prominent roles in the development of the new formalism. For an electron (or any other so-called particle) the state function f(x, t) takes two important limiting forms. One implies absolute ignorance of the electron's velocity. The function f(x, t) is then completely localized; a single electron is certain to be found at some specific point of space and thus displays the crucial characteristic of a classical particle. The other limiting form implies absolute ignorance of the electron's position. Strangely enough, f(x, t) then represents a sinusoidal wave. This circumstance accounts for de Broglie's great discovery and for the name "wave mechanics," which is often applied to the new quantum theory.

Heisenberg's famous uncertainty principle comes within the present context. The two extreme situations just mentioned illustrate it. For one case there was perfect knowledge of position and complete ignorance of momentum; for the other, the converse. Generally, gain in the knowledge of position may be shown to entail loss in the knowledge of momentum. To be specific, the product of the uncertainties, when expressed in suitable units, is of the order of the magnitude of Planck's constant but never smaller than h. This uncertainty relation springs directly from the use of operators to represent observables and therefore has its origin in the basic methodology of quantum theory.

Bohr's principle of complementarity is another interesting formulation of the same state of affairs. He holds that nature can be described in two complementary ways: (*a*) in terms of objects moving in space and time, this being essentially the method of classical physics; (*b*) in terms of the wave functions of quantum mechanics. One can never be wholly reduced to the other, and Bohr seems to regard both as necessary (complementing each other), for a complete description of experience.

Whatever view one wishes to take of quantum mechanical uncertainty, pessimism should be no part of it. Far from renouncing its hold on nature, the new theory grips nature all the more firmly while relinquishing its attachment in places that have become insecure.

Technical Papers

Nuclear Models

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Since there exists no exact theory of nuclear structure, one is forced to introduce a number of oversimplified nuclear models in order to explain the main features of the experimental material. The models can be classified into two distinct groups according to their fundamental viewpoints: (*a*) the independent particle viewpoint (I.P.); (*b*) the strong interaction viewpoint (S.I.).

Recently the I.P. models have been widely discussed in connection with the surprisingly successful application of shell structure to nuclear properties (*1*). One has observed abnormally large binding energies for nuclei for which either the neutron number or the proton number is equal to a series of so-called magic numbers. This phenomenon was interpreted by many authors by assuming that the nucleons move independently within a common potential trough. The energy levels in this trough are grouped in shells that are completely filled with particles (closed) when a "magic" number is reached. Very simple and general assumptions (e.g., spin orbit coupling) are sufficient to explain the observed values of the magic numbers. The physical properties of the different shells allow the prediction of more specific nuclear data, such as the occurrence of isomers, the spins and, in some cases, the magnetic moments and the quadrupole moments of nuclei in their ground states.

It must be emphasized that this picture is based upon a far-reaching assumption: The nucleons must be able to perform several revolutions on their orbits before they are disturbed and scattered by the interaction with neighbors. This condition is necessary for the existence of a well-defined energy and angular momentum in each separate orbit. The "mean free path" within nuclear matter must be of the order of several nuclear radii in order to justify the existence of separately quantized independent states for each particle.

The S.I. models are based upon the opposite assumption. They are all derived from the concept of the Compound nucleus. Bohr (*2*) has pointed out that, in most nuclear reactions, the incident particle, after entering the target nucleus, shares its energy quickly with all other constituents. This picture presupposes a mean free path of a nucleon that is much *shorter* than the nuclear radius. Nevertheless the Compound nucleus picture is very successful in accounting for the most important features of nuclear reactions. To mention a few examples: The existence of closely spaced and narrow resonances in slow neutron reactions (*2*), the success of the evaporation picture of nuclear reactions with fast particles (*3*), the large values ($\sim \pi R^2$) of reaction cross sections with fast neutrons (*4*).

The two viewpoints seem to be totally contradictory.

