dized derivatives play a significant part in the regulation of cell increase in number[1] has been stimulation of interest in the origin of the parent effective group.

The transformation of lifeless into living—the coming into expression in the chemical elements of those properties and combinations through which living substance is characterized is the becoming of growth.

Although there is an increasing literature dealing with the reactions of the biologically important sulfur groups *in vivo* and *in vitro,* and although there is much written of the ability of animals as well as plants to reduce the H_2S and even other sulfur compounds, not only elemental sulfur but also its highly oxidized derivatives such as sulfuric acid,[2, 3, 4] the beginning formation of the growth significant groups has yet to be traced.

Since soil is the source and plants the presumed medium for transformation of inorganic to organic—and since in root-tips there is concentration of SH and S.S. in organic form,[5] a first step in the search for the origin thereof would seem to be an inquiry into whether root-tips can make SH or S.S. from SO_4.

To do this the common bean (*Phaseolus vulgaris*) was sprouted on wet filter paper over absorbent cotton soaked in well-water in large Petri dishes. From the roots so obtained when 1 to 2 cm in length there was cut the distal 1.0 to 1.5 mm of meristematic tissue with a safety-razor blade. The tissue is recognizable by its yellowish tinge as distinct from the dead white of the proximal length. Some hundreds were cut and kept from drying for each trial. These were mashed and ground with a glass rod in the bottom of a 10 cc test-tube with a pinch of fine beach sand previously treated with cleaning solution and vigorously washed with well-water for 50 shakings. The mash was mixed with 1 cc well-water and centrifuged. Two to four drops of the supernatant fluid were added to each of two 10 cc test-tubes, the one containing 1 cc of a 0.1 per cent. $(NH_4)_2SO_4$ solution and the other plain well-water for control. Other controls were set up of sulfate solution and well-water alone; *viz.,* with no extract, usually two sets of each. After 18 to 24 hours the mixtures saturated with solid $(NH_4)_2SO_4$ were tested on the spot-plate for SH and S.S. with ammoniacal sodium nitroprusside.

Free SH was never found in these experiments which were repeated many times. The disulfide—or S.S. form—however, was detected more often than not after cyanide reduction in the extract-sulfate mixture. It was never found in the sulfate solution alone or well-water alone—and only infrequently in the extract-well-water mixture and then in very much less concentration than in the corresponding extract-sulfate mixture. A positive reaction was never obtained when the extract-sulfate mixture was boiled before incubation. This indicates that boiling destroyed some agent responsible for the reduction of SO_4 to S.S. The reaction was not inhibited in the unboiled extract-sulfate mixture on incubation in the presence of thymol.

These results suggest the presence of an enzyme in the root-tips of *Phaseolus vulgaris* which is capable of reducing SO_4 to S.S.

Although the possibility that the reduction derived from bacterial action is not completely eliminated, yet the fact that thymol did not inhibit its expression and the fact that a negative test was repeatedly given in the boiled extract-sulfate mixture despite opportunity for contamination, is evidence consistent with the enzyme postulate. At any rate the findings justify extension of the inquiry which will now be along more quantitative and other lines of obvious interest.

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EXACT PROBABILITIES IN CERTAIN CARD-MATCHING PROBLEMS

We consider three cases:

The 3 × 3 Case

With a deck of nine cards composed of 3 suits of 3 cards each, the number of possible "runs" is known to be 1,680. If each of these possible runs is "matched" against any fixed run (taken from a duplicate deck), the number of "hits" found may be called the "score" for that run. I have computed by direct elementary means the exact frequency with which each possible score will occur in a complete cycle of runs. This result is given in Table A:

TABLE A

Score	Freq.	Rel. freq.
0	56	.03333
1	216	.12857
2	378	.22500
3	435	.25893
4	324	.19286
5	189	.11250
6	54	.03214
7	27	.01607
8	0	.0
9	1	.00060
	1,680	1.00000

The mean of the distribution is 3, and the standard deviation is 1.5.

