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THE PROGRESS OF PHYSICS IN THE NINETEENTH CENTURY.¹

I.

Mr. President, Ladies and Gentlemen: You have honored me by requesting at my hands an account of the advances made in physics during the nineteenth century. I

¹ Paper read at the International Congress in St. Louis.

have endeavored, in so far as I have been able, to meet the grave responsibilities implied in your invitation; yet had I but thought of the overwhelmingly vast territory to be surveyed, I well might have hesitated to embark on so hazardous an undertaking. To mention merely the *names* of men whose efforts are linked with splendid accomplishments in the history of modern physics would far exceed the time allotted to this address. To bear solely on certain subjects, those for instance with which I am more familiar, would be to develop an unsymmetrical picture. As this is to be avoided, it will be necessary to present a straightforward compilation of all work above a certain somewhat vague and arbitrary lower limit of importance. Physics is, as a rule, making vigorous though partial progress along independent parallel lines of investigation, a discrimination between which is not possible until some catalysm in the history of thought ushers in a new era. It will be essential to abstain from entering into either explanation or criticism and to assume that all present are familiar with the details of the subjects to be treated. I can neither popularize nor can I endeavor to entertain, except in so far as a rapid review of the glorious conquests of the century may be stimulating.

In spite of all this simplicity of aim, there is bound to be distortion. In any brief account, the men working at the beginning of the century, when investigations were few and the principles evolved necessarily fundamental, will be given greater consideration than equally able and abler

investigations near the close, when workers (let us be thankful) were many, and the subjects lengthening into detail. Again, the higher order of genius will usually be additionally exalted at the expense of the less gifted thinker. I can but regret that these are the inevitable limitations of the cursory treatment prescribed. As time rolls on the greatest names more and more fully absorb the activity of a whole epoch.

METROLOGY.

Finally, it will hardly be possible to consider the great advances made in physics except on the theoretical side. Of renowned experimental researches, in particular of the investigations of the constants of nature to a degree of ever increasing accuracy, it is not practicable to give any adequate account. Indeed, the refinement and precision now demanded have placed many subjects beyond the reach of individual experimental research, and have culminated in the establishment of the great national or international laboratories of investigation at Sèvres (1872), at Berlin (1887, 1890), at London (1900), at Washington (1901). The introduction of uniform international units in cases of the arts and sciences of more recent development is gradually, but inexorably, urging the same advantages on all. Finally, the access to adequate instruments of research has everywhere become an easier possibility for those duly qualified, and the institutions and academies which are systematically undertaking the distribution of the means of research are continually increasing in strength and in number.

CLASSIFICATION.

In the present paper it will be advisable to follow the usual procedure in physics, taking in order the advances made in dynamics, acoustics, heat, light and electricity. The plan pursued will, therefore, specifically consider the progress in

elastics, crystallography, capillarity, solution, diffusion, dynamics, viscosity, hydrodynamics, acoustics; in thermometry, calorimetry, thermodynamics, kinetic theory, thermal radiation; in geometric optics, dispersion, photometry, fluorescence, photochemistry, interference, diffraction, polarization, optical media; in electrostatics, Volta contacts, Seebeck contacts, electrolysis, electric current, magnetism, electromagnetism, electrodynamics, induction, electric oscillation, electric field, radioactivity.

Surely this is too extensive a field for any one man! Few who are not physicists realize that each of these divisions has a splendid and voluminous history of development, its own heroes, its sublime classics often culled from the activity of several hundred years. I repeat that few understand the unmitigatedly fundamental character, the scope, the vast and profound intellectual possessions, of pure physics; few think of it as the one science into which all other sciences must ultimately converge—or a separate representation would have been given to most of the great divisions which I have named.

Hence even if the literary references may be given in print with some fullness, it is impossible to refer verbally to more than the chief actors and quite impossible to delineate sharply the real significance and the relations of what has been done. Moreover, the dates will in most instances have to be omitted from the reading. It has been my aim, however, to collect the greater papers in the history of physics, and the suggestion is implied that science would gain if by some august tribunal researches of commanding importance were formally canonized for the benefit of posterity.

ELASTICS.

To begin with elasticity, whose development has been of such marked influ-

ence throughout the whole of physics, we note that the theory is virtually a creation of the nineteenth century. Antedating Thomas Young, who in 1807 gave to the subject the useful conception of a modulus, and who seems to have definitely recognized the shear, there were merely the experimental contribution of Galileo (1638), Hooke (1660), Mariotte (1680), the elastic curve of J. Bernoulli (1705), the elementary treatment of vibrating bars of Euler and Bernoulli (1742), and an attempted analysis of flexure and torsion by Coulomb (1776).

The establishment of a theory of elasticity on broad lines begins almost at a bound with Navier (1821), reasoning from a molecular hypothesis to the equation of elastic displacement and of elastic potential energy (1822-1827); yet this startling advance was destined to be soon discredited, in the light of the brilliant generalizations of Cauchy (1827). To him we owe the six component stresses and the six component strains, the stress quadric and the strain quadric, the reduction of the components to three principal stresses and three principal strains, the ellipsoids and other of the indispensable conceptions of the present day. Cauchy reached his equations both by the molecular hypothesis and by an analysis of the oblique stress across an interface—methods which predicate fifteen constants of elasticity in the most general case, reducing to but one in the case of isotropy. Contemporaneous with Cauchy's results are certain independent researches by Lamé and Clapeyron (1828) and by Poisson (1829).

Another independent and fundamental method in elasticity was introduced by Green (1837), who took as his point of departure the potential energy of a conservative system in connection with the Lagrangian principle of virtual displacements. This method, which has been fruitful in the

hands of Kelvin (1856), of Kirchhoff (1876), of Neumann (1885), leads to equations with twenty-one constants for the ælotropic medium reducing to two in the simplest case.

The wave motion in an isotropic medium was first deduced by Poisson in 1828, showing the occurrence of longitudinal and transverse waves of different velocities; the general problem of wave motion in ælotropic media, though treated by Green (1842), was attacked with requisite power by Blanchet (1840-1842) and by Christoffel (1877).

Poisson also treated the case of radial vibrations of a sphere (1828), a problem which, without this restriction, awaited the solutions of Jaerisch (1879) and of Lamb (1882). The theory of the free vibrations of solids, however, is a generalization due to Clebsch (1857-58, 'Vorlesungen,' 1862).

