Conceptual Foundations of the Quantum Theory

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The World of Classical Physics

FIFTY YEARS AGO, when Planck discovered that light is not a continuous wave but a series of energy pulses, the world of science was amazed. Incredulous and perplexed at first, it hesitated to accept so strange a notion as the "quantum" but was forced nevertheless to acknowledge it as viable and healthy. Thus Planck's famous h survived its first few years as a curiosity and as a misfit, while most physicists tried to isolate it as far as possible from the more acceptable ideas that formed their stock in trade. Its originator himself, who may be quoted here, "tried immediately to weld the elementary quantum of action h somehow into the framework of the classical theory. But in the face of all such attempts, this constant showed itself obdurate. . . ." "Many of my colleagues," he goes on to say, "saw in this something bordering on tragedy" (M. Planck. *Scientific Autobiography.* New York: Philosophical Library [1949]).

Discovered as an exception to all rules, the quantum effect developed during the following two decades into a major threat to the long-cherished idea of continuity. Einstein's explanation of the photoelectric effect (1905) and Bohr's atomic theory (1913) exposed the workings of quantization in a far wider range than had been assumed originally, and the question arose whether all fundamental processes of nature might not take place discontinuously. Light energy, it was learned, was transmitted in pulses called photons; electrons jumped suddenly and unpredictably from atomic orbit to atomic orbit; molecules vibrated in discrete states of motion. Might it not be, then, that all phenomena ultimately partake of such fitfulness, that everything comes in lumps of smallest but finite size, that continuity is a myth?

These were among the most radical questions asked up to 1925. They reflect a readiness on the part of science to abandon some of the traditional ideas about the qualities of the universe and to replace them by features that seemed strange at the time; but the spirit behind the inquiry was still the desire to learn new things about a world of preconceived essences. Science at this time would grant the possibility of unexpected features, such as discontinuity, but would not doubt the basic methodological premises on which our whole conception of the world is based. It did not question complete objectivity of description, the independence of the known from the knower, nor did it cease to think in terms of mechanical models, nor doubt the predetermination of physical events. All these unorthodox doubts have been raised and confirmed in the past twenty-five years. They were precipitated by the remarkable series of incisive discoveries connected with the names of Bohr, Heisenberg, Schroedinger, Dirac, Born, Jordan, Compton, de Broglie, and many others.

Today, many scientists still think of the quantum theory as the doctrine of discontinuity, according to which all ultimate parts and properties of nature have some smallest measure. But it is in fact far more than this: a new way of thinking, a new view of physical reality, a new interpretation of the relation between the observer and the world. The sense in which this is true will be outlined in the following pages.

It is against the backdrop of "classical physics" that these profound changes have taken place, it behooves us to portray the backdrop first. The reader will find in this portrayal a statement of some of his firmest convictions, a summary of the scientific creed of centuries, indeed the gist of what he would regard as common sense. Only on realizing this does the magnitude of the departure that the quantum theory represents become apparent.

Our description of classical physics will concern itself with only one part of this wide field, the part called *classical,* or *Newtonian, mechanics,* which is at once its simplest chapter and the most typical representative of its procedures. It is a lineal descendant of astronomy and has inherited the grandeur, as well as the inexorability, of that ancient branch of science. Kant, whose philosophy can be called the metaphysical distillate of Newtonian mechanics, classed the starry heavens with the human conscience as the two deepest sources of our knowledge and our attitudes.

The motion of the stars is impressively continuous. They occupy every point of their path. The slowness of their motion accentuates its continuity. Clouds often obscure the stars, thus seeming to destroy their steady course. Yet this very interference and the ease with which it can be explained away by reference to the vagaries of the weather make the fact of continuity all the more sure and convincing.

Add to this the well-known circumstances that the path of a heavenly body has a mathematically simple form. Such simplicity is impaired if any points or pieces are missing from the mathematically perfect orbit, and in the same way, a star's motion can hardly sustain the blemish of having blind phases or of being jerky. Thus continuity, certified within limits by vision and supported by maxims of simplicity and perfection, is one of the clearest attributes of celestial motions.

Once the pattern is set, the scientist experiences little difficulty in comparing it with the more ordinary motions of his daily life. Analysis, a little more reliance on the essential simplicity and perfection of the world, restores harmony with the cosmos and reestablishes continuity as a ruling principle of science. Only atomic physics was able to impugn it.