The 4 × 4 Case

With a deck of sixteen cards composed of 4 suits of 4 cards each, the number of possible runs is known to be 63,063,000. The corresponding frequency table (computed by the same direct elementary process) is given as Table B.

TABLE B

Score	Freq.	Rel. freq.
0	748,521	.011,869,42
1	3,662,976	.058,084,39
2	8,607,744	.136,494,36
3	12,880,512	.204,248,32
4	13,731,616	.217,744,41
5	11,042,688	.175,105,66
6	6,928,704	.109,869,56
7	3,458,432	.054,840,91
8	1,395,126	.022,122,73
9	453,888	.007,197,37
10	122,016	.001,934,83
11	25,344	.000,401,88
12	4,824	.000,076,50–
13	512	.000,008,12
14	96	.000,001,52
15	0	.0
16	1	.000,000,02
	63,063,000	1.000,000,00

The mean is 4, and the standard deviation is $\sqrt{(3.2)}$, or 1.788,854,38.

The 5 × 5 Case

The case of greatest current interest concerns a deck of twenty-five cards composed of 5 suits of 5 cards each. With such a deck the number of possible runs is 623,360,743,125,120. The labor involved in a direct computation of the exact frequencies in this case appears to be prohibitive. By the use of more advanced methods, however, a solution for the 5 × 5 case has been worked out by my colleague, Dr. T. E. Sterne, and is presented in the following note.

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THE SOLUTION OF A PROBLEM IN PROBABILITY

If a deck of 25 playing-cards, composed of 5 suits of 5 cards each, is shuffled at random and then compared with any fixed sequence of a similar deck, then the number, *r*, of coincidences of suit can take any value, with the exception of 24, from 0 to 25 inclusive. The theoretical probability of *r*, which I call $p(r)$, is a quantity of some current interest. The simple Binomial Law is not strictly applicable, since the coincidences are not independent.

By considering the probabilities in a 5 × 5 contingency table, I have computed the exact values of $p(r)$ for a few large values of *r*. They are:

r	$p(r) 25!/(5!)^5$
21	44125
22	2500
23	250
24	0
25	1

I see no way to extend the exact computations practically to much smaller values of *r*. It is possible, however, to compute the moments of the distribution $p(r)$ exactly. The mean of *r* I find to be 5, and the second, third and fourth moments about the mean to be 25/6, 125/46 and 3625/69, respectively. All these values are exact. The distribution is thus of Pearson's Type I, and I have fitted a Type I curve to the above mean and moments with the results given in the second column of Table 1.

TABLE 1

r	$p(r)$	Binomial
0	.0043	.0038
1	.0247	.0236
2	.0730	.0708
3	.1374	.1358
4	.1852	.1867
5	.1919	.1960
6	1597	.1634
7	.1096	.1108
8	.0633	.0624
9	.0311	.0294
10	.0131	.0118
11	.0047	.0040
12	.0014	.0012
13	.0004	.0003
14	.0001	.0001
15	.0000	.0000
..	....	....
25	.0000	.0000

The values given by the Binomial Law appear in the third column, and appear to be fairly good approximations to the true values. The Binomial Law has the correct mean, but its second, third and fourth moments, 4, 2.4 and 48.16, are all somewhat too small. The Binomial Law would be exact were the coincidences of suit independent.

The values given in the second column above are expected to be correct to within a few units in the last place. Pearson Type I curves were fitted to the moments of the distributions of r in the 3 × 3 and 4 × 4 cases, and then compared with the exact values $p(r)$ computed for those cases by Professor Huntington. The accuracy of the Pearson curve in the 4 × 4 case was much greater than in the 3 × 3 case, and is shown in the comparison given in Table 2.

TABLE 2

r	True $p(r)$	Type I $p(r)$
0	.0119	.0112
1	.0581	.0570
2	.1365	.1374
3	.2042	.2059
4	.2177	.2176
5	.1751	.1741
6	.1099	.1095
7	.0548	.0552
8	.0221	.0224
9	.0072	.0073
10	.0019	.0019
11	.0004	.0004
12	.0001	.0001
13	.0000	.0000
..	....	....
16	.0000	.0000