SCIENCE

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A DECADE OF AMERICAN MATHEMATICS

THE year just closing carries with it into the past another calendar decade, and the fact suggests that I take up with an audience representing the mathematical section of the American Association for the Advancement of Science and the two other mathematical societies meeting with it, a sketch of the progress of our science in this country during the decade. In doing this, I am led to reflect, when I think of the struggle that has marked the period, that though it is difficult to see how a thoughtful and disinterested person can enthuse over international rivalries in territory, dominion, trade advantages or other details of national prestige which are pregnant with dangers of destruction far beyond any possible advantages gained, a desire for national preeminence in scientific attainment is most wholesome and valuable.

I wish I might, therefore, compare the work of America during the decade with that of other countries. But even if this were fair, in view of the handicap the war has imposed on other countries, it would inevitably entail a sitting in judgment on questions of value over a field so broad, with so large a body of workers, that I have hesitated to assume the competency or to appropriate the time requisite to a proper performance of the task.

Instead, I am restricting myself to a review of some aspects of the work of this country alone, seeking to find the directions it has taken, to find some of the respects in which it has been weak, and in which strong, and to draw a few conclusions as to strengthening it in the future.

As to an anlysis of the contributions made, you will agree that since over 1,200 articles

[1] Address delivered as retiring vice-president of Section A of the American Association for the Advancement of Science, at Chicago, Dec. 29, 1920.

have been published since 1910, a detailed examination of those articles would be impossible. I have, as a matter of fact, obtained what I believe to be a fairly complete list of these artcles, and made a rough classification of them according to subject matter. Perhaps a quantitative comparison, based on numbers of pages, would have been more informative than one based merely on numbers of titles, but this too would have been open to criticism, and somewhat more difficult to obtain and digest. If you will bear in mind the meaning of the figures given, I have little fear that you will over-estimate their significance, or infer that I have any disposition to propose *any* quantitative test as the sole measure of the excellence of an individual's scientific output. On the contrary, I should prefer six pages of Fredholm's in the *Proceedings* of the Royal Academy of Science of Sweden of 1900 to scores of titles and many hundred pages that might be picked out from journals on the other side, or this side, of the water.

The limitations on the statistical field before us must first be stated. It includes no historical, biographical, or philosophical contributions, and only such in applied mathematics as were contributed by men primarily mathematicians, or appeared in journals devoted entirely to mathematics. It does not, moreover, contain articles contributed to journals of primarily didactic emphasis. Otherwise, it is intended to be complete, and contains contributions to a considerable number of foreign periodicals.

I wish to consider first the distribution of effort amongst various sub-fields of mathematics, and then to comment on some other aspects of interest presented by the data collected. In the matter of classification, in addition to certain customary headings, I have endeavored to separate out a few other classes of subjects of interest for the purposes in hand: first, certain topics whose present vitality and interest among mathematicians generally have been pointed out by Bliss, Van Vleck and others on occasions similar to this, and secondly some topics characteristically American in that Americans have taken a significant or preponderant part in their development. The distribution of titles among the headings selected follows, the numbers given being the percentages of the total number of titles found, or 1,258.

Algebra		
Groups	8.9	
Theory of numbers (including theory of irrationals)	6.0	
Theory of equations, matrices and determinants	4.8	
Higher complex algebras	1.8	
Modular analysis	1.7	
Invariants	1.4	
Combinatory analysis	1.1	
Probabilities and statistics	0.5	
		26.2
Analysis		
Theory of functions of one or more complex variables	6.4	
Theory of functions of a real variable	6.0	
Differential equations	4.3	
Sturmian problems, including Fourier Series	3.9	
General analysis, calcul fonctionel	2.9	
Integral equations	2.5	
Calculus of variations	2.3	
Analysis situs	2.1	
Theory of integration (Lebesgue, etc.)	2.0	
Difference equations	1.2	
Functions of infinitely many variables	1.1	
Point sets	1.0	
Other analysis	5.3	
		41.0
Geometry		
Metric differential geometry	6.0	
Analytical geometry of curves and surfaces	5.7	
Geometry of hyperspace	2.0	
Geometric transformations	1.4	
Projective differential geometry	1.7	
Configurations of a finite number of elements	1.0	
Geometry of forms	0.9	
Modular geometry	0.8	
Congruences and complexes	0.6	
Projective geometry (other than differential)	0.6	
Non-Euclidian geometry	0.5	
Other geometry	1.0	
		22.2