Elasticity received a final phenomenal advance through the long continued labors of de St. Venant (1839-55), which in the course of his editions of the work of Moigno, of Navier (1863), and of Clebsch (1864), effectually overhauled the whole subject. He was the first to adequately assert the fundamental importance of the shear. The profound researches of de St. Venant on the torsion of prisms and on the flexure of prisms appeared in their complete form in 1855 and 1856. In both cases the right sections of the stressed solids are shown to be curved and the curvature is succinctly specified; in the former Coulomb's inadequate torsion formula is superseded and in the latter flexural stress is reduced to a transverse force and a couple. But these mere statements convey no impression of the magnitude of the work.

Among other notable creations with a special bearing on the theory of elasticity there is only time to mention the invention and application of curvilinear coordinates by Lamé (1852); the reciprocal theorem

of Betti (1872), applied by Cerruti (1882) to solids with a plane boundary—problems to which Lamé and Clapeyron (1828) and Boussinesq (1879–85) contributed by other methods; the case of the strained sphere studied by Lamé (1854) and others; Kirchhoff's flexed plate (1850); Rayleigh's treatment of the oscillations of systems of finite freedom (1873); the thermo-elastic equations of Duhamel (1838), of F. Neumann (1841), of Kelvin (1878); Kelvin's analogy of the torsion of prisms with the supposed rotation of an incompressible fluid within (1878); his splendid investigations (1863) of the dynamics of elastic spheroids and the geophysical applications to which they were put.

Finally, the battle royal of the molecular school following Navier, Poisson, Cauchy and championed by de St. Venant, with the disciples of Green headed by Kelvin and Kirchhoff—the struggle of the fifteen constants with the twenty-one constants, in other words—seems to have temporarily subsided with a victory for the latter through the researches of Voigt (1887–89).

CRYSTALLOGRAPHY.

Theoretical crystallography, approached by Steno (1669), but formally founded by Haüy (1781, 'Traité,' 1801), has limited its development during the century to systematic classifications of form. Thus the thirty-two type sets of Hessel (1830) and of Bravais (1850) have expanded into the more extensive point series involving 230 types due to Jordan (1868), Sohncke (1876), Federow (1890) and Schoenflies (1891). Physical theories of crystalline form have scarcely been unfolded.

CAPILLARITY.

Capillarity antedated the century in little more than the provisional, though brilliant, treatment due to Clairaut (1743). The theory arose in almost its present state

of perfection in the great memoir of Laplace (1805), one of the most beautiful examples of the Newton-Boscovichian (1758) molecular dynamics. Capillary pressure was here shown to vary with the principal radii of curvature of the exposed surface, in an equation involving two constants, one dependent on the liquid only, the other doubly specific for the bodies in contact. Integrations for special conditions include the cases of tubes, plates, drops, contact angle, and similar instances. Gauss (1829), dissatisfied with Laplace's method, virtually reproduced the whole theory from a new basis, avoiding molecular forces in favor of Lagrangian displacements, while Poisson (1831) obtained Laplace's equations by actually accentuating the molecular hypothesis; but his demonstration has since been discredited. Young in 1805 explained capillary phenomena by postulating a constant surface tension, a method which has since been popularized by Maxwell ('Heat,' 1872).

With these magnificent theories pounded for guidance at the very threshold of the century, one is prepared to anticipate the wealth of experimental and of detailed theoretical research which has been devoted to capillarity. Among these the fascinating monograph of Plateau (1873), in which the consequences of theory are tested by the behavior both of liquid lamellæ and by suspended masses, Savart's (1833), and particularly Rayleigh's, researches with jets (1879–83), Kelvin's ripples (1871), may be cited as typical. Of peculiar importance, quite apart from its meteorological bearing, is Kelvin's deduction (1870) of the interdependence of surface tension and vapor pressure when varying with the curvature of a droplet.

DIFFUSION.

Diffusion was formally introduced into physics by Graham (1850). Fick (1855),

appreciating the analogy of diffusion and heat conduction, placed the phenomenon on a satisfactory theoretical basis, and Fick's law has since been rigorously tested, in particular by H. F. Weber (1879).

The development of diffusion from a physical point of view followed Pfeffer's discovery (1877) of osmotic pressure, soon after to be interpreted by van't Hoff (1887) in terms of Boyle's and Avogadro's laws. A molecular theory of diffusion was thereupon given by Nernst (1887).

DYNAMICS.

In pure dynamics the nineteenth century inherited from the eighteenth that unrivaled feat of reasoning called by Lagrange the '*Mécanique Analytique*' (1788), and the great master was present as far as 1813 to point out its resources and to watch over the legitimacy of its applications. Throughout the whole century each new advance has but vindicated the preeminent power and safety of its methods. It triumphed with Maxwell (1864), when he deduced the concealed kinetics of the electromagnetic field, and with Gibbs (1876-78), when he adapted it to the equilibrium of chemical systems. It will triumph again in the electromagnetic dynamics of the future.

Naturally there were reactions against the tyranny of the method of '*liaisons*.' The most outspoken of these, propounded under the protection of Laplace himself, was the celebrated '*mécanique physique*' of Poisson (1828), an accentuation of Bosovich's (1758) dynamics, which permeates the work of Navier, Cauchy, de St. Venant, Boussinesq, even Fresnel, Ampère and a host of others. Cauchy in particular spent much time to reconcile the molecular method with the Lagrangian abstractions. But Poisson's method, though sustained by such splendid genius, has, nevertheless, on more than one occasion—in capillarity, in

elastics—shown itself to be untrustworthy. It was rudely shaken when, with the rise of modern electricity, the influence of the medium was more and more pushed to the front.

Another complete reconstruction of dynamics is due to Thomson and Tait (1867), in their endeavor to gain clearness and uniformity of design, by referring the whole subject logically back to Newton. This great work is the first to make systematic use of the doctrine of the conservation of energy.

Finally, Hertz (1894), imbued with the general trend of cotemporaneous thought, made a powerful effort to exclude force and potential energy from dynamics altogether—postulating a universe of concealed motions such as Helmholtz (1884) had treated in his theory of cyclic systems, and Kelvin had conceived in his adynamic gyrostatic ether (1890). In fact the introduction of concealed systems and of ordered molecular motions by Helmholtz and Boltzmann has proved most potent in justifying the Lagrangian dynamics in its application to the actual motions of nature.

