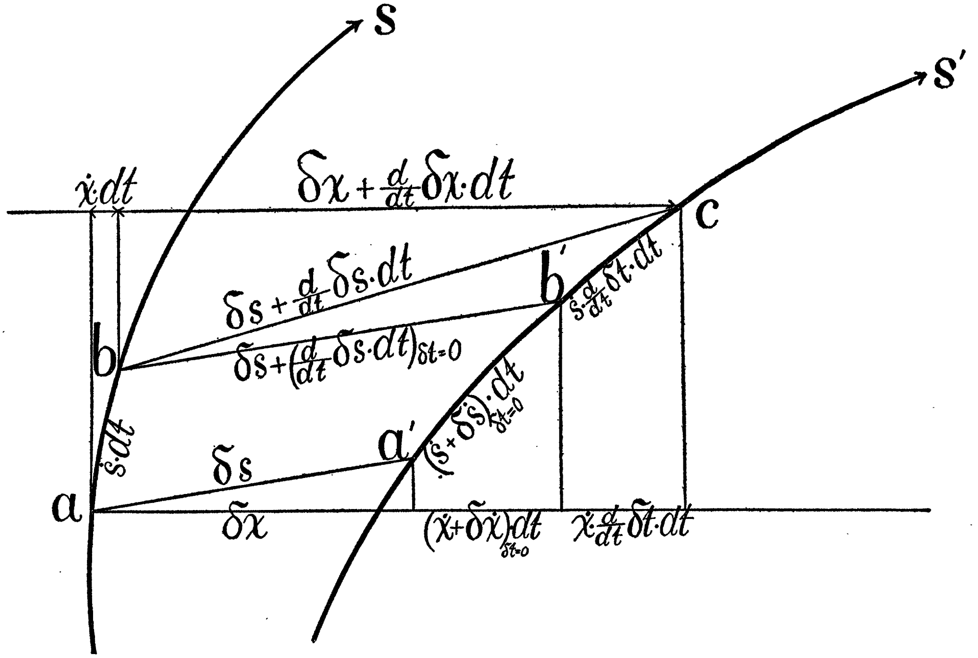
SPECIAL ARTICLES

VARIATIONS GRAPHICALLY

THE usual developments by which the calculus of variations is rigorously established, however cumbersome, are nevertheless satisfactory in so far as the reader knows what the aim is. But with a student, as a rule, they remain hazy. He acquiesces, of course, but he loses faith and the cloud may not be lifted during the whole of his subsequent course in the motion along it. Any two points, a and a', b and b', may therefore be regarded cotemporaneous at pleasure. We may express this by putting $\delta t = 0$, as in the figure. Any variation is possible, but the motion along s' must nevertheless be regarded as continuous; *i. e.*, the experimental motion is conceived as taking place, any assistance from without being admitted. The figure then shows at once, if we pass from a to b' in the two ways,



dynamics. I may therefore ask for indulgence if I publish the following simple treatment, because it has borne fruit and is intelligible to anybody who understands the equation $s = vt$.

Let s be the curve along which the motion of a particle actually takes place. Suppose it is to our advantage to consider what would happen if the motion proceeded along any other infinitely near curve s', selected at random but with the object stated. The notation would be less cumbersome without the differential coefficients $\dot{x}$, etc., but it is more direct to use them.

1. $\delta t = 0$. There are two cases. In the first, the curve s' is quite arbitrary, and so is

$$\delta x + (\dot{x} + \delta\dot{x})\,dt = \dot{x}dt + \delta x + \frac{d}{dt}\delta x \cdot dt,$$

or

$$\delta\dot{x} = \frac{d}{dt}\delta x \qquad (1)$$

the obvious meaning of the last equation.

2. δt *not zero.* In the second case the path s' is still arbitrary, but it may be regarded as a smooth wire along which a bead of the given mass slips by the same forces that move it naturally and without the wire along s. The two motions here are necessarily continuous and both are prescribed. Hence cotemporaneous points a and a', b and b', are prescribed, and an interval of time δt must elapse

in the second case if bc is to be any arbitrary displacement comparable to bb' above, § 1.

If aa' are chosen cotemporaneous, since both motions are continuous, the rate at which the interval will grow from nothing at a to δt at c, dt second later is

$$\frac{d}{dt}\delta t;$$

and the distance passed along the curve in this time excess,

$$\frac{d}{dt}\delta t \cdot dt$$

is therefore

$$\dot{x}\left(\frac{d}{dt}\delta t\right)dt$$

as the figure shows. Hence obviously as before

$$\delta x + (\dot{x} + \delta\dot{x})dt + \dot{x}\frac{d}{dt}\delta t \cdot dt = \dot{x}dt + \delta x + \frac{d}{dt}\delta x \cdot dt,$$

or

$$\frac{d}{dt}\delta x = \delta\dot{x} + \dot{x}\frac{d}{dt}\delta t. \qquad (2)$$

It is also obvious that if we sum up the increments vectorially, from a to c in the two directions the same proposition will hold with regard to s;

$$\frac{d}{dt}\delta s = \delta\dot{s} + \dot{s}\frac{d}{dt}\delta t.$$

3. The important transformation

$$\frac{d}{dt}(\dot{x}\delta x) = \ddot{x}\delta x + \dot{x}\frac{d}{dt}\delta x$$

by which one passes from D'Alembert to Hamilton or to least action, respectively (see Webster's "Dynamics," which, by the way should be the text-book of every American university, patriotic or not), is a mere interpretation of the last term by the aid of equation (1) in the first case, of equation (2) in the second.

Finally with regard to variations in general it is clear that if ϕ is to have but one value at each point in space and is to vary at a single definite rate in each direction from that point, it is immaterial whether one uses the differentials, dx, dy, dz, meaning thereby that in a complete differentiation we must get back to the initial surface or region $\phi = c$; or the variations δx, δy, δz, meaning that, in general, our progress may terminate in any infinitely near region $\phi = d$, at pleasure, the same differential coefficients must be used. For along x, ϕ can not vary in any other way than at a rate, $\partial\phi/\partial x$, whether our absolute progress is to be dx or δx.

All this is simple enough, but with my students it has made the difference between the spiritless acceptation of what somebody else is supposed to understand and the satisfaction of an actual grasp of the subject.

C. BARUS

BROWN UNIVERSITY,
PROVIDENCE, R. I.

MOSQUITO HABITS AND MOSQUITO CONTROL

UNTIL recently it was the general impression that all mosquitoes are blood-suckers and essentially alike in habits. Since the discovery of their relation to disease mosquitoes have been extensively studied, both systematically and biologically. While the study of mosquito biology has not by any means kept pace with the systematic work, a great deal has been learned about mosquito habits and it is now clear that there is great diversity of habits within the group.

To any one who has followed the literature, or become directly acquainted with the remarkable specializations in mosquito habits, it must be obvious that no control work can be carried out successfully and economically without intimate knowledge of the habits of these insects. Many persons, however, who are concerned with mosquitoes in a practical way, either directly in control work or as its advocates, have failed to appreciate this and hold the antiquated ideas. Work done on such a shallow basis must in many cases end in failure and disappointment.

Two striking examples, which have recently come to my notice, illustrate very well how such shortcomings lead to error. Sir Rubert W. Boyce, dean of the Liverpool School of Tropical Medicine, is the author of an interesting and excellent work which appeared recently under the title "Mosquito or Man?" While the book is written on broad lines it nevertheless contains specific statements, and