Applied mathematics		
Mechanics of continua (including potential theory and electro-magnetic phenomena)	4.7	
Kinematics and geometrical mechanics	1.7	
Celestial mechanics	1.7	
		8.1
Postulate theories		2.5
		100.0

It will be noticed that algebra and analysis constitute about two thirds of the whole, though this is not surprising in view of their large variety of phases and methods. Their share, however, is larger than in most countries, doubtless because of the prevailing tendencies in the countries to which our mathematicians went for training during the closing decades of the last century. I can not help feeling that a more even balance would be desirable, because of the considerable suggestive help of the more intuitive branches of mathematics. Particularly does it seem regrettable that mathematical physics has not received more attention from mathematicians. It is true that some work has escaped a place in the data of the present study because it has not found its way into mathematical periodicals. For instance, a former member of the ordnance department has told me that he has in his possession over a hundred copies, mostly unpublished blue-prints, of articles on ballistics. But in view of the reputed practical temperament of the American people, in view of the racial traditions we might naturally have inherited from Great Britain, in view of its service to mathematics through its great suggestiveness of interesting problems, and in view of the service of mathematics, through mathematical physics, to physics and engineering, it does seem clear that a greater cultivation of this field in this country is most desirable. In fact, it might almost be considered as characteristic of the decade that this desideratum has been repeatedly and forcefully pointed out.

One reason for the situation which exists is to be found in our tendency to early and over specialization. Our physics departments are apt to load their students with their own courses, with emphasis on the experimental side, often content to have their graduates equipped with the calculus and a formal course in differential equations; while, on the other hand, little physics is usually required of students concentrating in mathematics. This is in part due to lack of mutual confidence, and in part to the student's own haste to receive his degree. Instruction in mathematical physics should be given by mathematical physicists. But until we have produced a more adequate supply of these, mathematician and physicist must cooperate. We can at least offer courses in those parts of mathematics which are of fundamental importance to physics, and in which details of rigor are replaced by cautions, in case of real danger, and in which a sympathetic attitude toward a desire to find out how nature works replaces a disdain for everything aside from the mathematical game, the instructor bearing in mind that the physicist has always the appeal to experiment with which to check his logic. On the other hand, it is probable that lecture courses in physics would be more frequented by students of mathematics if an attempt were consistently and constantly made to draw a clear line between mathematical consequences of previously established results and fresh appeals to experiment or new physical hypotheses. The more this distinction can be made, and the more the physical assumptions can be simplified and gathered into groups at the beginning of course or topic, the more will the course be likely to appeal to the student with mathematical predisposition.

Returning to our table for a glance at the distribution of effort we find the place occupied by algebra even higher than we should expect. This is largely due to the work of two men, Dickson, in the theory of numbers, of groups, and in allied subjects of algebra, and Miller, in the theory of groups. Other investigators whose work has enriched this field include Blichfeldt, Carmichael, Vandiver, Bell and Lehmer, in the theory of numbers; Glenn, Carmichael, Coble, Curtiss, Bennett, Metzler,

Wedderburn and Rice in the theory of equations, matrices and determinants; Miss Hazlett and Glenn in invariants and in modular analysis; W. A. Manning and H. H. Mitchell in the theory of groups; E. B. Wilson and Shaw in vector theory and higher complex algebras; Rietz and Dodd in probabilities and statistics; White, Coble and Cole in combinatory analysis.