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The nuclear forces as we know them from the deuteron and from two-particle scattering experiments represent a strong interaction and therefore suggest the validity of the S.I. viewpoint. In fact, the known scattering cross sections of elementary particles at 20–30 mev (this is the order of the kinetic energy inside a nucleus) would indicate a mean free path of only 10^{-13} cm with nuclear matter. Hence the recent success of the I.P. shell model has led to speculations that envisage much weaker nuclear forces within a nucleus, compared to the ones observed between isolated pairs.

It is the purpose of this note to point out that the I.P. and the S.I. models are perhaps not as contradictory as it appears at first thought. It must be noted that the successful predictions of the shell model are always applied to the ground states or to the lowest excited states of nuclei. The regularities in the binding energies are properties of the ground state, the spin, and the magnetic and electric moments, too. The occurrence of isomers is a problem of the first excited state. Even the small neutron capture cross sections of "magic" nuclei (*5*) can be interpreted by assuming that the ground state of the target nucleus has an unusually low energy. Then the captured neutron forms a compound nucleus of an abnormally low excitation and its level density will also be abnormally low. This leads directly to a low capture cross section, since the neutron width is then much larger than the radiation width.

The applications of the S.I. models are restricted to problems involving high nuclear excitation. The Compound nucleus formed in a nuclear reaction is always excited, at least to an energy larger than the binding energy of the added nuclear particle (about 8 mev for protons or neutrons). Hence it seems that the strong interaction between nucleons within a nucleus is observed only at high nuclear excitations.

The failure of the S.I. viewpoint at low excitation energies does not necessarily imply that no strong interactions exist between nucleons. It is very probable that the Pauli principle prevents the strong interaction from exhibiting the expected effects. The interaction cannot produce the expected scattering within the nucleus, because all quantum states into which the nucleons could be scattered are occupied. Only at higher excitations, when not all of the lowest states are occupied, will scattering take place and prevent the formation of independent orbits.

It may be useful to discuss in this connection an analogous situation that one finds in the theory of the electron motion in solids. The electronic properties of metals and insulators can be described very successfully by assuming that the electrons move in a common potential field, the electric field of the ions in the lattice. The interaction between the electrons is completely neglected. The electronic states in the lattice field exhibit also a kind of shell structure, the Brillouin zones, and an insulator may be called a "magic" crystal for which the shells are completely filled.

The success of this description is perhaps also surprising in view of the fact that the interaction between electrons is by no means small. In fact, an electron with a few electron volts of energy that enters the metal from the outside is stopped in the metal within one or two interatomic distances, simply by the scattering with other metallic electrons. The mean free path of this electron within the metal is not greater than one interatomic distance, as can be shown with a simple calculation using the Rutherford scattering formula. In spite of this fact, the mean free path of the metallic electrons is very much greater than the interatomic distances; in fact, it is limited not at all by the interaction between the electrons but by the irregularities in the lattice. The reason is again found in the Pauli principle, which does not admit any scattering of electrons by electrons, because all states into which the scattering process may lead are occupied. This is not the case for the electron entering into the metal, since it possesses a surplus energy at least equal to the work function. Hence we find long mean free paths in the nonexcited state in spite of strong interaction, but short mean free paths in the highly excited state that is created when an electron enters from the outside.

The conditions of the electrons in a metal are obviously quite different from the conditions of the nucleons in a nucleus. There is no external field in the nucleus corresponding to the ionic field in the crystal. The common potential in the I.P. model is the average effect upon one single nucleon of all other constituents. However, the influence of the Pauli principle upon the mean free path of the electrons may serve as a useful analogy to understand the possibility of an I.P. picture in the presence of strong interaction between nucleons.

It should not be concluded from these considerations that the force between a pair of nucleons within a nucleus is necessarily equal to the force observed with an isolated pair. The failure to explain the saturation properties in heavy nuclei on the basis of the observed exchange character (Serber force) of the neutron proton forces suggests a different character of the nuclear potential within the nucleus. The recent successes of the shell model, however, do not necessarily imply that the forces between nucleons within the nucleus are very weak. In fact, this assumption would contradict the experimental evidence of the validity of the S.I. model in nuclear reactions.

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