Technical Papers

Charge Distribution in Nuclei from X-Ray Fine Structure[1]

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Since an atomic nucleus occupies a finite volume, electrons are less tightly bound than they would be to a point nucleus with the same charge. The amount of this reduction is greatest for an *s* electron, which has a large probability of being near the nucleus, less for a $p_{1/2}$ electron, and much less for a $p_{3/2}$ or *d* electron. This effect has previously been detected only as small differences in the electronic spectra of two or more isotopes, produced by small differences in the size of the isotopic nuclei. Experimental observations of this "isotope-effect" splitting are usually less than theoretical predictions by about a factor of two.

The entire shift rather than isotopic variations due to the finite nuclear volume may be determined from x-ray spectra. Thus it should appear in the separation of the L II and L III x-ray energy levels, which is the difference in binding energy of a $2p_{1/2}$ and a $2p_{3/2}$ electron. Conversely, if the separation is calculated allowing for interaction with other electrons (*1*), a discrepancy remains which can be used as a measure of the nuclear size.

The expected order of magnitude of the nuclear effect can be calculated by the method of Broch (*2*) using the normalization method for *p* electrons of Rosenthal and Breit (*3*). With the nucleus assumed to be, as indicated by nuclear scattering experiments and radioactivity, a sphere of radius $1 \cdot 5 \times 10^{-13} A^{1/3}$ cm, where A is the atomic mass, the fractional change in x-ray fine structure has been calculated. Since the distribution of charge in the nucleus is not known, the calculation has been made for the two extreme cases of a uniform charge density in the nucleus and a concentration of all nuclear charges on the surface. The effect increases very rapidly with Z, being as large as about 1/300 of the fine structure splitting for $Z = 90$. The calculated fractional change in the L II–L III separation may be represented very nearly by $f = C(e^{bZ} - 1)$, where $b = .0831$ and $C = 7 \cdot 32 \times 10^{-7}$ for the uniform charge density and $11 \cdot 2 \times 10^{-7}$ for the charge on the surface of the nucleus. Since the nuclear size correction varies so rapidly with Z, it may be distinguished from atomic effects, even when the exact magnitudes of some of these atomic effects are unknown.

Christy and Keller (*1*) have used the observed x-ray fine structure splittings to calculate the fine structure constant. In their calculation, screening of the L shell electrons by each other and by the K electrons was taken into account, but nuclear size was not considered. When their final value of $1/\alpha$ is corrected approximately for quantum electrodynamical interactions by using the anomalous g value of the electron (*4*), one obtains $1/\alpha = 137.11$. This is .05% higher than the value 137.043 given by other accurate measurements (*5*). When the nuclear size effect is included, assuming uniform charge distribution in the nucleus, the x-ray value of $1/\alpha$ is reduced to 137.04 in good agreement with other values.

When the effects discussed by Christy and Keller (*1*) and nuclear size are both taken into account the fine structure splitting is given by

$$\frac{\Delta\nu}{R} = \phi + BZ^2 + C\,\frac{\Delta\nu}{R}\left(e^{bZ} - 1\right),$$

where $\frac{\Delta\nu}{R}$ is the fine structure splitting in Rydbergs, and

$$\phi = \frac{2}{\alpha^2} S(\alpha Z) - 2\alpha^2 Z^3 f(\alpha Z).$$

The first term of ϕ is the Sommerfeld expression for the fine structure, and the second term is a correction calculated by Christy and Keller (*1*). B and C are constants which were determined by making a least square fit of the observed fine structure splittings[4] for $Z = 70$ to 92, with weights proportional to $1/Z^2$ and using $\frac{1}{\alpha} = 137.043$. In this manner we obtain $B = 4 \cdot 816 \times 10^{-4}$ and $C = -9 \cdot 55 \times 10^{-7}$. As a test of the sensitivity of the fit to C this parameter was varied and B re-evaluated by least squares. A 50% change in C in either direction doubles the error squared sum, so that a probable error of about 25% is to be expected.

Thus C deduced from the experimental values is somewhat larger than that for uniform charge density but only by the amount of the experimental error. If nuclear polarization effects are significant, they should decrease the observed value of C. It may hence be concluded that charge is either uniformly distributed in the heavier nuclei or slightly denser toward the surface. Perhaps precise x-ray measurements of some of the heavier elements could improve the accuracy with which this electron-nuclear interaction is known.

The related effect of isotope shift in optical spectra of heavy elements has a magnitude smaller by a factor of two than that calculated using a radius $r = 1.5 \times 10^{-13} A^{1/3}$ and uniform charge density (*6*). The present results exclude the possibility that the charge is concentrated enough toward the center of the nucleus to account for the discrepancy.

