

of variations depending upon the preferred orientation of the crystallites in the sample. From the slope of the ρ vs. $1/T$ curve in the intrinsic range, the width of the forbidden band is found to be 0.33 ± 0.01 ev.

The resistivity has also been measured in samples in the form of single crystals. At 27°C the resistivity is 0.56 ohm-cm in the direction perpendicular to the principal axis, and 0.29 ohm-cm in the parallel direction. Since the samples on which the measurements were taken were well into the intrinsic range at room temperature, these values, which are much higher than those previously reported (3, 4), may be taken as characteristic of pure tellurium.

A complete report, including measurement of the thermoelectric power and Ettingshausen coefficient, is being prepared for publication.

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Interpretation of the Double Reversal of the Hall Effect in Tellurium

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The double reversal of the Hall effect observed in high purity tellurium samples cannot be explained by conventional semiconductor theory. Bottom (1) has shown that the temperature T_r of the reversal at low temperature depends on the concentration of impurities N_{ex} and obeys the usual condition for the Hall constant to become zero.

$$\ln \frac{N_{\text{ex}}}{T_r^{3/2}} = \ln \left\{ A \frac{c^2 - 1}{c} \right\} - \frac{\Delta B}{2kT_r} \quad (1)$$

c is the ratio of electron to hole mobility, ΔB is the width of the energy gap, and A is defined by the relations

$$n' = AT^{3/2} e^{-\Delta B/2kT}, \quad (2)$$

where n' is the concentration of holes or electrons in a pure, intrinsic semiconductor, k is the Boltzmann constant, and T is the absolute temperature.

The higher reversal temperature, as Bottom (1) has shown, is independent of impurity content and is the same for all samples. This can be explained if one considers the number of lattice defects³ N_D/cm^3 , which

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² We call the number of impurities N_{ex} : the number at exhaustion when all impurities are ionized.

³ The author learned only after completion of his paper that this fact was first realized by W. Schottky (in a letter Feb. 2, 1950, to K. Lark-Horovitz). Schottky has even called this behavior a new type of "intrinsic conductivity," where the temperature dependence in the high temperature range corresponds to the formation and dissociation of a Te vacancy plus hole.

in turn depend on the temperature T and on the activation energy of the defect W .

$$N_D = \alpha N_L e^{-W/kT} \quad (3)$$

α is determined by the ratio of lattice defects N_D to lattice sites N_L and is of the order of unity. This equation describes the number of lattice defects introduced in germanium by quenching from elevated temperatures (3).

$$W = E + \frac{\delta E}{\delta T} T. \quad (4)$$

The temperature coefficient of W , suggested by Fan, takes into account a change in the binding energy due to thermal expansion.

The lattice defects are known to give rise to localized states which may act as acceptors (4-6). We can set, therefore, approximately

$$N = N_{\text{ex}} + N_D, \quad (5)$$

and N is constant in the temperature range where N_D is negligible, but will follow the exponential law (3) if N_{ex} becomes very small compared with N_D . The conductivity σ can now be written as

$$\sigma = eb_2[n'(c+1) + N_{\text{ex}} + N_D], \quad (6)$$

where e is the charge of the electron, b_2 the hole mobility, and the other letters have the significance indicated above.

Depending on the temperature range, the conductivity will first be determined from Eq. (2), and at higher temperatures, from Eq. (3). Middleton (7) found in one sample a second high temperature slope of the $\log \rho$ vs. $1/T$ plot. The slope was $8352/T$; and if this is to be attributed to the defects at these high temperatures, one obtains $E = 0.72$ ev.

The condition for zero Hall constant becomes now

$$n' \left(\frac{c^2 - 1}{c} \right) = N_{\text{ex}} + N_D. \quad (7)$$

For a quantitative analysis, the mobility ratio c has to be determined. It was obtained from the ratio of the negative maximum of the Hall constant R_{max} , and the Hall constant at exhaustion R_{ex} , and an analysis of the scattering mechanism in the exhaustion and transition ranges. In this way one obtains $c = 1.7$.⁴

Using Eq. (1) we obtain $\Delta B = 0.36$ ev, and $A = 1.55 \times 10^{15}$.

If in the high temperature range N_{ex} is small compared with N_D , one has for the second reversal at $T = 500^\circ \text{K}$

$$n' \frac{c^2 - 1}{c} = N_D. \quad (8)$$

For $N_{\text{ex}} > 3 \times 10^{17}$ per cc the Hall constant remains positive throughout the whole temperature range. For $N_{\text{ex}} \sim 1.2 \times 10^{17}$ per cc the Hall constant becomes very small and might reach zero at about $T = 480^\circ \text{K}$. Double reversal occurs only if $N_{\text{ex}} < 10^{17}$ per cc. The temperature of the lower reversal is then mainly governed by N_{ex} ; the temperature of the higher reversal occurs at $T = 500^\circ \text{K}$ independent of N_{ex} .

