twisted rails were very common; in fences four rails high, often one out of four and sometimes three out of four would be thus misshappen. As both pulp logs and rails were stripped of bark this twisting is very evident, especially with the latter when they are old and weathered.

The Gaspé is easy of access by auto from points in Maine and Quebec and with a guide familiar with the mountain forests this area would not be a difficult place to conduct investigations. A knowledge of French would be a desideratum, for, although the peninsula has been settled for some 300 years, practically all the population still speaks French only.

BEIRNE B. BRUES

SCIENTIFIC BOOKS

The Theory of Groups and Quantum Mechanics. By HERMANN WEYL. Translated from the second (revised) German edition by H. P. Robertson. New York, E. P. Dutton and Company. 422 pp. $6.00.

THE major importance of this authoritative work on group theory and quantum mechanics has become well known to theoretical physicists since the publication of the first German edition almost four years ago. Besides providing the most modern and stimulating account of the theory of groups as a branch of mathematical discipline, Weyl's latest book gives a masterly account of the applications to quantum mechanics. The first edition suffered from an extreme condensation which made it difficult to read. The second edition includes much new material corresponding to the advances particularly in quantum electrodynamics and in the theory of chemical valence. Moreover, the new edition is characterized by considerable improvement in clarity of presentation. This, together with the fact that the work is now available to American readers in an excellent translation, should give a great impetus to the assimilation of these ideas by American physicists.

Since the subject-matter of the book is quite abstruse, the reviewer feels that a technical criticism of its contents is of less value in a journal of general interests such as SCIENCE than a short essay on the general nature of group theory and its rôle in theoretical physics.

What, then, is group theory? A group consists of any set of elements in particular mathematical operations, which possesses a few simple properties. Each element of the group is an operation performed on some object. The single operation, which is equivalent in its effects to the successive performance of two operations of the group, is also counted as an element of the group. The inverse operation or operation that "undoes" the effect of any element of the group is further counted in the group. Two elements that "undo" each other in this way are called reciprocal elements. The identical operator, or the operation which leaves the object unaltered, also must occur in the group. The mode of combination of the elements must also obey the associative law, a simple general requirement which need not be stated explicitly.

It seems astonishing that the deductive method applied to such general postulates could produce a large body of theorems. But this is the fact, as mathematicians know. Of course, although they delight to express their results in the most abstract form possible, they know too that the group concept arose in a much more concrete way than the preceding paragraph would indicate. Perhaps the subject can be traced to Lagrange's (1770) recognition of the relation of the operations of permuting a series of objects to the theory of algebraic equations. But the real development of the subject with the same application in mind began a century ago (1831) in the researches of Galois. He introduced the term "group" in mathematics in the present technical sense of the word.

The details of the theory and the kind of applications depend much on whether the group contains a finite or an infinite number of elements and if infinite, whether the elements constitute a continuous or a discrete aggregate. In elementary geometry finite groups are of interest in studying regular solids: here each element of the group is the operation of rotating the solid through some particular angle about a particular axis so that the solid after rotation is indistinguishable from its aspect before the rotation. Thus a cube may be rotated through 120 degrees about a body diagonal without altering its geometrical relation to other objects. Closely related to this application to regular solids is the use of the theory of finite groups of rotations to describe and classify the various types of crystal symmetry. Considerations of this sort made possible a theoretical crystallography (Schoenflies, 1891) which has come very much to the foreground since Laue's discovery (1912) of the diffraction of x-rays, which provided a laboratory means for finding the arrangement of atoms in crystals.

The theory of infinite groups, especially of continuous groups, that is, groups each element of which is associated with a value or set of values of one or more continuous variables, finds application in the more advanced parts of geometry. The essence of a system of geometry is contained in its means of characterizing the figures or designs that are counted as equivalent. Thus in ordinary geometry a circle and an ellipse are not counted as equivalent figures. But projective geometry counts as belonging to the same species any plane figure and the shadow cast by that figure when interposed between another plane and a point source of light. A circle may cast an elliptical shadow so an ellipse and a circle will be equivalent.[1] Other special systems of geometry are built on other relations of equivalence than that of the optical shadow just mentioned. But the general idea is much the same, as Klein first recognized in 1872 in his famous "Erlanger Programm." Here Klein first showed how the abstract theory of groups could serve as a unifying principle for the treatment of the numerous special advances in geometry made in the earlier part of the century.