Under the heading of analysis proper, in which the notion of limit plays a role, we find the theory of functions of complex variables taking first place. Our progress along these lines is largely due to Osgood, although there is also found a gratifying variety of contributions on conformal mapping, the theory of algebraic functions, and special analytic functions by Lefschetz, Gronwall, Haskins, and others. The theory of functions of a real variable would normally come higher on the list but for the fact that certain topics usually here included have been separated out, such as Fourier series, point-sets, etc. The theory of functions of a real variable is characterized by the fact that it has a larger number of individual contributors than the other topics, although the work of Blumberg deserves special notice. The field of differential equations, apart from Sturmian problems, has had, except for three fundamental papers of Birkhoff, comparatively little and scattered attention. Macmillan and Lipka have, however, written interesting papers on this topic. In the field of Sturmian problems including boundary value problems, oscillation and expansion problems, we may take distinct satisfaction in the valuable work of Bôcher, Richardson, Birkhoff, Jackson, and their pupils and followers. The calculus of variations, once characterized by Schwarz as the most interesting and difficult branch of mathematics, has had comparatively few devotees, but contributions of importance have been made by Bliss, certain of his pupils, by Dresden and E. V. Miles.

I wish now to speak briefly of certain comparatively new branches of analysis. Professor White once pointed out in an interesting statistical review of mathematical development[2] a distinct tendency to follow fashions. The reflection that men are apt to be stimulated by each other's work may rob this fact of some of its surprise, but the substantiality of the fact can not be denied. Of course when a new domain is opened up by fundamental discoveries, it is to be expected that sooner or later the event will be followed by a widespread and rapid development of that domain. An interesting example of a delay in such development is found in the fact that Fredholm's paper on integral equations, above alluded to, lay for two years unnoticed until the labors of Hilbert gave it its due prominence. New domains of the sort alluded to are at present: the still vital subject of integral equations, the related field of functions of infinitely many variables (though Hill and von Koch considerably antedate Fredholm), the theory of generalized integrals opened up by Lebesgue, general analysis, due to E. H. Moore, Fréchet and Volterra, and one or two other fields to be mentioned presently. It seems to me that in view of the general attention being given to these subjects, American interest in them has been distinctly less than it should have been. General analysis leads, with 2.9 per cent. of the total number of papers. The general analysis of Moore has been ably cultivated by his pupils, Hildebrand, Chittenden, and others, while the calcul fonctionel has had fewer devotees. But along the latter lines should be mentioned the papers of C. A. Fischer, Evans, and the two articles of Bliss inspired by his work in ballistics. The cultivation of integral equations in this country has been due to several influences. Besides the general theory of Moore, we find interest in the subject stimulated by Bôcher and Volterra, the contributions coming mainly from the pens of Mrs. Pell, Hurwitz, and Evans, respectively. The theory of functions of infinitely many variables receives more than one contribution each from but two authors, Hart and Daniell.

I have given a special place to analysis situs

[2] Science, new series, Vol. 42, pp. 105–113, 1915.

because it seems to me that the work of Poincaré and others, including Birkhoff in this country, has emphasized the growing importance of qualitative mathematics in dynamical problems. The showing here is good, including interesting contributions from Veblen, Alexander, Birkhoff, R. L. Moore, and Kline. The closely related topic of point sets, whose vital connection with geometry and dynamics was forcefully pointed out by Van Vleck in his address as retiring president of the American Mathematical Society in 1915, claims but one per cent. of the total number of titles, the articles on this subject coming from Van Vleck, R. L. Moore, Blumberg and Kline. In the theory of integration we find 1.2 per cent. of the titles, Bliss and Daniell being the principal contributors.