But continuity is only one facet of a more general supposition about nature, which is espoused in its totality by classical mechanics. The stars have attributes besides position and speed of motion; they exhibit brightness and color as well. Continuity is assigned to all of these, and the whole complex of phenomena named a star is expected to behave in a manner we might call *consistent*. On the lower plane of everyday experiences, consistency comes to mean continuity of an ever-widening set of properties, such as size, shape, temperature, energy content, and indeed all the refined attributes with which physical science endows its systems. And, beyond this, consistency requires an interrelation between all of these; hence the inexorability which the human interpreter once saw in the stars is implanted into lesser nature as determinism or causality.

Another aspect has been borrowed from the celestial bodies and invested in classical mechanics generally. It is the aloofness of the stars, their inaccessibility to designed experimentation. Human actions have no effect on them, their fate is independent of man's. To be sure, this stringent kind of independence cannot be carried into objects that the experimenter can manipulate, for he is clearly able to make them do some things they would not do without him. The classical physicist therefore lessens the rigor of celestial motions by advancing the notion of *interaction*. He supposes that object A can influence object B in a precisely determinable way, precise in the sense that he is always able to specify which is object A, which is object B, and which is the interaction. In this way, although he admits of interaction between the observer and what is being observed, he nevertheless retains the essential features of the grand conviction that "here am I," and "over there is the universe." These two are separate entities, engaged in the drama of being, which presents itself under the forms of a spectator and a spectacle. This spectator-spectacle distinction, generated by the early contemplation of the heavens, has continued to be a hall mark of classical mechanics and has characterized all thinking up to 1925. Its renunciation is still regarded as anathema by most scientists and many philosophers. The reason for this stand is a belief that the spectatorial doctrine is the only one that achieves objectivity and insures reliability of report. We now know from an analysis of Heisenberg's uncertainty principle that there are other ways of representing our experience which do not fall short of these desiderata.

It is well to pause and see more concretely what these vaguely worded conceptual premises of classical physics are. For contrast, then, I shall describe the kind of experience that would belie these premises and then raise some questions that might seem natural if such experience were encountered. Imagine the star, while being observed, to behave very much as it does in our world, remaining stationary or slowly moving so long as our eye or the telescope is trained on it. But assume that, when we turn our gaze away and then look again, the star has altered its position, only to behave regularly again while under observation. Certainly, this does not happen among the stars in our world but is conceivable and not contrary to any laws of thought. If it did happen, would we still say without qualification, "the star has position," "its motion is continuous," "the fate of the star is independent of the observer," and would we still maintain the spectatorial doctrine?

Or suppose an object had the ability to outdo a chameleon and change its color erratically on different occasions of observation. Would we then still assign to the object a color, although this color is highly indefinite? If our super-chameleon displayed a different color everytime we looked but retained it while our gaze is fixed upon it, would we regard ourselves as mere spectators or feel in some way responsible for the grotesque emergence of differences? Again, the physicist is not confronted with this situation and is likely to smile at the naïveté of such questions.

But there are fields of experience in which similar questions make sense and are answered. Take an example from psychology. People have at times the quality of being angry. If they possess sufficient self-control, a direct observation may not yield a clue to their anger, and a verbal inquiry may be necessary to ascertain it. The results of inquiries at different times are quite likely to be erratic, and it is certainly not a foregone conclusion that the subject's states of mind are independent of the fact that an inquiry has been conducted. The social and biological sciences abound with similar examples, where the spectator-spectacle view becomes artificial and can be maintained only by an appeal to classical physics, made in the hope that this discipline will prove successful where it has not been tested.

If we take the facts of our simple tests on a person's anger without embellishment, we find, in the first place, that he is sometimes angry and sometimes not. We would therefore not speak of anger as a property which a man always possesses, but one which he *may* exhibit. In the physical world, objects are assumed to have certain properties (like position and size of stars), which they *must* possess at all times. Galileo spoke of them as primary qualities and distinguished them from secondary qualities that arise in the act of perception (e.g., color). Nowadays this distinction is difficult to maintain, chiefly because it is impossible in many instances to prove that a quality owes its occurrence to the perceptory process. Still, there is a difference between a quality that is merely a *latent* or possible attribute, and the position of a physical object, which is assumed to be inalienably *possessed* though its value may change. Thus, for the sake of fixing attention, let us speak of possessed properties when referring to such determinate and intuitively objective qualities as the position of a tree, the mass of a stone, the velocity of an automobile, on the one hand; of latent properties when referring to such transient qualities as the anger of a person, the value of a commodity, and (perhaps) the life of a virus, on the other.