The specific contributions of the first rank which dynamics owes to the last century, engrossed as it was with the applications of the subject, or with its mathematical difficulties, are not numerous. In chronological order we recall naturally the statics (1804) and the rotational dynamics (1834) of Poincot, all in their geometrical character so surprisingly distinct from the cotemporary dynamics of Lagrange and Laplace. We further recall Gauss's principle of least constraint (1829), but little used, though often in its applications superior to the method of displacement; Hamilton's principle of varying action (1834) and his characteristic function (1834, 1835), the former obtainable by an easy transition from D'Alembert's prin-

ciple and by contrast with Gauss's principle, of such exceptional utility in the development of modern physics; finally the development of the Leibnitzian doctrine of work and *vis viva* into the law of the conservation of energy, which more than any other principle has consciously pervaded the progress of the nineteenth century. Clausius's theorem of the 'Virial' (1870) and Jacobi's (1866) contributions should be added among others.

The potential, though contained explicitly in the writings of Lagrange (1777), may well be claimed by the last century. The differential equation underlying the doctrine had already been given by Laplace in 1782, but it was subsequently to be completed by Poisson (1827). Gauss (1813, 1839) contributed his invaluable theorems relative to the surface integrals and force flux, and Stokes (1854) his equally important relation of the line and the surface integral. Legendre (published 1785) and Laplace (1782) were the first to apply spherical harmonics in expansions. The detailed development of volume surface and line potential has enlisted many of the ablest writers, among whom Chasles (1837, 1839, 1842), Helmholtz (1853), C. Neumann (1877, 1880), Lejeune-Dirichlet (1876), Murphy (1833) and others are prominent.

The gradual growth of the doctrine of the potential would have been accelerated, had not science to its own loss overlooked the famous essay of Green (1828) in which many of the important theorems were anticipated, and of which Green's theorem and Green's function are to-day familiar reminders.

Recent dynamists incline to the uses of the methods of modern geometry and to the vector calculus with continually increasing favor. Noteworthy progress was first made in this direction by Moebius (1837-43, 'Statik,' 1838), but the power

of these methods to be fully appreciated required the invention of the 'Ausdehnungslehre,' by Grassmann (1844), and of 'quaternions,' by Hamilton (1853).

Finally the profound investigations of Sir Robert Ball (1871, et seq., 'Treatise') on the theory of screws with its immediate dynamical applications, though as yet but little cultivated except by the author, must be reckoned among the promising heritages of the twentieth century.

On the experimental side it is possible to refer only to researches of a strikingly original character like Foucault's pendulum (1851) and Fizeau's gyrostat; or like Boys's (1887, et seq.) remarkable quartz-fiber torsion-balance, by which the Newtonian constant of gravitation and the mean density of the earth originally determined by Maskelyne (1775-78) and by Cavendish (1798) were evaluated with a precision probably superior to that of the other recent measurements, the pendulum work of Airy (1856) and Wilsing (1885-87), or the balance methods of Jolly (1881), König and Richarz (1884). Extensive transcontinental gravitational surveys like that of Mendenhall (1895) have but begun.

HYDRODYNAMICS.

The theory of the equilibrium of liquids was well understood prior to the century even in the case of rotating fluids, thanks to the labors of Maclaurin (1742), Clairaut (1743) and Lagrange (1788). The generalizations of Jacobi (1834) contributed the triaxial ellipsoid of revolution and the case has been extended to two rotating attracting masses by Poincaré (1885) and Darwin (1887). The astonishing revelations contained in the recent work of Poincaré are particularly noteworthy.

Unlike elastics, theoretical hydrodynamics passed into the nineteenth century in a relatively well-developed state. Both types of the Eulerian equations of motion

(1755, 1759) had left the hands of Lagrange (1788) in their present form. In relatively recent time, H. Weber (1868) transformed them in a way combining certain advantages of both, and another transformation was undertaken by Clebsch (1859). Hankel (1861) modified the equation of continuity, and Svanberg and Edlund (1847) the surface conditions.

Helmholtz in his epoch-making paper of 1858 divided the subject into those classes of motion (flow in tubes, streams, jets, waves) for which a velocity potential exists and the vortex motions for which it does not exist. This classification was carried even into higher orders of motion by Craig and by Rowland (1881). For cases with a velocity potential, much progress has been made during the century in the treatment of waves, of discontinuous fluid motion, and in the dynamics of solids suspended in frictionless liquids. Kelland (1844), Scott Russel (1844) and Green (1837) dealt with the motion of progressive waves in relatively shallow vessels, Gerster (1804) and Rankine (1863) with progressive waves in deep water, while Stokes (1846, 1847, 1880) after digesting the contemporaneous advances in hydrodynamics, brought his powerful mind to bear on most of the outstanding difficulties. Kelvin introduced the case of ripples (1871), afterwards treated by Rayleigh (1883). The solitary wave of Russel occupied Boussinesq (1872, 1882), Rayleigh (1876) and others; group waves were treated by Reynolds (1877) and Rayleigh (1879). Finally the theory of stationary waves received extended attention in the writings of de St. Venant (1871), Kirchhoff (1879) and Greenhill (1887). Early experimental guidance was given by the classic researches of C. H. and W. Weber (1825).

The occurrence of discontinuous variation of velocity within the liquid was first fully appreciated by Helmholtz (1868),

later by Kirchhoff (1869), Rayleigh (1876), Voigt (1885) and others. It lends itself well to conformal representations.

The motions of solids within a liquid have fascinated many investigators and it is chiefly in connection with this subject that the method of sources and sinks was developed by English mathematicians, following Kelvin's method (1856) for the flow of heat. The problem of the sphere was solved more or less completely by Poisson (1832), Stokes (1843), Dirichlet (1852); the problem of the ellipsoid by Green (1833), Clebsch (1858), generalized by Kirchhoff (1869). Rankine treated the translatory motion of cylinders and ellipsoids in a way bearing on the resistance of ships. Stokes (1843) and Kirchhoff entertain the question of more than one body. The motion of rings has occupied Kirchhoff (1869), Boltzmann (1871), Kelvin (1871), Bjerknes (1879) and others. The results of C. A. Bjerknes (1868) on the fields of hydrodynamic force surrounding spheres, pulsating or oscillating, in translatory or rotational motion accentuate the remarkable similarity of these fields with the corresponding cases in electricity and magnetism and have been edited in a unique monograph (1900) by his son. In a special category belong certain powerful researches with a practical bearing, such as the modern treatment of ballistics by Greenhill and of the ship propeller of Ressel (1826), summarized by Gerlach (1885, 1886).