While geometry does not seem to have had its full share of attention, we are well represented in differential geometry, largely because of the labors of Eisenhart, Wylczynski, Kasner, G. M. Green, Graustein and others. Further branches of geometry in which Americans have labored productively are: the geometry of algebraic varieties, in which Lefschetz has done notable work; the geometry of special classes of curves and surfaces, cultivated by Snyder, White, Emch, Sisam, Ranum, Roe and others; the geometry of forms; modular geometry, by Dickson, Glenn and Coble; the geometry of hyperspace, by C. L. Moore and Eiesland; transformations, by Snyder, Sharpe and others; non-Euclidean geometry; while a number of memoirs on different aspects of geometry have been contributed by Coolidge.

The work in mathematical physics has been due almost solely to four men: Bateman, Gronwall, Webster and Roever. Progress in celestial mechanics is to be credited almost entirely to F. R. Moulton and his pupils and to Birkhoff. The work of Birkhoff in the field of dynamical systems has been conspicuous.

In postulate theory, the papers of Huntington, R. L. Moore, and Scheffer have aroused much interest.

The above review lays claim only to being a sketch, and doubtless overlooks single papers of real importance. It does, however, give some idea of the fields being cultivated, and of the more prominent figures in them. Not as an afterthought, but with singular pleasure, do I allude to several developments which are peculiarly American, in that they were largely initiated and cultivated on this side of the ocean: to Moore's general analysis already touched upon, and to Wilczynski's projective differential geometry, ably initiated by him and carried on by himself and pupils, and to the all too short-lived Green—to Kasner's geometrical mechanics, and to the theory of linear difference equations in the hands of Birkhoff, Carmichael and their pupils. While one might wish a more extensive cultivation of such branches as are largely indigenous, it seems to me that their very existence furnishes some evidence of the vitality of American mathematics, and a foundation for predictions that its importance is on the increase. Doubtless there are other evidences of the same sort of thing that have escaped my attention.

A few further remarks on the statistics gathered may be of interest. The 1,258 titles found were the contributions of 325 persons. Nearly half of this number contributed but one paper each. I think it fair to assume that two thirds of the latter had recently received their doctorates and were writing their first and last paper at the same time. This large "mortality" indicates a great waste of intellectual capital, and deserves careful consideration. Some of it means the diversion of energy of able investigators into the instruction of pupils who might be predicted to be unproductive, and some of it is due to the crushing out of scientific enthusiasm in really able young mathematicians by an unsympathetic or over-exacting environment. The "treatment indicated" must be decided upon in the individual cases.

I have heard the advice given to young scientists, that if they wish to show the greatest productivity, the best way to accomplish this is by a high degree of specialization. The results of the present study bear this out, though not to the extent one might anticipate.

For one third of the twelve most productive mathematicians have papers scattered over a half dozen different fields each. Doubtless the advice given is particularly apt in cases where industry is a more predominant characteristic than elements of genius. But it does seem as if actual productive experience in different domains did, in some cases, add to the mathematician's power, by suggesting ideas, analogies and methods. It seems to me also to have a steadying effect on one's sense of values. He who sits in judgment on the value of scientific work is in a precarious position. To be sure, it must be done, by editors, if not by others. I have sought for some time a satisfying criterion of values. Probably no absolute criterion exists. The best working test I have been able to find, both in my own judgment and in that of those mathematicians with whom I have discussed the question, is to be found in the degree of relationship of the investigation to be judged with other branches of mathematical or allied sciences whose vitality and interest are recognized. If this solution is at all an acceptable one, it is at once clear how experience in different fields may enhance the value of the worker's product.

It may be of interest to note, in these times of agitation for cooperative research, that less than 3 per cent. of the titles listed were of joint papers, and of these not one bore evidence of being inspired by the movement. While cooperative investigation in mathematics should have all the trial it can get, it is evident that men are not likely to take to it naturally in any great degree, though there have clearly been instances in which investigators, brought together by community of interest, have distinctly enhanced their product by collaboration.