In this language, classical mechanics may be characterized by saying that it regards all properties it uses in the description of experience primarily as possessed by objects. What was called continuity, consistency, independence, is seen to be included in this generalization. Sensory experience, as it explores our far and near surroundings, justifies the point of view of classical mechanics.

The Microcosm

Classical mechanics has largely come to be identified with common sense. As we transport ourselves to the world of atomic magnitudes, which we now propose to do, we shall seem in some measure to be violating common sense. Hence there is need to indicate at once on what grounds we are entitled to abandon this time-tested criterion of truth. And here it may come as a shock to the reader to be told that so-called common sense has never had a shred of validity in the face of new and revolutionary theories of nature. The latter have ever had to assert themselves in the face of reactionary beliefs parading under that guise, and when these new theories succeeded, common sense readily adjusted itself to include them, as it should; for this overrated principle of truth is nothing more than the popular residue of accepted scientific theories and embodies their familiar features. It never leads, it always follows, scientific discovery. D'Alembert unmasked it in his motto for the scientists: *"Allez en avant, la foi vous viendra."*

To survey the facts of the microcosm—i.e., the observations that have led to the construction of the quantum theories in their present form—we assume our sense organs to be replaced by more sensitive devices, such as the electronics expert can actually build, devices which allow us to perceive very small distances, very short intervals of time, and extremely light objects. Although somewhat idealized, this assumption is not pure fiction; nor is such apparatus a fanciful dream. The fact that devices of this kind have actually been built and used saves our story from being imaginary and makes it relevant.

In this atomic world we perceive no coherent objects. Our "eyes" are now sensitive to the single darts of light (photons) cast off by single luminous atoms. Hence our microcosm is not uniformly illuminated and filled with moving things; it presents a speckled kind of vision with bright patches emerging here and there from utter darkness, different patches having different durations. Distant objects of large size and mass exhibit a kind of uniform glow and suggest some cohesion in this chaotic scheme of things, but the smaller dots near by give very little indication of uniformity or pattern.

An unimaginative observer restricted to this world would hardly postulate persistent bodies present at all times; he might indeed doubt the existence of entities except at the moment of vision. He would not find it plausible to speak of the flow of time, regarding "emergence of sensed intervals" as a more satisfactory phrase. To him, continuous space might seem a farfetched abstraction, and if he were to postulate the presence of objects, he would hardly suppose them to have definite positions at all instants of time. Certainly, he would have little occasion to invent the differential calculus.

On closer examination the microcosm reveals some degree of coordination. Patches of light are not completely random but appear in more or less ordered sequences. There are times when nothing can be seen, and then again the visual field is dotted with perceptions. Furthermore, these perceptions often indicate a preferred location in space—though they rarely mark a point. The physicist who knows about the microcosm, when noting this modicum of regularity, will of course take it as the occasion for postulating the existence of objects, vaguely localizable in space and somehow progressing from one place to another; his instinct for causation is thus satisfied. But if he had never seen a perfectly continuous path he would hardly regard them as moving in our sense—position and velocity would doubtless be what I have previously called *latent properties* of an object. He would encounter as much difficulty in the idea of continuous motion as we ordinarily do in the notion of discontinuous emergence in our world.

Properties of objects are like anger of a person in another respect. Our microcosmic observer will see things under two conditions only: Either he illuminates them by means of an external light source, or he waits for them to emit photons. In either case will their manifestations be random. (To be sure, there are possibilities for "tying down" an atom by letting it move in a very small space in a most erratic manner—but such cases are the exception and not the rule.) The macroscopic physicist feels uneasy about this, and he advances the none-too-ingenious conjecture that he, himself, or incidental circumstances, are to blame for the randomness. When things are illuminated, he reasons, they are being bombarded by photons, and their impacts are the random cause of lawless appearances. When atoms are self-luminous, recoils from ejected photons propel the emitters about in unpredictable fashion. But how is he going to tell? No observation is possible without the agencies of external or self-illumination; a control experiment is out of the question. The psychologist can at least ask a subject, "Are you angry because I am asking you this question?" The physicist can ask his atom only, "Are you angry?" Despite all this, it does no harm if the physicist tries to explain the erratic behavior of the atomic world by a reference to causative agencies; it affords him comfort and makes the microcosm seem less strange. The fact is, however, that he is then indulging in a bit of metaphysical speculation, and he ought to be aware of it.