This application of group theory to geometry brings us close to another leading concept which is of great importance to modern geometry and theoretical physics: the notion of invariance, and associated ideas like covariance. When a rigid body is displaced we say that its length is unaltered, that is, the length is unaltered or invariant under the operation of displacement. The group of all displacements includes any translation and any rotation. Length is an invariant of the group. When we say that length of an actual physical body is invariant we are making a physical assertion which may or may not be true of real bodies. But any way this may serve as an illustration of the concept of invariance.

Now it is fairly evident that such invariant quantities are the important elements in a physical theory, and thus we see how Klein's "Erlanger Programm" reaches over into physics: the physicist must learn to recognize the invariant elements in situations that confront him, and he must learn the nature of the group of operations under which that element is an invariant.

Klein himself recognized these things, and so we find that a set of his lectures delivered in 1915–1917 on the history of mathematics bears the subtitle, "Die Grundbegriffe der Invariantentheorie und ihr Eindringen in die mathematische Physik."[2] Aside from development of ordinary vector analysis and quaternions which are as much geometry as mathematical physics, the notion of invariant formulation of physical laws first found expression in the theory of relativity.

It is a triviality to say time is a fourth dimension in a world of space-time, if the statement carries no more content than that t is a fourth independent variable to go with x, y and z. What is the extra something which puts content into the concept of space-time? It is the specification of the group of transformations in space-time with which important physical invariants are associated. The crisis which resulted in the theory of relativity may be described thus: mechanics and electrodynamics each could be given invariant formulations, but the two branches of physics, while having common elements, seemed to be associated with different groups.

The group associated with classical mechanics is known as the Galilean group, while that of electrodynamics is the Lorentz group. Experiments like that of Michelson and Morley decided in favor of the Lorentz group as more fundamental for physics, so mechanics had to be altered to fit. In the Lorentz group, length of a rigid body is not an invariant, so the length appears different to different observers —the famous Lorentz-Fitzgerald contraction.

The general theory of relativity is simply a further development of the preceding ideas. For the examples thus far cited, the operations or transformations are linear in the coordinates of the points involved. Let them be general analytic transformations and let invariants of such an extended group be studied and the mathematical substratum of the general theory lies at hand. Of course to make a physical theory these mathematical results need coordination to the world with physical postulates, but the theory of groups provides the basis.

Now at last to quantum theory. In the examples heretofore cited, where the subject-matter is relatively less abstract, the development actually went ahead pretty much without the aid of the theory of groups. After it was completed the group theory provided the most systematic and unified standpoint from which to regard the results. But that was not the case with quantum mechanics. Here the applications of group methods (first by Wigner, then soon afterward by Heisenberg, von Neumann, Weyl and others) actually preceded development in other ways and first gave some of the most important new concrete results of quantum mechanics. The main applicability of group theory to quantum mechanics touches on two invariantive properties of the equations: An atomic system is invariant under ordinary rotations of the coordinate axes, hence the applicability of the continuous group of rotations as in geometrical problems; moreover, the different electrons in a many electron system are dynamically indistinguishable so the finite group whose elements are permutations of the electrons finds use here.

The theory of groups should be thought of as more than a tool for solving special problems, however,

[1] Actually the relation of projectivity is more general than that of perspectivity, but that does not alter the argument.