In addition to publications in journals, there are the books which have appeared during the decade. The number of books on higher mathematics which Americans have published in this time barely exceeds three score. Titles of American books in the lists of current publications in higher mathematics are as needles in a haystack. The value to American mathematics of authoritative and up to date handbooks by American authors has been sufficiently emphasized to need no further comment here. The books which have appeared include a sufficient number of treatises of such excellence as to leave no doubts as to the capabilities of authorship in this country. Three, at least, of them, have been translated into French or German; two, written in these languages, enjoy large sales here and abroad. The problem is an economic one, and the desirability of subsidy encouragement has been pointed out. All I wish to do here, is to suggest the help each individual can give by buying such books whenever possible, and by recommending their purchase by libraries.

Our sketch of the decade should not terminate without mention of the fact that a half dozen Americans have been elected to foreign academies, and that in three instances Americans have been the recipients of prizes or medals from such organizations, and in a further instance, of an honorable mention. The foreign recognition in these cases has been amply merited. The point is, of course, that by reason of national pride, of habit, of language barriers, recognition must of necessity lag behind merit. But some conscious effort on our part may well be directed toward the attainment of deserved recognition. It seems incontrovertible that if a bit of mathematics is worth writing, it is worth writing to be read. Otherwise the author is guilty of usurping pages in the journals and space on library shelves to no purpose but the gratification of vanity. This is not the place to enter upon a discussion of style, but one or two aspects of the matter have forced themselves upon my attention in connection with the present study. Style as a whole is, and doubtless should be, individual, and its development is largely merely a matter of conscious purpose. Its fundamental element for the mathematician is, of course, clarity. But when one looks over the standard reviews of mathematical literature, and notices the extent to which the reviewer takes his cue from the author's opening lines—frequently contenting himself with citing the author's own estimate of his work, it becomes clear that particular emphasis in

writing should be laid upon the opening paragraph. I suggest that without exception the introduction should be framed about the thought "this is an interesting and worthwhile contribution" (if the author has not this conviction, he ought not to ask it of the editor); that it ought therefore to give the historical setting of the problem in hand, an indication of its relationship with other problems of established importance or of recognized interest, either in mathematics or in allied sciences, with a clear indication of the novelty claimed in results or methods or presentation. If the author will then outline his general course of reasoning, he will further enhance the readability of his paper. Similar remarks apply to the presentation of papers at meetings of scientific societies. A paper should be made interesting, or be read by title. As long as the author conceives it to be an act of merit merely to go through the form of presenting a paper, so long will he deserve to have his audience melt away to talk of really interesting things in the corridors. It is, to be sure, a difficult task to make a highly technical subject of general interest, but it can be done by a placing of emphasis more on setting and less on detail, and the effort is a scientific duty.

I have alluded to the question of values. There is another side to this question, or perhaps rather another aspect of values that must not be overlooked. I have heard the question raised as to where or when our American LaGranges are to appear. There is little doubt but that young men of a high degree of genius exist in this country in every generation. That more of them do not find their way into mathematics, or, having found their way, do not continue on and develop there, is, to a considerable extent, a matter of environment. By cultivating a background of productive scholarship, by cultivating an appreciation of productive scholarship, something can be done toward producing a favorable environment. Ususally men of genius are as sensitive to appreciation, as responsive to encouragement, as any one else. The geniuses must have their audience of appreciative scientists, the less gifted producers must have their audience of interested readers, and the science as a whole must have a hold on popular respect.

Now nothing enhances a man's mathematical interests like a share in the development of the science, even though the share have but slight intrinsic importance. It seems to me therefore that it is desirable to have means of publication of papers of minor importance—it being understood that in respect to content or method some novelty and merit is present—both because the encouragement thus given may at any time be the occasion of stimulating effort destined to become of high value, and because the interest engendered is likely, at the least, to become a support to the more effective producers.

Just as an enormous impetus to mathematical work in this country was nearly coincident with the foundation of the New York Mathematical Society, so also I think we may reasonably look for a distinct impetus from the founding of the Mathematical Association of America, whose successful launching has been one of the important scientific events of the decade, and which has already brought to light a lively group of mathematical interests beyond the hope even of the founders.