Closer study of phenomena discloses order imposed on randomness even in the Lilliputian world. The *mean* positions of what was construed as objects seem to obey definite laws. That is to say, if a list were made of the appearances in space of a group of luminous dots, and their mean were computed in the manner by which one obtains the center of a population, this center would move more or less in accordance with macroscopic laws. In fact, nature often relieves us of the need for this computation by doing it herself. We have already said that the more distant objects of the microcosm show coherence and a measure of consistency. This is because they are made up of many atoms and large masses, which consolidate their moods into relative certainties.

It is a long journey from the atomic to the celestial sphere, from apparent caprice to the majestic imperturbability of the stars. And yet one can pass from one to the other without changing one's philosophical equipment. The *statistical* regularity, which we noted in the microcosm and which is entirely compatible with individual randomness, can condense itself to practical lawfulness in the domain of large and heavy objects, just as a probability can tend to the limiting value one. This is indeed what happens: Newtonian (classical) mechanics can be shown to be the "limiting form" of quantum mechanics. The universe is therefore still of one piece. Note, however, that the story is not reasonable when told the other way around. Quantum theory is not a limiting form of classical physics, for it cannot be readily conceived how mechanical lawfulness could degenerate into statistical behavior, unless the latter had been embryonically present at the start: Statistical regularity is the more general concept and must be regarded as primary. This confers upon the quantum theory the status of logical priority over classical mechanics and over common sense.

Summarizing, we may say that sensorylike experience in the microcosm lays bare the precariousness of assuming physical properties, like positions and velocities, to be necessarily and at all times possessed by physical systems. They may become latent observables.[1] In particular, continuity and consistency are not suitable attributes of individual atomic behavior.

[1] The word *observable,* introduced by Dirac, has come to mean any physical property which can be observed or measured.

Representation of Experience in Classical Mechanics

The conceptual tools used by the physicist must be in accord with the nature of the experience he wishes to represent. In view of the regularity of stars and stones and all the other objects of the macro-world he must employ a very determinate kind of description, which I shall briefly illustrate. The simplest object, or physical system, is a *particle,* and complex bodies are assumed to be composed of particles. A particle has a definite mass and is assumed to occupy a definite point of space at every instant of time. The *state* of a particle—by this we mean a collection of attributes or properties, all possessed in the present instance, which are just sufficient to allow prediction of future behavior—is given when a position and a velocity (or, better, momentum) are specified. Since these are functions of the time, both must be allowed to vary. All this leads quite naturally to one result: position and velocity are *functions* of time, since a function is that mathematical construct which can be made to vary continuously and then carries exact values for all values of its argument.

By the same token, all other "observables," such as energy, angular momentum, etc., are regarded as functions of the time, either directly or indirectly through their dependence on position and velocity. To use an earlier terminology, they are invariably possessed observables, having a meaning independent of observation at all times. States, in this classical scheme of things, are causally related in the following way. There are available laws of motion, which are differential equations of such character that their solutions are made definite by constants of integration, which are precisely the quantities employed to define a state. In the case of a particle, position and momentum, x and p, determine a state. Newton's second law regulates the motion. It can be integrated to yield x and p, but the integral will be indefinite. When x_1 and p_1, values at some specific time, are given, x and p become determinate and prescribe the motion for all times. Thus the state, x_1 and p_1, of the system at time t_1 is the "cause" of all later states.

One characteristic feature of this formalism is its rigid link-up with observation. A state specifies one possible position and one possible momentum; conversely, a single measurement of each of these observables performed upon a system suffices to determine its state. There is a unique correspondence between a "state variable" and an individual measurement of it; actually this is a typical feature introduced by the tacit use of possessed observables. Somehow, our whole notion of physical reality is colored by this fortunate appearance of a unique correspondence between simple acts of perception and significant theoretical description. Yet the quantum theorist must recognize the limitations of this view in order to keep the prospects of future theoretical developments unencumbered.

