AO}, P_{BO}, etc., will denote the frequencies of M II segregations of the loci, *A, B*, etc. which result from crossovers occurring between the locus concerned and its centromere; P_{AB} will signify the frequency of M II segregations of *A* and *B* to yield tetratype asci; p_{AB} will denote the recombinant frequency between *A* and *B*.

Perkins (*4*) and Whitehouse (*5*) noted that in the cross $AB \times ab$, the frequency of tetratype asci (i.e., *AB, Ab, aB, ab*) is given by

$$P_{AB} = P_{AO} + P_{BO} - \frac{3}{2} P_{AO} P_{BO}, \tag{1}$$

if *A* and *B* are independent. Perkins (*4*) pointed out that with the aid of another locus, *C*, not linked to either *A* or *B*, equations analogous to (1) can be written for P_{AC} and P_{AB}. The resulting set of three simultaneous equations may then be solved for P_{AO}, P_{BO}, and P_{CO}, as was done by Whitehouse (*6*). The three genes can thus be localized with reference to their respective centromeres in terms of their M II segregation frequencies.

It is perhaps worth noting that this procedure cannot be applied if two of the three loci are linked and in the same arm. Thus, if *A* and *B* are so linked, the three probabilities P_{AB}, P_{AC}, and P_{BC} are no longer independent and one obtains instead of (1)

$$P_{AB} = \frac{P_{BO} - P_{AO}}{1 - \frac{3}{2} P_{AO}}, \tag{2}$$

which does not form a solvable set of simultaneous equations with the other two relations that can be written for P_{AC} and P_{BC}.

Data obtained by the proper application of these or analogous methods can provide consistent information in terms of distances from the centromere. Difficulty arises, however, when it becomes necessary to compare such map distances with those obtained by the conventional method, which depends on the frequency of recombinant strands. It has been assumed by the authors mentioned above, as well as by others, that

$$p_{AB} = \frac{1}{2} P_{AB}, \tag{3}$$

where, as above, p_{AB} refers to recombinant frequency between *A* and *B* and P_{AB} the corresponding M II frequency.

The reasoning often offered to justify this conversion is that only one half of all crossovers that occur are observed in ordinary recombinant analysis, since only one chromatid out of any given tetrad is recovered. Although they employ this conversion factor, both Rizet and Engelmann (*7*) and Papazian (*8*) have pointed out that it at best represents an approximation which can be valid only over short map distances. That this contention is correct is evident from the fact that the limit approached by p_{AB} as the number of chiasmata between *A* and *B* increases is ½, whereas the limit of P_{AB} is $\frac{2}{3}$. Thus for long map distances relation (3) would yield a value of 0.33 for p_{AB} instead of the 0.5 to be expected.

The accurate conversion of M II frequencies into recombinant map units requires the derivation of the explicit relation between P_{AB} and p_{AB}. A relation of this kind can be obtained from the corresponding mapping functions. In the absence of chiasmata interference, these functions can be simply deduced, since for any fixed average the number of chiasmata will be distributed according to the terms of the Poisson series. Haldane (*9*) has shown that under these conditions the frequency of recombinant strands is given by

$$p_{AB} = \frac{1}{2}\left(1 - e^{-2m'}\right) \tag{4}$$

where m' is the *average* number of chiasmata per *two strands* occurring between the relevant loci. Mather (*10*) has demonstrated that the proportion of a set of tetrads which will exhibit M II segregation if each tetrad has experienced *precisely* r chiasmata between *A* and *B* is given by

$$P_{AB}(r) = \frac{2}{3}\left[1 - \left(-\frac{1}{2}\right)^{r}\right]. \qquad (5)$$

As Rizet and Engelmann (*7*) pointed out, to convert Eq (5) to a mapping function analogous to (4), Haldane's assumption of a Poissonian distribution must be adopted. Consider a set of tetrads in which an average of m chiasmata has occurred between A and B per tetrad. The fraction which will have experienced precisely r chiasmata in this region will be given by $\frac{m^r}{r!}e^{-m}$. Consequently, the fraction P_{AB} which will exhibit M II segregations between A and B will be given by

$$P_{AB} = \sum_{n=0}^{\infty} \frac{m^r}{r!} e^{-m} P_{AB}(r). \qquad (6)$$

On substituting from Eq (6) for $P_{AB}(r)$ and performing the indicated summation, Eq (4) reduces to

$$P_{AB} = \frac{2}{3}\left(1 - e^{-\frac{3}{2}m}\right). \qquad (7)$$

It should be noted that the M II frequency mapping function provided by Eq (7) behaves very much like its recombinant analog of Eq (4). For small values of m, P_{AB} is equal to m, just as for small values of m', p_{AB} equals m'.