Various ways of external encouragement of mathematical science in America have recently been discussed. They include the items I have mentioned, the encouragement of publication both of books and periodicals, they include prizes for important contributions, and they include recommendations for diminishing the distractions which hamper the scientist in the way of excessive instruction and administration. While I do not wish to suggest that more effective means exist, nor to imply that such steps ought not to be seconded most heartily, I do wish to point out that each individual can throw his added influence into the scale and help materially and immediately, first by efforts to produce, in the faith that such efforts will certainly result in his being more vitally a scientist and a more enthusiastic teacher of his subject; second, by cultivating a discriminating sense of value, and endeavoring to throw his pro-

ductive efforts into the most important channels which promise him some success; and thirdly by realizing his duty to make the value and interest of his own work, and of his science in general, appeal as widely as possible.

O. D. KELLOGG

SUMMARY OF A REPORT OF THE PERMANENT SECRETARY CONCERNING THE AFFAIRS OF THE ASSOCIATION, SUBMITTED TO THE EXECUTIVE COMMITTEE AT ITS MEETING, APRIL 24, 1921

THE following paragraphs present the main features of the permanent secretary's report for the period from October 1, 1920, to March 31, 1921.

In accordance with a vote of the Council at Chicago, Doctor Sam F. Trelease was appointed assistant secretary, beginning January 1. The assistant secretary has thus far been engaged mainly in editorial work on the new membership list.

The new volume of the Summarized Proceedings is far advanced and will soon appear from the press. It is planned to be more useful and satisfactory than the earlier volumes. It will contain the constitution and by-laws of the association, the summarized reports of seven annual meetings—from 1914 to 1920 (with citation references to SCIENCE for the important official publications), and the complete membership list corrected to the date of printing. The list contains about 12,000 names and addresses. Subscriptions for the new volume were booked at the price of $1.00 to members, until December 1, 1920, since which date the price to members has been $1.50. Over 1,600 volumes have been paid for in advance. (The present price will be maintained until the date of actual publication, after which it will become $2 to members and $2.50 to nonmembers. Subscriptions and remittances should be sent to the Permanent Secretary of the American Association for the Advancement of Science, Smithsonian Institution, Washington, D. C.)

The American Mathematical Society, which was invited to become affiliated with the association at the Chicago meeting, has ratified this affiliation and is now an affiliated society. The roll of the society includes 313 members of the association, of which number 107 are association fellows. The society is therefore entitled to two representatives in the council of the association.

Two state academies of science, the Michigan Academy and the Oklahoma Academy, have been added to the list of affiliated academies through their election by the council at the Chicago meeting. Each affiliated academy is entitled to a representative in the association council.

(With the two academies that were affiliated by the action of the Executive Committee on April 24—the North Carolina Academy and the Maryland Academy—there are now twelve affiliated academies, named as follows: Illinois, Iowa, Kansas, Kentucky, Maryland, Michigan, Nebraska, New Orleans, North Carolina, Oklahoma, and Wisconsin.)

The arrangement for the affiliation of academies allows the academy to collect the annual association dues of its national members (members who are also members of the American Association) and allows it to retain, for its expenses all association entrance fees obtained through its efforts and also one dollar of each payment of association annual dues collected by it. The permanent secretary's office supplies each affiliated academy with printed and addressed statement cards for all of its national members and these are sent to the members of the academy, so as to be received October 1 of each year (the beginning of the association fiscal year). For each $5 payment received in response to this billing the academy transmits $4 to the office of the permanent secretary, who then orders the free journal for each member so paying. (The journal can not be ordered until the $4 remittance is in the hands of the permanent secretary.)—Immediately after its affiliation each newly affiliated academy receives from the permanent secretary's office a payment amounting to one dollar for each one of its national members who has already paid his association dues for the current year. When a member of the association becomes a member of an affiliated academy after its affiliation the acad-