⁴ We have assumed this mobility ratio as constant up to the temperature of the second Hall reversal.

Fig. 1 illustrates this behavior. We have plotted N_D vs $1/T$, giving the temperature dependence of defects. We have also plotted n' vs. $1/T$, which gives essentially the intrinsic slope and $n' \frac{c^2-1}{c}$, which is parallel to the line representing n' . The concentration of impurities at exhaustion is indicated for various tellurium samples measured by Bottom and others (8).

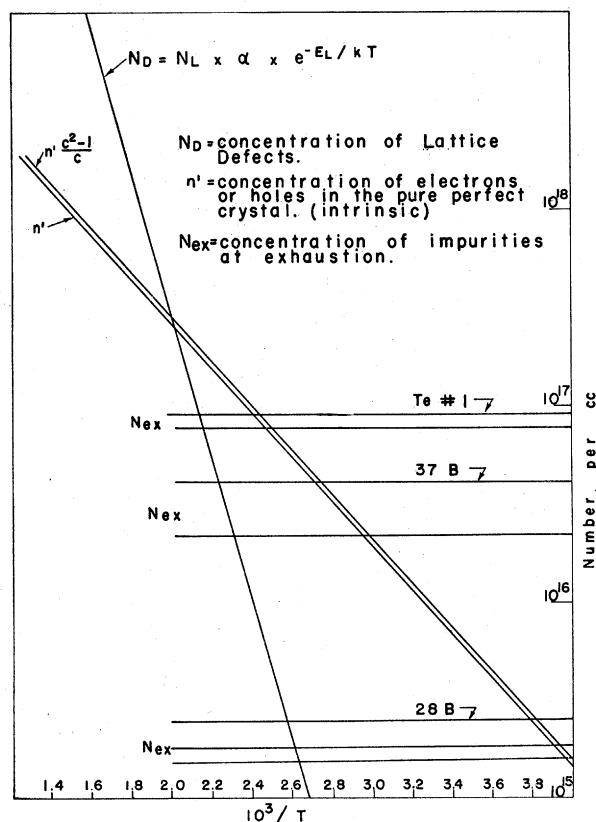


FIG. 1.

The intersection of the lines $N_{ex} = \text{constant}$ with $n' \times \frac{c^2-1}{c}$ gives the low temperature reversal. The intersection of N_D with $n' \frac{c^2-1}{c}$ gives the high temperature reversal.

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Red Blood Cell Studies: Ashby Curves¹

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Important information about the nature and life span of red cells is obtained by the differential agglutination method of Ashby (1). All curves at present available can be explained by assuming (a) that normal red cells undergo a period of wearing or aging which precedes their eventual breakdown, and (b) that the worn or aged red cells are broken down by a random, first order process. The wearing times of individual cells would in general be different, but in the present treatment it is assumed that all the red cells of any one person have the same wearing time. It is also assumed that the rate of breakdown of worn cells is characteristic of their environment.

If it is further assumed that a representative steady state population is transfused, it follows that the number of wearing cells remaining at time t is

$$w = \begin{cases} Y(0)(t_1 - t)/(t_1 + K_1), & 0 \leq t \leq t_1 \\ 0, & t_1 \leq t, \end{cases}$$

if $Y(0)$ = number of transfused erythrocytes, t_1 = their wearing time (which for simplicity is assumed to be the same for all transfused cells), and K_1 = the average lifetime of worn cells in the donor.

On letting y = the number of worn cells remaining at time t ,

$$K_2 \frac{dy}{dt} = \begin{cases} K_2 Y(0)/(t_1 + K_1) - y, & 0 \leq t \leq t_1 \\ -y, & t_1 \leq t \end{cases}$$

if K_2 = the average lifetime of worn cells in the recipient. These K 's are therefore the reciprocals of the corresponding turnover rates for the random breakdown. Since $y = K_1 Y(0)/(t_1 + K_1)$ at $t = 0$ and since $Y = y + w$, the final solution of the differential system is

$$Y = \begin{cases} \left(\frac{Y(0)}{t_1 + K_1} \right) \cdot [t_1 - t + K_2 + (K_1 - K_2) \exp(-t/K_2)], & 0 \leq t \leq t_1 \\ Y(t_1) \exp(-(t - t_1)/K_2), & t_1 \leq t \end{cases} \quad (1)$$

where $Y(t_1) = \left(\frac{Y(0)}{t_1 + K_1} \right) [K_2 + (K_2 - K_1) \exp(-t_1/K_2)]$,

so that if $t_1 \gg K_2$, $Y(t_1) \approx K_2 Y(0)/(t_1 + K_1)$.

For donors in diseased states (such as sickle cell anemia, congenital hemolytic anemia, and acquired hemolytic anemia) in which $\left(\frac{t_1 = 0}{\text{approx}} \right)$, Eq. (1) becomes:

$$Y = Y(0) \exp(-t/K_2). \quad (2)$$

Normal cases are readily interpreted on the basis

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