The derivation of relations (3), (5), and (7) assumes the absence of both chiasmata and chromatid interference. Under these conditions the average number of chiasmata per four strands is equal to twice the average number of chiasmata per two strands. Thus 2 m' of Eq (4) is equal to m of Eq (7). These equations may therefore be solved for m, and, from the resulting equality, the desired relation between p_{AB} and P_{AB} is obtained:

$$p_{AB} = \frac{1}{2}\left[1 - \left(1 - \frac{3}{2}P_{AB}\right)^{2/3}\right]. \qquad (8)$$

Eq (8) can be used to convert M II frequencies between the limits of 0 and $\frac{2}{3}$ into the corresponding recombinant map values. For small values of P_{AB}, the quantity $\left(1 - \frac{3}{2}P_{AB}\right)^{2/3}$ is approximated by $(1 - P_{AB})$. Under these conditions relation (8) reduces to the generally used approximation described by Eq (3). The range of P_{AB} values over which this approximation is useful may be seen from Table 1, in which are tabulated corresponding p_{AB} values calculated with the aid of Eq (6). The difference in map units between the calculated values and those obtained from the approximation is indicated in the last column. For values of P_{AB} in excess of 0.2 the approximation becomes progressively more inaccurate. Because of the nature of the function, the extent of the divergence increases rapidly as P_{AB} approaches 0.67.

TABLE 1

COMPARISON OF THE $P/2$ APPROXIMATION FOR RECOMBINANT MAP DISTANCES WITH THOSE OBTAINED FROM EQUATION (6)

P	$P/2$	p	Difference in map units
0	0	0	0
0.10	0.050	0.056	0.6
.20	.100	.106	0.6
.30	.150	.165	1.5
.40	.200	.228	2.8
.50	.250	.302	5.2
.60	.300	.392	9.2
0.67	0.335	0.501	16.6

There are a number of instances in the literature of tetrad analysis in which discrepancies exist between map distances calculated from M II frequencies and those determined by recombinant proportions on identical segregating tetrads. Such discordances may arise from one or both of the following sources: (*a*) the application of the $\frac{1}{2}$ approximation in the conversion of M II frequencies to recombinant distances in situations where this yields an inaccurate estimation; (*b*) a departure from randomness in chromatid crossing over.

The use of Eq (8) would be expected to remove the discrepancy in those falling into the first category. It might, however, increase the degree of divergences that are due to aberrancies in chromosomal mechanics. It is of course the latter that are of greatest interest, and tetrad analysis derives its primary importance principally because it can detect such abnormalities from a comparison of M II and recombinant frequencies on the same material.

As an instance of the first type we may take an example from Whitehouse's (*1*) calculations of map distances in *Neurospora*. The locus for fluffy yielded 57.5% M II segregations and, using the approximation, was placed 28.8 map units away from its centromere. This represents relatively poor agreement with the map distance which is obtained from recombinant frequencies with two other loci, peach and tuft, lying between fluffy and the centromere. Correcting for double crossovers in the region between peach and fluffy, the recombinant values would place fluffy at 37.1 units from the centromere. The discrepancy is virtually removed, however, by using Eq (8) for the conversion, since it is found that an M II frequency of 57.5% corresponds to a recombinant map distance of 37.0.

Of greater interest is the existence of cases in which the correction fails to eliminate the discrepancy completely. Zickler (*11*), working with *Bombardia lunata*, reported a recombinant frequency of 37.1% between the loci for sex and color, whereas the corresponding M II frequencies were 62.7% and 57.7%, respectively. Zickler was of the opinion that these results were inconsistent with normal meiosis, since the M II frequencies yielded 60.2 or 2.5 as the alternative map distances between the two loci, and neither agreed with the recombinant value of 37.1. Ludwig (*12*) and Ryan (*13*) pointed out, however, that if one corrects for double crossovers between the two, assuming the loci are on opposite sides of the centromere, a recombinant value of 42% is obtained from the M II frequencies. This reduces the discrepancy to 5 map units. Both authors, however, used the approximation in converting the M II frequencies. Employing the more precise relation increases the discrepancy to 11 map units. It would appear, then, that Zickler's data probably do require explanation, although it is clearly not necessary, as Ryan (*13*) pointed out, to accept his hypothesis of M II segregation of the centromeres.