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[4] Obtained from the tabulation of $\lambda L\beta_1$, and $\lambda L\alpha_2$ in Y. Cauchois and H. Hulubei, *Longueurs d'Orde des Emissions X et des Discontinuities d'Absorption X*. Paris: Hermann et Cie (1947). (One of the values for Pb is misquoted in this table.)

An alternative explanation for the smallness of the isotope shifts is obtained if one realizes that the last added neutrons should be approximately uniformly distributed over a region including a dense core of nuclear matter and a less dense region at the nuclear surface about 1.5×10^{-13} cm thick. Then only the part of the neutron wave function inside the core radius is effective in displacing protons outward and so causing isotope shifts. The isotope shift is hence about 50% of that calculated when the gradual decrease in nuclear density at the surface is neglected.

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An Artificial Kidney—A Simplified Apparatus

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The object of this report is to present and to describe an apparatus suitable for routine clinical use as an artificial kidney. It is of small size, is simple in design and construction, and has uncomplicated and safe operation.

The ideal indications for use of an artificial kidney may be found in acute reversible conditions causing retention or a critical increase of the crystalloid metabolins, or of other diffusible toxic products in the blood. It becomes necessary then to remove the toxic products by artificial means to sustain life for the period of repair. Such conditions may be mechanical or pathologic, of renal, extrarenal, or mixed origin. They include the nephritides of acute toxemias, of infection, pregnancy, and poisons; the nephroses such as the bichloride of mercury kidney; trauma to kidneys and ureters causing anuria; urinary obstructions; anuria following blood transfusions, severe burns; poisoning caused by diffusible chemicals, such as the barbiturates, sulfonamides; crystalloid and electrolyte imbalance.

The extrarenal routes attempted for excretion of retention products include skin, peritoneal lavage, gastrointestinal tract, kidney transplantation, crossed blood transfusion, plasmapheresis, perfusion through an isolated loop of intestine, and vividialysis or vividiffusion (artificial kidney).

The artificial kidney, a method of vividialysis, has proved the most effective and practical means for the extrarenal excretion of retention products. This work began with Abel, Rowntree, and Turner, in 1913 (*1*), and was followed by Haas (*2*) and Thalhimer (*3*). With the use of heparin in place of the more toxic hirudin, and cellophane as the dialyzer in place of collodion, celluidin, or peritoneal membrane, further progress continued with Kolff (*4*), Murray, Delorme, and Thomas (*5*), Lam and Ponka (*6*), Alwall (*7*), Skeggs and Leonard (*8*), Vanatta, Muirhead, and Grollman (*9*), and Merrill, Smith, Callahan, Thorn, and Walter (*10*).

The various apparatus differ in design, but are basically similar and have one common principle: blood of the subject is introduced through a cellophane tube which is immersed in a bath of perfusion fluid. The diameter of the tube ranges from 0.1 cm to 2.54 cm, with lengths of 10–45 m. The tubing is arranged in parallel series, wound spirally about a vertical stationary drum, or spirally about a horizontal revolving drum. The volume of the bath fluid has varied from 25 to 100 liters, and the fluid is stationary, or it is agitated by a motor-driven propeller. A blood volume of 0.5–1.0 liter is contained in the tubing. To reduce the blood volume and the size of the apparatus, Alwall (*7*) maintains compression of the cellophane tubing by a special device; Skeggs (*8*) employs a series of "kidney units," each consisting of a single layer of sheet cellophane between two corrugated rubber pads.

These systems are necessarily large or complicated and present serious operational and technical difficulties. In circulating a large volume of blood through such extensive extracorporal systems, blood clotting, sterility, hemolysis, pyrogen reactions, blood flow, blood viscosity, hemorrhage into the tubing, and requirements of large amounts of heparin become major problems and increase the operational risk, limiting the use of the artificial kidney as a routine clinical procedure. Although the therapeutic value of the artificial kidney is well established, the treatment is considered radical and heroic.

Our apparatus is shown in Fig. 1. It consists of a silicone-coated glass chamber, with a detachable upper portion. There are conduits attached for the entrance and exit of blood, and for the entrance and exit of a prepared rinsing or perfusion fluid. Twenty-three ft of ¾-in. cellophane tubing are incorporated in the chamber in concentric layers. The cellophane tubing requires no support and is simply placed in the chamber with its free ends connected to a reservoir containing the rinsing fluid. A continuous circulation of the rinsing fluid is first established through the cellophane tubing in the chamber. The fluid leaves the reservoir through conduit *"A"*, and is pumped through conduit *"B"* into the cellophane tubing in the chamber, and leaves the chamber through conduit *"C"* to return to the reservoir, providing a continuous flow of the rinsing fluid through the cellophane tubing. This circulation of the fluid can be accomplished by gravity; however, since this would require two reservoirs and manual attention, it has been found preferable to use