We have now seen how naturally our large-scale experience can be comprised under such maxims as these: Physical things consist of particles. Particles have certain determinate properties at all times. Their states are suitable collections of observable quantities, and such observables are represented by mathematical functions. Each observable can in principle be measured through a single act of observation.

The mathematician might wish to express the classical situation by speaking of a simple *isomomorphism* between our description of nature and our immediate experience.

Representation of Experience in the Microcosm

Clearly, so straightforward a scheme will not work in the erratic world of the atom, and physicists have found it necessary to adopt a less familiar formalism. To say that the new theory is less simple is hardly fair, for simplicity is largely a matter of taste and prior conditioning. Let us see what requirements the new formalism ought to satisfy.

We still wish to populate the world of space and time with objects or, to use a more neutral phrase, with physical systems. It goes without saying that these need not be of the material variety (e.g., electromagnetic fields)—need not, in fact, even carry energy (e.g., the sinusoidal component of a group of waves). But a physical system is still the carrier of observables. On the other hand, these observables are not necessarily of the possessed variety, may not have values under all conditions and at all times. Continuous functions are therefore *not* their natural representatives. Yet the observables must provide a link with measurements—though perhaps not a unique link, since measurements are known to scatter—and measurements yield numbers. Consequently, whatever the representative of an observable turns out to be, it must provide numbers to be checked against observations. Whether the state of a physical system may continue to be a collection of observables as it was in classical mechanics, will have to be decided by the available mathematical opportunities; there is, at any rate, no logical requirement that this must be the case.

By the bounty of providence, there is not one, there are at least three formalisms that satisfy the requirements just cited. The matrix theory advanced by Heisenberg, the operator calculus discovered by Schroedinger and perfected by Dirac and Born, the theory of vectors in Hilbert space proposed by von Neumann, are equally satisfactory from most points of view and lead to the same verifiable results. It is therefore superfluous to describe them all, and I shall limit my account to what is essentially the Schroedinger-Dirac-Born method, restricting it further, of course, to conceptual structure without detail.

Rather central in this quantum mechanical scheme of things is the notion of a *mathematical operator*. Though the word sounds forbidding, it signifies something very simple indeed. Let f (x) be some function of the variable x, such as $ax+b$, or sin x. To operate on f(x) means to change it into some other function, say, g(x); hence anything that changes f(x) is called an operator. Multiplication by x, which changes $ax+b$ into ax^2+bx, is an operation; and the symbol $\times x$ that represents it is an operator. So are the integral sign and the symbol for differentiation. It is customary to write a capital letter for an operator. Thus, if Q stands for $\frac{d}{dx}$, and $f(x)=\sin x$, the equation $\frac{d}{dx}(\sin x)=\cos x$ may be written

$$Qf(x)=g(x), \tag{1}$$

g(x) being the new function cos x. Almost any operation can be written in this form, which has the advantage of displaying the mathematical elements that are important here: the *operator* Q, the *operand* f, and the *result* g. The choice of operators in mathematics is, of course, extremely large.

Many of them have this rather interesting property: When applied to *certain* functions, they will simply multiply them by a constant factor. Thus the operator $\frac{d^2}{dx^2}$ when applied to sin x or to cos x, merely changes the sign of the function; i.e., multiplies it by -1. A function F(x) which is "immune" to an operator Q (i.e., is only multiplied by a constant when acted on by Q) is called an *eigenfunction* of Q. In symbols,

$$QF(x)=qF(x). \tag{2}$$

The constant q is said to be an *eigenvalue* of the operator Q.

Suppose that an operator Q is given. We can then write down its eigenvalue equation (2) and find the function $F(x)$ and the corresponding q. But it turns out that there are, in general, many different F's, each with its own eigenvalue q, for which equation (2) has satisfactory[2] solutions. Hence we may say that a mathematical operator Q "generates" a set of eigenfunctions F and a set of eigenvalues q. A specific q may belong to several F's, but there are in general innumerable q's for every Q.

TABLE 1

	Classical mechanics	Quantum mechanics
Observables	x(t), p(t), etc.	Q, P, etc.
State	{x, p}	f(x, t)
Observed values	Values of x, p, etc. at different times	Eigenvalues of Q, P, etc.