The numerous contributions of Kelvin (1888, 1889) in particular have thrown new light on the difficult but exceedingly important question of the stability of fluid motion.

The century, moreover, has extended the working theory of the tides due to Newton (1687) and Laplace (1774), through the labors of Airy, Kelvin and Darwin.

Finally the forbidding subject of vortex motion was gradually approached more and more fully by Lagrange, Cauchy (1815, 1827), Svanberg (1839), Stokes (1845); but the epoch-making integrations of the differential equations together with singularly clear-cut interpretations of the whole subject are due to Helmholtz (1858). Kelvin (1867, 1883) soon recognized the importance of Helmholtz's work and extended it, and further advance came in particular from J. J. Thomson (1883) and Beltrami (1875). The conditions of stability in vortex motion were considered by Kelvin (1880), Lamb (1878), J. J. Thomson and others, and the cases of one or more columnar vortices, of cylindrical vortex sheets, of one or more vortex rings simple or linked, have all yielded to treatment.

The indestructibility of vortex motion in a frictionless fluid, its open structure, the occurrence of reciprocal forces, were compared by Kelvin (1867) with the essential properties of the atom. Others like Fitzgerald in his cobwebbed ether and Hicks (1885) in his vortex sponge have found in the properties of vortices a clue to the possible structure of the ether. Yet it has not been possible to deduce the principles of dynamics from the vortex hypothesis, neither is the property which typifies the mass of an atom clearly discernible. Kelvin invokes the corpuscular hypothesis of Lesage (1818).

VISCOSITY.

The development of viscous flow is largely on the experimental side, particularly for solids, where Weber (1835), Kohlrausch (1863, et seq.) and others have worked out the main laws. Stokes (1845) deduced the full equations for liquids. Poiseuille's law (1847), the motion of small solids in viscous liquids, of vibrating plates and other important special cases, has yielded to treatment. The coefficients of

viscosity defined by Poisson (1831), Maxwell (1868), Hagenbach (1860), O. E. Meyer (1863), are exhaustively investigated for gases and for liquids. Maxwell (1877) has given the most suggestive and Boltzmann (1876) the most carefully formulated theory for solids, but the investigation of absolute data has but begun. The difficulty of reconciling viscous flow with Lagrange's dynamics seems first to have been adjusted by Navier.

AEROMECHANICS.

Aerostatics is indissolubly linked with thermodynamics. Aerodynamics has not marked out for itself any very definite line of progress. Though the resistance of oblique planes has engaged the attention of Rayleigh, it is chiefly on the experimental side that the subject has been enriched, as, for instance, by the labors of Langley (1891) and Lilienthal. Langley (1897) has, indeed, constructed a steam propelled aeroplane which flew successfully; but man himself has not yet flown.

Moreover, the meteorological applications of aerodynamics contained in the profound researches of Guldberg and Mohn (1877), Ferrel (1877), Oberbeck (1882, 1886), Helmholtz (1888, 1889) and others, as well as in such investigations as Sprung's (1880) on the inertia path, are as yet rather qualitative in their bearing on the actual motions of the atmosphere. The marked progress of meteorology is on the observational side.

ACOUSTICS.

Early in the century the velocity of sound given in a famous equation of Newton was corrected to agree with observation by Laplace (1816).

The great problems in acoustics are addressed in part to the elastician, in part to the physiologist. In the former case the work of Rayleigh (1877) has described the present stage of development, interpreting

and enriching almost every part discussed. In the latter case Helmholtz (1863) has devoted his immense powers to a like purpose and with like success. König has been prominently concerned with the construction of accurate acoustic apparatus.

It is interesting to note that the differential equation representing the vibration of strings was the first to be integrated; that it passed from D'Alembert (1747) successively to Euler (1779), Bernoulli (1753) and Lagrange (1759). With the introduction of Fourier's series (1807) and of spherical harmonics at the very beginning of the century, D'Alembert's and the other corresponding equations in acoustics readily yielded to rigorous analysis. Rayleigh's first six chapters summarize the results for one and for two degrees of freedom.

Flexural vibration in rods, membranes and plates become prominent in the unique investigations of Chladni (1787, 1796, 'Akustik,' 1802). The behavior of vibrating rods has been developed by Euler (1779), Cauchy (1827), Poisson (1833), Strehlke (1833), Lissajous (1833), Seebeck (1849), and is summarized in the seventh and eighth chapters of Rayleigh's book. The transverse vibration of membranes engaged the attention of Poisson (1829). Round membranes were rigorously treated by Kirchhoff (1850) and by Clebsch (1862); elliptic membranes by Mathieu (1868). The problem of vibrating plates presents formidable difficulties resulting not only from the edge conditions, but from the underlying differential equation of the fourth degree due to Sophie Germain (1810) and to Lagrange (1811). The solutions have taxed the powers of Poisson (1812, 1829), Cauchy (1829), Kirchhoff (1850), Boussinesq (1871-79) and others. For the circular plate Kirchhoff gave the complete theory. Rayleigh systematized the results for the quadratic plate and the

general account makes up his ninth and tenth chapters.

Longitudinal vibrations which are of particular importance in case of the organ pipe, were considered in succession by Poisson (1817), Hopkins (1838), Quet (1855); but Helmholtz in his famous paper of 1860 gave the first adequate theory of the open organ pipe, involving viscosity. Further extension was then added by Kirchhoff (1868), and by Rayleigh (1870, et seq.), including particularly powerful analysis of resonance. The subject in its entirety, including the allied treatment of the resonator, completes the second volume of Rayleigh's 'Sound.'

On the other hand, the whole subject of tone quality, of combination and difference tones, of speech, of harmony, in its physical, physiological and esthetic relations, has been reconstructed, using all the work of earlier investigators by Helmholtz (1862), in his masterly 'Tonempfindungen.' With rare skill and devotion König contributed a wealth of siren-like experimental appurtenances.

