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BIOLOGY AND MATHEMATICS.[1]

THAT which is most characteristic of the present epoch in the history of man is undoubtedly the vast and beneficent growth of science.

In things apart from science, other races at times long past may be compared to the most civilized people of to-day.

The lyric poetry of Sappho has never been equaled. The epic flavor of Homer, even after translation, comes down to us unsurpassed through the ages.

Dante, the voice of ten silent centuries, may wait another ten centuries before his medieval miracle of song finds its peer.

The Apollo Belvidere, the Venus of Milo, the Laocoon, are the glory of antique, the despair of modern sculpture. To mention oratory to a schoolboy is to recall Demosthenes and Cicero, even if he has never pictured Cæsar, that greatest of the sons of men, quelling the mutinous soldiery by his first word, or with outstretched arm, in Egypt's palace window, holding enthralled his raging enemies, gaining precious moments, *time,* the only thing he needed to enable him to crush them under his dominant intellect.

There is no need for multiplying examples. The one thing that gives the present generation its predominance is science.

All criticisms of life made before science had taken its present place, or attempting to ignore its prominence, are obsolete, as are of necessity any systems founded on pre-scientific or anti-scientific conceptions.

[1] Address before the Ohio Academy of Science.

Now the latest of the great sciences is biology, and it could be so widely interpreted as to include many of the others, for example, physiology, psychology, sociology; but chiefly it takes for itself the broad general beginnings.

These older sciences were really engaged upon narrow domains, narrow ramifications in the universe of biology; and the general has helped the preexistent special by giving the broader conceptions connoted by comparative physiology, comparative psychology, comparative sociology.

Since Wöhler, the distinction between organic and inorganic matter has become merely schematic; but the line drawn at life has resisted obliteration.

It is true that my friend Professor Herrera has said:

> I conceive the human organism as a machine containing some five or six liters of blood employed in appropriating to itself the nutritious principles of food, absorbing oxygen, and carrying it to the nerve to make it vibrate by discharges of carbon dioxide.
>
> Life is now to be defined as the result of the physicochemical action of protoplasmic currents, the cause of such currents being diffusion, heat and some other secondary factors.

But until some one sees such currents set up in some way differing from the natural transmission of preexistent life, a thing which no one at present even hopes for, the old boundary remains undisturbed.

If any benefit is obtainable from a physico-chemical nomenclature and notation, science will not object to their use.

Suppose, then, we put it in the boldest form, that biology is now engaged in the creation of an available representation of the activities and laws of activity of these wonderful protoplasmic currents.

The definition then would be something like this: Biology is the science created to give understanding and mastery of the protoplasmic activities on this earth; to make easy the explanation and description of such activities and the transmission of this mastery.

The association, the suggestion, is immediate:

Beyond the microtome, the microscope, the statistics of observation, of experiment, of what instrument of world-conquest must the new science avail herself? The answer is patent: of mathematics, that giant pincers of scientific logic which showed Newton the moon as simply a bigger apple trying to fall straight down on his head, flashed out in the mind of Adams the unseen planet Neptune, told Rayleigh that the chemists had always been breathing vast quantities of argon without knowing it, pointed to Mendelieev the places of unknown chemical elements, and through Helmholtz and his pupil Hertz has given us the Lenard rays, the Roentgen rays, radium itself, and wireless telegraphy based on Hertzian waves.

In mathematics, the part which is being recognized as pure deductive logic is ever greater. The residuum takes from biological advance itself new form and new statement.

After the questions, what are facts? what is reality? questions not to be answered either by biology or mathematics, there come, if we decide to retain as rough working hypotheses the expressions, fact, reality, subsequent questions, such as what then is a *geometric* fact, a *geometric* reality?

These latter questions involve a wrestling with primitive origins in physiological psychology, now entangled with metaphysical constructions, all being studied at present with help of the biologically given hypothesis of evolution.

To note the essential interrelation of biology and mathematics it is only needful to recall that evolution postulates a world independent of man, preceding man, and teaches the production of man from lower biologic forms by wholly natural causes.

If this be so, then skipping the fundamental puzzle as to how a living thing gets any conscious knowledge, any subjective representation of that independent world, it remains of the very essence of the doctrine of evolution that man's knowledge of this independent world, having come by gradual betterment, trial, experiment, adaptation, and through imperfect instruments, for example, the eye, can not be metrically exact.

In the easiest measurements it is said we can not even with the best microscopes go beyond one one millionth of a meter; that is, we are limited to seven significant figures at most. What is the meaning then of the mathematics which, as in the case of the evaluation of π, has gone to seven hundred places of significant figures?

