DISCUSSION AND CORRESPONDENCE

ARCHIMEDES AND TRIGONOMETRY

ARCHIMEDES (287–212 B. C.) is commonly regarded as the greatest mathematician of antiquity, but it is only recently that he has been credited by modern writers on the history of mathematics with fundamental developments in trigonometry. Our text-books on this subject usually refer to the Greek astronomer, Hipparchus, who lived about a century later than Archimedes, as the founder of trigonometry. It should therefore be of wide interest to note here that recent discoveries relating to the works of Archimedes, especially to one devoted to the heptagon, seem to establish the fact that we now have more substantial reasons for regarding Archimedes as the founder of trigonometry than we have for giving this credit to Hipparchus. In particular, Archimedes seems to have known rules which are equivalent to our common formulas for the sine and the cosine of the sum and the difference of two angles.

In a recent number of the *Archiv für Geschichte der Mathematik, der Naturwissenschaften und der Technik,* Volume 10, page 432, the well-known German writer on the history of elementary mathematics, Johannes Tropfke, discusses at considerable length some of the necessary modifications relating to the history of trigonometry which result from the recent discoveries as regards the work of Archimedes. He points out, in particular, that Archimedes knew the rule which is now commonly expressed by the following formula:

$$\sin\frac{A}{2}=\sqrt{\frac{1-\cos A}{2}}$$

The ancient Hindu and Arabian writers regarded this rule as one of the most important ones of trigonometry, and it has been assumed heretofore that it was probably due to Ptolemy, who used it without referring to its earlier use by others. It is very interesting to note therefore that it now seems to be due to Archimedes, who lived about four hundred years earlier than Ptolemy. The recent discoveries to which we have referred tend also to give additional support to the view that the so-called Heron formula for the area of a plane triangle in terms of its sides is due to Archimedes, and this constitutes another substantial reason for regarding him as the founder of trigonometry.

It should be added that these discoveries relate to Arabian translations and to references found in Arabian works, and it is well known that many such references are unreliable. In the present instance there are, however, many indirect evidences which tend to support these references. At any rate it is interesting to know that discoveries which promise so much for the history of trigonometry have recently been made, especially since the work of writers who preceded Archimedes seems to show clearly that the fundamental rules to which we referred above could not have been known in their times, and hence it does not seem likely that they could have been known long before the times of Archimedes. Such discoveries also tend to emphasize the important fact that the rapid advances which are being made in the history of science are apt to affect our views as regards some of the oldest and most fundamental developments.

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REPRODUCTION RATE IN WILD RATS

As a contribution to the subject dealt with by Dr. Donaldson in SCIENCE for March 20, 1925, "Control of Rat Population," we submit the following:

In 1922 the board of health of New Bedford, Mass., at the suggestion of the U. S. Public Health Service, undertook a rodent survey of the town which was carried on for a period of about two years. Two trappers were employed to trap rats in the business portion of the city and along the water front. The rats were brought to the laboratory of the board of health and examined for lesions of plague by one of us (C. S. S.). Since the whole number examined was small (about 6,000) it was possible to extend the examination beyond the mere search for plague. Careful observations of pregnancy in the female and of the number of fetuses were recorded. Of the total number of female rats examined, seven per cent. were found to be pregnant. This percentage varied little from season to season.

The female rat becomes sexually mature at about three months of age and remains fertile until about twenty months of age. The average life-span of the rat is about three years. Thus the period of fertility is approximately one half of the whole life-span. If the rats which were trapped were equally distributed as to age, approximately one half of the females brought to the laboratory would be fertile. The period of gestation in the rat is about twenty-one days. Pregnancy is grossly visible in the exposed uterus for about sixteen days. If the female rats which were trapped were all reproducing at the rate of one litter per year and there was a purely random distribution as above-mentioned, there would be about one chance in forty-five ($16/365 \times \frac{1}{2}$) of finding pregnancy in any female, that is, about 2 per cent. of the females would be pregnant. The fact that 7 per cent. of the rats were found to be pregnant seems to indicate that

rats under the conditions obtaining in New Bedford were bearing at the rate of from three to four litters per year.

Limitation of numbers in nature is thus seen to be accomplished by a restriction of the reproduction rate as well as through the action of natural enemies. In other words, a physiological limit is imposed probably through the influence of nutrition. Trapping or poisoning merely serves to increase the available food supply for survivors.

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VITAMIN B

Now that the vitamin originally known as "Vitamin B" has been definitely shown to be made up of at least two other vitamins, this laboratory has devised a method of separating these vitamins from each other and thus leaving them in a liquid form which can be easily used.

When these two fractions are fed alone and separately from the basal diet there is only a slight stimulation of growth for two or three weeks and then a gradual decline, but when the two are fed together there is a moderate rate of growth. This rate of growth, however, is not what one expects from the amount used (equivalent to 1.0 gm of original yeast daily) nor is the rate of growth comparable to that produced by the original yeast. In looking about for an explanation it was thought that possibly the yeast-residue contained the missing factor. A check lot of rats was given this yeast-residue, but instead of a good growth there was slight growth for about two weeks and then a decline and death. However, when these two other vitamins were added to the yeast-residue and the same fed to rats, excellent growth resulted. This new substance found in the yeast-residue after the two other vitamins have been removed meets all the definitions of a vitamin. It appears to be thermostable, and insoluble in water. It activates the two other vitamins of the vitamin-B complex and causes a greater growth than the two alone.

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WHAT IS OSMOSIS?

THE recent edition of the text-book of General Botany by Holman and Robbins states:

> The word osmosis has been given such a variety of meanings by those who have used it that it has lost any precise meaning whatever. As far as possible we shall avoid its use.

That it has been badly used, or rather misused, is evident, but should it be abandoned for this reason? The word is applied to a very definite physical phenomenon and has an exact meaning regardless of whether it has been misused or not.

Examination of standard text-books of botany reveals the source of the present status of the term. A single illustration will suffice. One widely used text states:

> Diffusion through a membrane is osmosis. When two fluids (liquids or gases) of different densities are separated by a porous membrane, diffusion through the membrane will take place until equilibrium results. The diffusion will be more rapid from the less dense to the more dense fluid.

The use of the term *dense* in this connection is meaningless. Students usually interpret it in terms of viscosity, a condition unrelated to osmosis. If it be considered to mean specific gravity then again the definition will not account for the direction of movements of all materials, for the dissolved salts of an egg, to use the illustration given in connection with this definition, will diffuse out into the water in which the egg is immersed at the same time water diffuses in. In this case, however, the movement is from the egg or "denser" medium, to the water or "less dense" medium. Therefore, the "law" is not applicable and consistent for the relatively simple example used to illustrate the process.

Any discussion introducing such terms as "dense solution," "less dense solution," "weaker solution," "stronger solution," and similar descriptions of the concentration of materials in a solution is entirely misleading and can but cause confusion.

A clear statement of the principles involved should remove all possibility of misunderstanding and restore to usage a term too important to be discarded.

In the first place osmosis follows the simple law of diffusion in that the direction of major movement of any material is determined by the concentration, *i.e.*, the number of molecules or ions of that material. The general direction of movement is always from a region where the diffusing material is higher in concentration of particles. The particles of the diffusing material are moving in *both* directions through the membrane, but more are moving away from the position in which most are found than are returning.

Materials move independently of each other, no matter how heterogeneous the solution in contact with the membrane may be. In no case are the diffusing particles dependent on the movement of water or any other material for their own movement.

Osmosis, then, can be defined briefly and accurately as diffusion through a membrane, the direction of