Acousticians have been fertile in devising ingenious methods and apparatus, among which the tuning fork with resonator of Marloye, the siren of Cagniard de le Tour (1819), the Lissajous curves (1857), the stroboscope of Plateau (1832), the manometric flames of König (1862, 1872), the dust methods of Chladni (1787) and of Kundt (1865-68), Melde's vibrating strings (1860, 1864), the phonograph of Edison and of Bell (1877), are among the more famous.

HEAT: THERMOMETRY.

The invention of the air thermometer dates back at least to Amontons (1699), but it was not until Rudberg (1837), and more thoroughly Regnault (1841, et seq.) and Magnus (1842) had completed their work on the thermal expansion and compressi-

bility of air, that air thermometry became adequately rigorous. On the theoretical side Clapeyron (1834), Helmholtz (1847), Joule (1848), had in various ways proposed the use of the Carnot function (1894) for temperature measurement, but the subject was finally disposed of by Kelvin (1849, et seq.) in his series of papers on temperature and temperature measurement.

Practical thermometry gained much from the measurement of the expansion of mercury by Dulong and Petit (1818), repeated by Regnault. It also profited by the determination of the viscous behavior of glass, due to Pernet (1876) and others, but more from the elimination of these errors by the invention of the Jena glass. It is significant to note that the broad question of thermal expansion has yet no adequate equation, though much has been done experimentally for fluids by the magnificent work of Amagat (1869, 1873, et seq.).

HEAT CONDUCTION.

The subject of heat conduction from a theoretical point of view was virtually created by the great memoir of Fourier (1822), which shed its first light here, but subsequently illumined almost the whole of physics. The treatment passed successively through the hands of many of the foremost thinkers, notably of Poisson (1835, 1837), Lamé (1836, 1839, 1843), Kelvin (1841-44) and others. With the latter (1856) the ingenious method of sources and sinks originated. The character of the conduction is now well known for continuous media, isotropic or not, bounded by the more simple geometrical forms, in particular for the sphere under all reasonable initial and surface conditions. Much attention has been given to the heat conduction of the earth, following Fourier, by

Kelvin (1862, 1878), King (1893) and others.

Experimentally, Wiedemann and Franz (1853) determined the relative heat conduction of metals and showed that for simple bodies a parallel gradation exists for the cases of heat and of electrical conductivity. Noteworthy absolute methods for measuring heat conduction were devised in particular by Forbes (1842), F. Neumann (1862), Ångström (1861-64), and a lamellar method applying to fluids by H. F. Weber (1880).

CALORIMETRY.

Practical calorimetry was virtually completed by the researches of Black in 1763. A rich harvest of experimental results, therefore, has since accrued to the subjects of specific, latent and chemical heats, due in particularly important cases to the indefatigable Regnault (1840, 1845, et seq.). Dulong and Petit (1819) discovered the remarkable fact of the approximate constancy of the atomic heats of the elements. The apparently exceptional cases were interpreted for carbon silicon and boron by H. F. Weber (1875), and for sulphur by Regnault (1840). F. Neumann (1831) extended the law to compound bodies and Joule (1844) showed that in many cases specific heat could be treated as additively related to the component specific heats.

Among recent apparatus the invention of Bunsen's ice calorimeter (1870) deserves particular mention.

THERMODYNAMICS.

Thermodynamics, as has been stated, in a singularly fruitful way interpreted and broadened the old Leibnitzian principle of *vis viva* of 1686. Beginning with the incidental experiments of Rumford (1798) and of Davy (1799) just antedating the century, the new conception almost leaped into being when J. R. Mayer (1842, 1845)

defined and computed the mechanical equivalent of heat, and when Joule (1843, 1845, et seq.) made that series of precise and judiciously varied measurements which mark an epoch. Shortly after Helmholtz (1847), transcending the mere bounds of heat, carried the doctrine of the conservation of energy throughout the whole of physics.

Earlier in the century Carnot (1824), stimulated by the growing importance of the steam engine of Watt (1763, et seq.), which Fulton (1806) had already applied to transportation by water and which Stephenson (1829) soon after applied to transportation by land, invented the reversible thermodynamic cycle. This cycle or sequence of states of equilibrium of two bodies in mutual action is, perhaps, without a parallel in the prolific fruitfulness of its contributions to modern physics. Its continued use in fifty years of research has but sharpened its logical edge. Carnot deduced the startling doctrine of a temperature criterion for the efficiency of engines. Clapeyron (1834) then gave the geometrical method of representation universally used in thermodynamic discussions to-day, though often made more flexible by new coordinates as suggested by Gibbs (1873).

To bring the ideas of Carnot into harmony with the first law of thermodynamics it is necessary to define the value of a transformation, and this was the great work of Clausius (1850), followed very closely by Kelvin (1851) and more hypothetically by Rankine (1851). The latter's broad treatment of energetics (1855) antedates many recent discussions. As early as 1858 Kirchhoff investigated the solution of solids and of gases thermodynamically, introducing at the same time an original method of treatment.

The second law was not generally accepted without grave misgiving. Clausius,

indeed, succeeded in surmounting most of the objections, even those contained in theoretically delicate problems associated with radiation. Nevertheless, the confusion raised by the invocation of Maxwell's 'demon' has never quite been calmed; and while Boltzmann (1877, 1878) refers to the second law as a case of probability, Helmholtz (1882) admits that the law is an expression of our inability to deal with the individual atom. Irreversible processes as yet lie quite beyond the pale of thermodynamics. For these the famous inequality of Clausius is the only refuge. The value of an uncompensated transformation is always positive.

The invention of mechanical systems which more or less fully conform to the second law has not been infrequent. Ideas of this nature have been put forward by Boltzmann (1866, 1872), by Clausius (1870, 1871) and more powerfully by Helmholtz (1884) in his theory of cyclic systems, which in a measure suggested the hidden mechanism at the root of Hertz's dynamics. Gibbs's (1902) elementary principles of statistical mechanics seem, however, to contain the nearest approach to a logical justification of the second law—an approach which is more than a dynamical illustration.

The applications of the first and second laws of thermodynamics are ubiquitous. As interesting instances we may mention the conception of an ideal gas and its properties; the departure of physical gases from ideality as shown in Kelvin and Joule's plug experiment (1854, 1862); the corrected temperature scale resulting on the one hand, and the possibility of the modern liquid air refrigerator of Linde and Hampson (1895) on the other. Difficulties encountered in the liquefaction of incoercible gases by Cailletet and Pictet (1877) have vanished even from the hydrogen coercions

of Olezewski (1895) and of Dewar and Travers.