[2] Is it permissible to praise one book while reviewing another? This book of Klein's is delightful reading and the most valuable mathematical historical work that I have seen.

and it is in the connection now to be mentioned that the theory probably is most important for quantum theory and for theoretical physics in general. In spite of successes the present laws of quantum physics have their defects, and so it is necessary to find new and better ones. In the search for these, certain broad general requirements are laid down by the restrictions of invariance under different groups. The theoretical physicist need not waste time looking for laws outside the bounds thus set. More positively he finds on looking inside these bounds that the researches of pure mathematicians there have already provided a store of possibilities that await application to physics. This is the line of attack which will probably prove most fruitful in future modifications of the present quantum theory. It is for stimulation in this direction that a study of Weyl's book is perhaps most profitable.

To conclude, I can not refrain from giving wider circulation to an interesting historical point connected with the modern theories of chemical valence. In 1878, Sylvester, then professor of mathematics at Johns Hopkins, was worried about a lecture he had promised to give. He writes, "Casting about, as I lay awake in bed one night, to discover some means of conveying an intelligible conception of the objects of modern algebra to a mixed society . . . and impressed as I had long been with a feeling of affinity if not identity of object between the inquiry into compound radicals and the search for "Grundformen" or irreducible invariants, I was agreeably surprised to find, of a sudden, distinctly pictured on my mental retina a chemico-graphical image . . ." He then goes on to expound a very close analogy between his algebraic researches in invariants and the theory of chemical valence. He concludes with the sentence, "Thus we see that chemistry is the counterpart of a province of algebra as probably the whole universe of fact is, or must be, of the universe of thought."

This paper[3] does not seem to have attracted much attention: it was probably too mathematical for the chemists and too chemical for the mathematicians. It is mentioned, however, by Study,[4] in the Encyclopädie der Mathematischen Wissenschaften, who is somewhat inclined to sneer. "Formal analogies to the chemical structural formulas which occur in the theory of invariants of binary (and other) algebraic forms, have given rise to the fantastic hope," he writes, "that chemistry has something to gain from this branch of algebra. M. Noether has rightly, in a report on such attempts, pointed out the superficiality of the whole relationship and the lack of an actual analogy."

But the point of the story is that just exactly Sylvester's analogy and detailed calculations now play a rôle in quantum mechanical theories of valence, as Weyl has shown. The moral is obvious.

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SCIENTIFIC APPARATUS AND LABORATORY METHODS

AN INEXPENSIVE MICROGRAPHIC PROJECTOR

A PROJECTION and drawing apparatus which embodies many desirable features not found in commercial machines can be made after the design in Figs. 1, 2 and 3 accompanying this article.

An ordinary microscope is fastened to a vertical stand and inverted so that the light from a powerful lamp passes through the microscope projecting the image on a table to which the stand is attached.

The support is made of such material as is available to every laboratory. Ordinary ¾ inch galvanized pipes (B) are attached to a 47″ x 10″ x 1¼″ board by means of the collars (B′). The lamp is bolted to the wooden carrier (C) and the microscope is attached to a similar carrier (E) by means of a bolt and a strip of metal (D). Back and top views of a carrier are shown in Fig. 2, to show the location of the grooves which fit around the pipes (B). The carriers are held in place by tightening the bolts with wing nuts. The stand is held in a vertical position by the stove bolts (F) which hold it to the boards (H). These boards are fastened to the table by means of wood screws. The table top is made of white pine.

The microscope should be equipped with a substage condenser and interchangeable oculars of different magnifications. A loose ocular is held in place by inserting a very small piece of paper between it and the draw tube. A Bausch and Lomb light with a six-volt ribbon filament bulb gives very satisfactory illumination.

To operate the microscope and lamp, attach as illustrated. A slide is clamped on the microscope stage in the usual manner. A mechanical stage may be used to good advantage. Next the lamp is adjusted to obtain the maximum illumination. For low power work better illumination is secured when the substage condenser is removed. The image is brought into focus on a sheet of white paper on the table top. The size of the image may be regulated by changing the distance between the microscope (E) and the paper. To absorb the heat from the lamp a water

[3] Sylvester, ''Mathematical Papers,'' 3, 148.

[4] Encylopädie, VI, 389.