After these preliminaries we are ready to state the basic ideas of quantum mechanics in reasonably precise form. Reference is made to Table 1 which

[2] That is, solutions satisfying physical conditions.

contains the nucleus of the following discussion. The column labeled classical mechanics presents a summary of what was said in the preceding section. Position x and momentum p are typical observables of a particle; its state is defined when x and p are known; and the values of x and p revealed in measurements are the values which the observables possess at the time t in question. This last remark will seem trivial to a person who fails to distinguish between possessed and latent properties.

In quantum mechanics an observable is represented by a mathematical operator Q, or P. (We use capital letters for operators.) An operator does not possess numerical values and cannot be measured. But latently it contains numbers, for it can generate them by means of equation (2). And here occurs the miracle of quantization: The discrete values of energy, action, light quanta, and so forth, discovered at the beginning of the century, all happen to be eigenvalues of certain operators! Quantization may indeed be considered a by-product of the indirect description of nature in terms of operators; the roots of quantization lie deeper, therefore, than the mere incidence of discontinuities in the physical world suggests.

The operators themselves have to be found by trial and error, although classical mechanics gives some valuable clues. The business of the theoretical physicist has therefore changed its scope. Whereas he previously sought for functions (such as $\frac{1}{2}mv^2 =$ kinetic energy) to represent his observables, he now searches for operators with suitable eigenvalues. (It should be noted, however, that a function is a specific form of an operator.)

The top and bottom entries in the last column of Table 1 are now explained, but the row labeled "State" is still mysterious. Since operators, and hence observables, do not in general "have" values, a unique collection of observed values cannot define a state in quantum theory. Here the greatest departure from classical physics is made. A state is represented by a function of the coordinates and the time, $f(x, t)$.

But what does a function, $f(x, t)$, have to do with the state of a particle? It certainly means nothing with respect to the behavior of a point moving continuously along a path. Let us remember, however, the idiosyncrasies of the microcosm, which did not contain such uniformly moving points. It presented erratic appearances of "luminous patches," held together by *statistical* coordination. The state function $f(x, t)$ represents this statistical coordination. It tells, in fact, what the probability is that our system shall emerge at the place designated by x at the time t. Hence it conveys fully all the significant elements of our sensory experience in the microcosm; it provides the maximum of available information. When $f(x, t)$ is given, the physicist can compute the *probabilities* for all events (observations) of which he can possibly become aware; but he cannot predict exactly *what* will happen. Indeed, if the present form of the quantum theory is correct—and its immense success leaves little doubt as to its essential truth—precise prediction of all individual events in the microcosm is forever impossible.

Uncertainty

The preceding developments raise important philosophic questions. Very little space can be devoted to them here. We have said that evidence for the persistence of objects in the atomic world is decidedly less obvious than in ordinary experience. Nevertheless, it is our habit to ascribe even the fitful and transient occurrences in the microcosm to the existence of enduring objects. In a sense, we postulate fireflies. And this practice is wholly above reproach, for it has never led us astray. But can we attribute the usual macroscopic qualities to microcosmic objects?

Clearly, it would be nonsense to speak of the color of an electron, since the electron is smaller than a wavelength of visible light. Equally precarious is the assignment of definite size and shape to this physical entity, because there are no unique experimental or theoretical procedures for ascertaining size and shape of such an object. But many of us hesitate to say the same about the electron's position. Yet it appears from every angle that position, too, has become a latent observable in its relevance to atomic entities, and we must not say that an electron *has* position at all times. The discoveries of quantum mechanics have forced us to become suspicious of the indiscriminate way in which the classical physicist assigned intuitable attributes to all parts of his domain. His trust in mechanical models now appears misplaced. Perhaps God is a mathematician and favors mathematical models.

Is the electron a particle or a wave? The vantage point we have now reached permits us to wonder idly why it should be either. If our knowledge of a firefly were confined to its spasmodic emissions of light, if we could not grasp and feel and handle it, we might not wish to speculate or pronounce judgment upon its mechanistic essence. It would remain an object of physical interest, describable in terms of what we know about it, both empirically and by the agency of valid theory. In the case of the firefly the assumption that it be corpuscular would in fact be a fruitful one; above all, it would never lead to contradiction with experience. In the case of an electron this assumption does lead to contradiction. So does the allegation that it be a wave. Hence it is simply neither. Nor is this a logical paradox, for wave and particle are not even exhaustive mechanical alternatives, let alone the only possible forms of physical reality. One of the lessons of quantum mechanics is its reminder that mathematical models are as good as mechanical ones.