Again, the broad treatment of fusion and evaporation, beginning with James Thomson's (1849) computation of the melting point of ice under pressure, Kirchhoff's (1858) treatment of sublimation, the extensive chapter of thermo-elastics set on foot by Kelvin's (1883) equation, are further examples.

To these must be added Andrews's (1869) discovery of the continuity of the liquid and the gaseous states foreshadowed by Cagniard de la Tour (1822, 1823); the deep insight into the laws of physical gases furnished by the experimental prowess of Amagat (1881, 1893, 1896), and the remarkably close approximation amounting almost to a prediction of the facts observed which is given by the great work of van der Waals (1873).

The further development of thermodynamics, remarkable for the breadth, not to say audacity, of its generalizations, was to take place in connection with chemical systems. The analytical power of the conception of a thermodynamic potential was recognized nearly at the same time by many thinkers: by Gibbs (1876), who discovered both the isothermal and the adiabatic potential; by Massieu (1877), independently in his 'fonctions caractéristiques'; by Helmholtz (1882), in his 'Freie Energie'; by Duhem (1886) and by Planck (1887, 1891), in their respective thermodynamic potentials. The transformation of Lagrange's doctrine of virtual displacements of indefinitely more complicated systems than those originally contemplated, in other words the introduction of a virtual thermodynamic modification in complete analogy with the virtual displacement of the 'mecanique analytique,' marked a new possibility of research of which Gibbs made the profoundest use. Unaware of

this marshaling of powerful mathematical forces, van't Hoff (1886, 1888) consummated his marvelously simple application of the second law; and from interpretations of the experiments of Pfeffer (1877) and of Raoult (1883, 1887) propounded a new theory of solution, indeed a basis for chemical physics in a form at once available for experimental investigation.

The highly generalized treatment of chemical statics by Gibbs bore early fruit in its application to Deville's phenomenon of dissociation (1857), and in succession Gibbs (1878, 1879), Duhem (1886), Planck (1887), have deduced adequate equations, while the latter in case of dilute solutions gave a theoretical basis for Guldberg and Waage's law of mass action (1879). An earlier independent treatment of dissociation is due to Horstmann (1869, 1873).

In comparison with the brilliant advance of chemical statics which followed Gibbs, the progress of chemical dynamics has been less obvious; but the outlines of the subject have, nevertheless, been succinctly drawn in a profound paper by Helmholtz (1886), followed with much skill by Duhem (1894, 1896) and Natanson (1896).

KINETIC THEORY OF GASES.

The kinetic theory of gases at the outset, and as suggested by Herapath (1821), Joule (1851, 1857), Krönig (1856), virtually reaffirmed the classic treatise of Bernoulli (1738). Clausius in 1857-62 gave to the theory a modern aspect in his derivation of Boyle's law in its thermal relations, molecular velocity and of the ratio of translational to total energy. He also introduced the mean free path (1858). Closely after followed Maxwell (1860), adducing the law for the distribution of velocity among molecules, later critically and elaborately examined by Boltzmann (1868-81). Nevertheless, the difficulties relating to the partition of energy have not yet been sur-

mounted. The subject is still under vigorous discussion, as the papers of Burbury (1899) and others testify.

To Maxwell (1860, 1868) is due the specifically kinetic interpretation of viscosity, of diffusion, of heat conduction, subjects which also engaged further attention from Boltzmann (1872-87). Rigorous data for molecular velocity and mean free path have thus become available, and van der Waals (1873) added a final allowance for the size of the molecules.

Less satisfactory has been the exploration of the character of molecular force for which Maxwell, Boltzmann (1872, et seq.), Sutherland (1886, 1893) and others have put forward tentative investigations.

The intrinsic equation of fluids discovered and treated in the great paper of van der Waals (1873), though partaking of the character of a first approximation, has greatly promoted the coordination of most of the known facts. Corresponding states, the thermal coefficients, the vapor pressure relation, the minimum of pressure-volume products, and even molecular diameters are reasonably inferred by van der Waals from very simple premises. Many of the results have been tested by Amagat (1896).

The data for molecular diameter furnished by the kinetic theory as a whole, viz., the original values of Loschmidt (1865), of van der Waals (1873) and others, are of the same order of values as Kelvin's estimates (1883) from capillarity and contact electricity. Many converging lines of evidence show that an approximation to the truth has surely been reached.

RADIATION.

Our knowledge of the radiation of heat, diathermacy, thermocrosis, was promoted by the perfection which the thermopyle reached in the hands of Melloni (1835-53). These and other researches set at rest forever all questions relating to the identity

of heat and light. The subject was, however, destined to attain a much higher order of precision with the invention of Langley's bolometer (1881). The survey of heat spectra, beginning with the laborious attempts of Herschel (1840), of E. Becquerel (1843, 1870), H. Becquerel (1883) and others, has thus culminated in the magnificent development shown in Langley's charts (1883, 1884, et seq.).

Kirchhoff's law (1860), to some extent anticipated by Stewart (1857, 1858), pervades the whole subject. The radiation of the black body, tentatively formulated in relation to temperature by Stefan (1879) and more rigorously by Boltzmann (1884), has furnished the savants of the Reichsanstalt with means for the development of a new pyrometry whose upper limit is not in sight.

Among curious inventions Crooke's radiometer (1874) and Bell's photophone may be cited. The adaptation of the former in case of high exhaustion to the actual measurement of Maxwell's (1873) light pressure by Lebedew (1901) and Nichols and Hull (1903) is of quite recent history.

The first estimate of the important constant of solar radiation at the earth was made by Pouillet (1838); but other pyrheliometric methods have since been devised by Langley (1884) and more recently by Ångström (1886, et seq.).

VELOCITY OF LIGHT.