References

1. WHITEHOUSE, H. L. K. *New Phytologist,* **41**, 23 (1942).
2. MATHER, K., and BEALE, G. K. *J. Genetics,* **43**, 1 (1942).
3. LINDEGREN, C. C. *Hereditas,* Suppl. Vol., 338 (1949).
4. PERKINS, D. D. *Genetics,* **34**, 607 (1949).
5. WHITEHOUSE, H. L. K. *New Phytologist,* **48**, 212 (1949).
6. ———. *Nature,* **165**, 893 (1950).
7. RIZET, G., and ENGELMANN, C. *Rev. cytol. biol. vegetales,* **11**, 201 (1949).
8. PAPAZIAN, H. G. *Genetics,* **36**, 441 (1951).
9. HALDANE, J. B. S. *Cytologia,* **3**, 54 (1931).
10. MATHER, K. *J. Genetics,* **30**, 53 (1935).
11. ZICKLER, H. *Planta,* **22**, 573 (1934).
12. LUDWIG, W. *Z. ind. Abs. Vererbungsu,* **73**, 332 (1937).
13. RYAN, F. J. *Bull. Torrey Botan. Club,* **70**, 605 (1943).

Manuscript received April 28, 1952.[1]

---

[1] After the present manuscript was accepted for publication a paper by Papazian (*Genetics,* **37**, 175 [1952]) appeared in which relation (8) was derived by a somewhat different argument.

Comments and Communications

Scientific Personnel in the USSR

ANALYSIS of recent Soviet official reports[1] indicates a rapid rate of growth in that country's professional labor force, both in absolute numbers and in relation to total nonagricultural employment. In 1952 the number of persons employed in professional positions who were graduates of higher educational institutions in industry, agriculture, and health totaled 860,000 as compared with 358,000 in 1937. As a proportion of total employment excluding collective-farm workers, this figure increased from about 1.3% in the earlier year to 2.0% in the later.

The 1952 professional labor force includes about 475,000 engineers and natural scientists in manufacturing, construction, transportation, and communications, plus some 145,000 in agriculture. It comprises about 240,000 physicians, dentists, and pharmacists. Professors and instructors in higher educational institutions numbered 80,000 in 1950. In that year, teachers in primary and secondary schools totaled 1,600,000, but two thirds of them had only a secondary school education.

Whereas the over-all enrollment in Soviet primary and nonspecialized secondary schools (37 million in 1950) has been little greater than before the war, that in technical and other specialized secondary schools has risen rapidly. By 1948 it had reached 1,094,000 as compared with 823,000 in 1940; the numbers of graduates in the two years were 252,000 and 164,000, respectively. Comparable data are unavailable for 1952, although the rise in technical school enrollments and graduations has unquestionably continued.

In higher education, the trend has been comparable. By 1948 the number of full-time undergraduate and graduate students aggregated 734,000, compared with 583,000 in 1940. The 1952 figure approximated 974,000. Furthermore, the number of persons taking part-time or correspondence courses has jumped from 229,000 in 1940 to half a million in 1950. The number graduating was 102,000 in 1940, 122,000 in 1948, and about 127,000 in 1950. Graduate students working for strictly research degrees (*Kanditat* and *Doktor*) totaled 21,000 in 1950, compared to a 1941 *goal* of 13,300. On the other hand, the number of higher educational institutions has increased somewhat more slowly than enrollment: 782 in early 1941; 880 in 1950.

Data are unfortunately lacking for a current breakdown of enrollment and graduation in higher education by fields, but it is clear that major fluctuations have taken place in recent years. Thus the immense shortages of medical personnel caused by World War II led to a very high output (presumably based on an accelerated program) in 1948 and 1949. In those years the net increases in Soviet physicians, graduate dentists, and graduate pharmacists were 24,000 and 26,000, respectively. For 1950, however, the number

TABLE 1

Speciality	Percentage of total	
	Enrollment	Annual graduates
Industry	24.4*	20.9*
Transportation and Communications	5.6*	5.9*
Agriculture	8.2*	7.8*
Medicine	18.6	16.3
Education	37.3	42.6
Including foreign-language specialists	2.9	2.5
Art	1.3	1.4
Economics	1.9	2.6
Law	1.1	1.0
Others	1.6	1.5
Total	100.0	100.0

* At least one fifth administrators and political officers.

---

[1] *Tretii Pyatiletnii Plan Razvitiya Narodnoye Khozyaistro SSSR* ("Third Five-Year Plan for the Development of the National Economy of the USSR"), 184, 238; *Gosudarstvennyi Plan Razvitiya Narodnoye Khozyaistvo SSSR na 1941 g* ("State Plan for the Development of the National Economy of the USSR for 1941"), 612–46; *Narodnoye Khozyaistvo: Sbornik* ("National Economy. A Collection") **3**, 413, 445 (1950); **4**, 132, 138, 140–2, 423–4 (1951); *Izvestiya,* Oct. **7**, 1952.