This, then, seems to be the resolution of the wave-particle dualism which perturbed the adolescence of the quantum theory. Yet it is no mere historical accident that the ideas of wave and particle played prominent roles in the development of the new formalism. For an electron (or any other so-called particle) the state function f(x, t) takes two important limiting forms. One implies absolute ignorance of the electron's velocity. The function f(x, t) is then completely localized; a single electron is certain to be found at some specific point of space and thus displays the crucial characteristic of a classical particle. The other limiting form implies absolute ignorance of the electron's position. Strangely enough, f(x, t) then represents a sinusoidal wave. This circumstance accounts for de Broglie's great discovery and for the name "wave mechanics," which is often applied to the new quantum theory.

Heisenberg's famous uncertainty principle comes within the present context. The two extreme situations just mentioned illustrate it. For one case there was perfect knowledge of position and complete ignorance of momentum; for the other, the converse. Generally, gain in the knowledge of position may be shown to entail loss in the knowledge of momentum. To be specific, the product of the uncertainties, when expressed in suitable units, is of the order of the magnitude of Planck's constant but never smaller than h. This uncertainty relation springs directly from the use of operators to represent observables and therefore has its origin in the basic methodology of quantum theory.

Bohr's principle of complementarity is another interesting formulation of the same state of affairs. He holds that nature can be described in two complementary ways: (*a*) in terms of objects moving in space and time, this being essentially the method of classical physics; (*b*) in terms of the wave functions of quantum mechanics. One can never be wholly reduced to the other, and Bohr seems to regard both as necessary (complementing each other), for a complete description of experience.

Whatever view one wishes to take of quantum mechanical uncertainty, pessimism should be no part of it. Far from renouncing its hold on nature, the new theory grips nature all the more firmly while relinquishing its attachment in places that have become insecure.

Technical Papers

Nuclear Models

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Since there exists no exact theory of nuclear structure, one is forced to introduce a number of over-simplified nuclear models in order to explain the main features of the experimental material. The models can be classified into two distinct groups according to their fundamental viewpoints: (*a*) the independent particle viewpoint (I.P.); (*b*) the strong interaction viewpoint (S.I.).

Recently the I.P. models have been widely discussed in connection with the surprisingly successful application of shell structure to nuclear properties (*1*). One has observed abnormally large binding energies for nuclei for which either the neutron number or the proton number is equal to a series of so-called magic numbers. This phenomenon was interpreted by many authors by assuming that the nucleons move independently within a common potential trough. The energy levels in this trough are grouped in shells that are completely filled with particles (closed) when a "magic" number is reached. Very simple and general assumptions (e.g., spin orbit coupling) are sufficient to explain the observed values of the magic numbers. The physical properties of the different shells allow the prediction of more specific nuclear data, such as the occurrence of isomers, the spins and, in some cases, the magnetic moments and the quadrupole moments of nuclei in their ground states.

It must be emphasized that this picture is based upon a far-reaching assumption: The nucleons must be able to perform several revolutions on their orbits before they are disturbed and scattered by the interaction with neighbors. This condition is necessary for the existence of a well-defined energy and angular momentum in each separate orbit. The "mean free path" within nuclear matter must be of the order of several nuclear radii in order to justify the existence of separately quantized independent states for each particle.

The S.I. models are based upon the opposite assumption. They are all derived from the concept of the Compound nucleus. Bohr (*2*) has pointed out that, in most nuclear reactions, the incident particle, after entering the target nucleus, shares its energy quickly with all other constituents. This picture presupposes a mean free path of a nucleon that is much *shorter* than the nuclear radius. Nevertheless the Compound nucleus picture is very successful in accounting for the most important features of nuclear reactions. To mention a few examples: The existence of closely spaced and narrow resonances in slow neutron reactions (*2*), the success of the evaporation picture of nuclear reactions with fast particles (*3*), the large values ($\sim \pi R^2$) of reaction cross sections with fast neutrons (*4*).

The two viewpoints seem to be totally contradictory.

[1] On leave from Massachusetts Institute of Technology.