Data for the velocity of light, verified by independent astronomical observations, were well known prior to the century; for Römer had worked as long ago as 1675, and Bradley in 1727. It remained to actually measure this enormous velocity in the laboratory, apparently an extraordinary feat, but accomplished simultaneously by Fizeau (1849) and by the aid of Wheatstone's revolving mirror (1834) by Fou-

coults (1849, 1850, 1862). Since that time precision has been given to this important constant by Cornu (1871, 1873, 1874), Forbes and Young (1882), Michelson (1878, et seq.) and Newcomb (1885). Foucault (1850), and more accurately Michelson (1884), determined the variation of velocity with the medium and wave length, thus assuring to the undulatory theory its ultimate triumph. Grave concern, however, still exists, inasmuch as Michelson and Morley (1886) by the most refined measurement, and differing from the older observations of Fizeau (1851, 1859), were unable to detect the optical effect of the relative motion of the atmosphere and the luminiferous ether predicted by theory.

Römer's observation may in some degree be considered as an anticipation of the principle first clearly stated by Döpler (1842), which has since become invaluable in spectroscopy. Estimates of the density of the luminiferous ether have been published, in particular by Kelvin (1854).

GEOMETRIC OPTICS.

Prior to the nineteenth century geometric optics, having been mustered before Huyghens (1690), Newton (1704), Malus (1808), Lagrange (1778, 1803) and others, had naturally attained a high order of development. It was, nevertheless, remodeled by the great paper of Gauss (1841) and was thereafter generalized step by step by Listing, Möbius (1855), and particularly by Abbe (1872), postulating that in character, the cardinal elements are independent of the physical reasons by which one region is imaged in another.

So many able thinkers like Airy (1827), Maxwell (1856, et seq.), Bessel (1840, 1841), Helmholtz (1856, 1867), Ferraris (1877, 1880) and others have contributed to the furtherance of geometric optics, that definite mention is impossible. In other

cases again, profound methods like those of Hamilton (1828, et seq.), Kummer (1859), do not seem to have borne correspondingly obvious fruit. The fundamental bearing of diffraction on geometric optics was first pointed out by Airy (1838), but developed by Abbe (1873) and after him by Rayleigh (1879). An adequate theory of the rainbow, due to Airy and others, is one of its picturesque accomplishments (1838).

The so-called astronomical refraction of a medium of continuously varying index, successively treated by Bouguer (1739, 1749), Simpson (1743), Bradley (1750, 1762), owes its recent refined development to Bessel (1823, 1826, 1842), Ivory (1822, 1823, et seq.), Radau (1884) and others. Tait (1883) gave much attention to the allied treatment of mirage.

In relation to instruments the conditions of aplanatism were examined by Clausius (1864), by Helmholtz (1874), by Abbe (1873, et seq.), by Hockin (1884) and others, and the apochromatic lens was introduced by Abbe (1879). The microscope is still well subserved by either the Huyghens or the Ramsden (1873) eyepiece, but the objective has undergone successive stages of improvement, beginning with Lister's discovery in 1830. Amici (1840) introduced the principle of immersion; Stephenson (1878) and Abbe (1879), homogeneous immersion; and the Abbe-Zeiss apochromatic objective (1886), the outcome of the Jena-glass experiments, marks, perhaps, the high-water mark of the art for the microscope. Steinheil (1865, 1866) introduced the guiding principle for photographic objectives. Alvan Clark carried the difficult technique of telescope lens construction to a degree of astonishing excellence.

SPECTRUM. DISPERSION.

Curiously, the acumen of Newton (1866,

1704) stopped short of the ultimate conditions of purity of spectrum. It was left to Wollaston (1802), about one hundred years later, to introduce the slit and observe the dark lines of the solar spectrum. Fraunhofer (1814, 1815, 1823) mapped them out carefully and insisted on their solar origin. Brewster (1833, 1834), who afterwards (1860) published a map of 3,000 lines, was the first to lay stress on the occurrence of absorption, believing it to be atmospheric. Forbes (1836) gave even greater definiteness to absorption by referring it to solar origin. Foucault (1849) pointed out the coincidence of the sodium lines with the D group of Fraunhofer, and discovered the reversing effect of sodium vapor. A statement of the parallelism of emission and absorption came from Ångström (1855) and with greater definiteness and ingenious experiments from Stewart (1860). Nevertheless, it was reserved to Kirchhoff and Bunsen (1860, 1861) to give the clear-cut distinctions between the continuous spectra and the characteristically fixed bright-line or dark-line spectra upon which spectrum analysis depends. Kirchhoff's law was announced in 1861 and the same year brought his map of the solar spectrum and a discussion of the chemical composition of the sun. Huggins (1864, et seq.), Ångström (1868), Thalén (1875), followed with improved observations on the distribution and wave-length of the solar lines; but the work of these and other observers was suddenly overshadowed by the marvelous possibilities of the Rowland concave grating (1882, et seq.). Rowland's maps and tables of the solar spectrum as they appeared in 1887, 1889, et seq., his summary of the elements contained in the sun (1891), each marked a definite stage of advance of the subject. Mitscherlich (1862, 1863) probably was the first to recognize the banded or channeled spectra

of compound bodies. Balmer (1885) constructed a valuable equation for recognizing the distribution of single types of lines. Kayser and Runge (1887, et seq.) successfully analyzed the structure of the spectra of alkaline and other elements.

The modernized theory of the grating had been given by Rayleigh in 1874 and was extended to the concave grating by Rowland (1892, 1893) and others. A general theory of the resolving power of prismatic systems is also due to Rayleigh (1879, 1880) and another to Thollon (1881).

The work of Rowland for the visible spectrum was ably paralleled by Langley's investigations (1883, et seq.) of the infrared, dating from the invention of the bolometer (1881). Superseding the work of earlier investigators like Fizeau and Foucault (1878) and others, Langley extended the spectrum with detailed accuracy to over eight times its visible length. The solar and the lunar spectrum, the radiations of incandescent and of hot bodies, were all specified absolutely and with precision. With artificial spectra Rubens (1892, 1899) has since gone further, reaching the longest heat waves known.

A similarly remarkable extension was added for the ultra-violet by Schumann (1890, 1892), contending successfully with the gradually increasing opacity of all known media.

Experimentally the suggestion of the spectroheliograph by Lockyer (1868) and by Janssen (1868) and its brilliant achievement by Hale (1892) promise notable additions to our knowledge of solar activity.

Finally, the refractions of absorbing media have been of great importance in their bearing on theory. The peculiarities of metallic reflection were announced from his earlier experiments (1811) by Arago in 1817 and more fully investigated by Brewster (1815, 1830, 1831). F. Neumann

(1832) and MacCullagh (1837) gave sharper statements to these phenomena. Equations were advanced by Cauchy (1836, et seq.) for isotropic bodies, and later with greater detail by Rayleigh (1872), Ketteler (1875, et seq.), Drude (1887, et seq.) and others. Jamin (1847, 1848) devised the first experiments of requisite precision and found them in close agreement with Cauchy's theory. Kundt (1888) more recently investigated the refraction of metallic prisms.

Anomalous dispersion was discovered by Christiansen in 1870, and studied by Kundt (1871, et seq.). Sellmeyer's (1872) powerful and flexible theory of dispersion was extended to include absorption effects by Helmholtz (1874), with greater detail by Ketteler (1879, et seq.), and from a different point of view by Kelvin (1885). The electromagnetic theory lends itself particularly well to the same phenomena, and Koláček (1887, 1888), Goldhammer (1892), Helmholtz (1892), Drude (1893) and others instanced its adaptation with success.

PHOTOMETRY, FLUORESCENCE, PHOTOCHEMISTRY.

The cosine law of Lambert (1760) has since been interpreted in a way satisfying modern requirements by Fourier (1817, 1824) and by Lommel (1880). Among new resources for the experimentalist the spectrophotometer, the Lummer-Brodhun photometer (1889) and Rood's flicker photometer (1893, 1899) should be mentioned.

Fluorescence, though ingeniously treated by Herschel (1845, 1853) and Brewster (1846, et seq.), was virtually created in its philosophical aspects by Stokes in his great papers (1852, et seq.) on the subject. In recent years Lommel (1877) made noteworthy contributions. Phosphorescence has engaged the attention of E. Becquerel (1859), among others.

The laws of photochemistry are in large

measure due to Bunsen and Roscoe (1857, 1862). The practical development of photography from its beginnings with Daguerre (1829, 1838) and Niépce and Fox-Talbot (1839), to its final improvement by Maddox (1871) with the introduction of the dry plate, is familiar to all. Vogel's (1873) discovery of appropriate sensitizers for different colors has added new resources to the already invaluable application of photography to spectroscopy.

INTERFERENCE.

The colors of thin plates treated successively by Boyle (1663), Hooke (1665), and more particularly by Newton (1672, 'Opticks,' 1704), became in the hands of Young (1802) the means of framing an adequate theory of light. Young also discovered the colors of mixed plates and was cognizant of loss of half a wave-length on reflection from the denser medium. Fresnel (1815) gave an independent explanation of Newton's colors in terms of interference, devising for further evidence, his double mirrors (1816), his biprism (1819) and eventually the triple mirror (1820). Billet's plates and split lens (1858) belong to the same classical order, as do also Lloyd's (1837) and Haidinger's (1849) interferences. Brewster's (1817) observation of interference in case of thick plates culminated in the hands of Jamin (1856, 1857) in the useful interferometer. The scope of this apparatus was immensely advanced by the famous device of Michelson (1881, 1882), which has now become a fundamental instrument of research. Michelson's determination of the length of the meter in terms of the wave-length of light with astounding accuracy is a mere example of its accomplishments.

Wiener (1890) in his discovery of the stationary light wave introduced an entirely new interference phenomenon. The method was successfully applied to color

photography by Lippmann (1891, 1892), showing that the electric and not the magnetic vector is photographically active.

The theory of interferences from a broader point of view, and including the occurrence of multiple reflections, was successively perfected by Poisson (1823), Fresnel (1823), Airy (1831). It has recently been further advanced by Feussner (1880, et seq.), Sohncke and Wangerin (1881, 1883), Rayleigh (1889) and others. The interferences along a caustic were treated by Airy (1836), but the endeavor to reconstruct geometric optics on a diffraction basis has as yet only succeeded in certain important instances, as already mentioned.

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(To be continued.)

SCIENTIFIC BOOKS.

Species and Varieties, their Origin by Mutation. By HUGO DE VRIES; edited by DANIEL TREMBLY MACDOUGAL. Chicago, The Open Court Publishing Co.

De Vries's great work 'Die Mutationstheorie' marks an epoch in biology as truly as did Darwin's 'Origin of Species.' The revolution that it is working is less complete, perhaps, because there has remained no such important doctrine as that of continuity to be established. But there was need of a revolution in our method of attacking the problems of evolution. Ever since Darwin's time most biologists have been content to *discuss* and argue on the *modus operandi* of evolution. The data collected by Darwin have been quoted like scriptural texts to prove the truth of the most opposed doctrines. We have seen biologists divided into opposing camps in defense of various isms, but of the collection of new data and, above all, of experimentation we have had little. The great service of de Vries's work is that, being founded on experimentation, it challenges to experimentation as the only judge of its merits. It will attain its highest usefulness only if it creates a wide-

spread stimulus to the experimental investigation of evolution.

To be read, nowadays, a book must be brief. Much of the success of the 'Origin of Species' was due to the mass of material which was left out. The very bulkiness of de Vries's original work must prevent its being read as widely as it deserves. There was needed a briefer presentation of de Vries's views and one in English, and this need has fortunately been filled by de Vries himself in the work now under review. This book should be read with care by every biologist; the brief synopsis of its contents which alone is possible here can in no wise make such a reading unnecessary.

After an introductory chapter, the fundamental distinction between elementary species and varieties is discussed. Elementary species are forms that are distinct in several characters from their close allies and breed true. They are thought to have arisen from their parent form in a progressive way, *i. e.*, by the addition of a new characteristic. In this they are distinguished from retrograde varieties on the one hand and mere fluctuations of characters on the other.

The subject of retrograde varieties (constituting the third section of the book) assumes great importance in de Vries's system. They are varieties in the new (restricted) sense. They differ from the parent species usually in the absence of some single character; for example, the white flower variety of a plant or the hairless form of an ordinarily hirsute species. The eliminated characters are of a few, definite, constantly recurring kinds. In this respect varieties differ from elementary species whose differential characters are most varied. Moreover, varieties are subordinate to some elementary species, whereas elementary species are coordinate.

In self-fertilization varieties behave in a characteristic way. They are frequently constant. Even varieties that are intermediate between the parent species and other varieties may be as stable as the latter. Indeed, we know that certain garden varieties have been bred in their present form for two or three centuries. Such varieties may, however,