If then we are to hold to evolution, science must be a construction of the animal and human mind; for example, geometry is a system of theorems deduced in pure logical way from certain unprovable assumptions precreated by auto-active animal and human minds.

So also is biology. But here the assumptions are more fluctuating, and many of them are still on trial.

Since every science strives to characterize as to size, number, and, where possible, spatial relations, the phenomena of its domain, each has need of the ideas and methods of mathematics. One of the fundamental ideas of mathematics is the idea of variation, the variable, qualitative and quantitative variability.

When related quantities vary, one may vary arbitrarily. This is called the independent variable. Others may vary in dependence upon the first. Such are called dependent variables or functions of the independent variable. The change of the variables may be continuous or discontinuous. The blind prejudice for the assumption of continuity is so profound as to be unconscious.

But if biologists did but know it, the characteristics, peculiarities and methods of investigation for continuous functions differ essentially from those for discontinuous functions.

Our calculus assumed continuity in all its functions, and also that differentiability was a necessary consequence of this continuity.

Lobachevski, the creator of the non-Euclidean geometry, emphasized the distinction between continuity and differentiability, therein also being half a century in advance of his contemporaries.

The mathematicians of the eighteenth century did not touch the question of the relation between continuity and differentiability, presuming silently that every continuous function is *eo ipso* a function having a derivative.

Ampère tried to prove this position, but his proof lacked cogency. The question about the relation between continuity and differentiability awoke general attention between 1870 and 1880, when Weierstrass gave an example of a function continuous within a certain interval and at the same time having no definite derivative within this interval (non-differentiable).

Meanwhile, Lobachevski, already in the thirties, showed the necessity of distinguishing the 'changing gradually' (in our terminology, continuity) of a function and its 'unbrokenness' (now, differentiability).

With especial precision did he formulate this difference in his Russian memoir of 1835: 'A Method for Ascertaining the Convergence,' etc.

A function changes gradually when its increment diminishes to zero together with the increment of the independent variable. A function is unbroken if the ratio of these two increments, as they diminish, goes over insensibly into a new function, which consequently will be a differential coefficient. Integrals must always be so divided into intervals that the elements under each integral sign always change gradually and remain unbroken.

In more detail Lobachevski treated this question in his work, 'On the Convergence of Trigonometric Series,' in which are also contained very interesting general considerations on functions.

It seems, he writes, that we can not doubt the truth that everything in the world can be represented by numbers, nor the truth that every change and relation in it can be expressed by analytic functions. At the same time a broad view of the theory admits the existence of a dependence only in the sense that we consider the numbers united with one another as if given together.

Now biology deals largely with aggregates of individuals, and then, like the pure theory of numbers, its variables are discrete, and must change by jumps of at least one individual.

A mathematics proper to such investigations has not been accessible to the biologist, for not only has his calculus been founded solely on continuity, but also his geometry has been developed for him on continuity assumptions from the very beginning.

The very first proposition of Euclid is to describe an equilateral triangle on a given sect (a given finite straight line). It begins: 'Let AB be the given sect. From the center A with radius AB describe the circle BCD. From center B with radius BA, describe the circle ACE. From the point C, at which the circles cut one another, etc.' But the whole demonstration is the assumption of this point C. Why must the circles intersect? Not one word is given in proof of this, which is the whole problem.

You may say the circle is a continuous aggregate of points. If so, then the circle can not represent a biologic aggregate of individuals.

Geometry can be treated without any continuity assumption, without continuous circles, in fact without compasses.

Such a geometry, a geometry for biologists, is my own 'Rational Geometry,' the very first text-book of geometry in the world without any continuity assumption.

How biology has been misled in its mathematics you will realize when you recall that geometry and calculus have been the basis of mechanics, mechanics the basis for astronomy and physics, physics the basis for physical chemistry, while even the theory of probability had no discontinuous mathematics specially its own.

Therefore, biologists had clapped over their eyes spectacles of green continuity, and these spectacles colored biologic theories with the following characteristics as enumerated by the Russian Bugaiev: (1) the continuity of phenomena; (2) the permanence and unchangeableness of their laws; (3) the possibility of characterizing a phenomenon by its elementary manifestations; (4) the possibility of unifying elementary phenomena into one whole; (5) the possibility of sketching precisely and definitely a phenomenon for a past or future moment of time.

These ideas make the very essence, the framework, the skeleton of modern biologic theories. They have forced their way in and imbedded themselves as being necessary to make possible the application of the methods of continuity-mathematics to the investigation of nature. They follow out the fundamental characteristics of continuous analytic functions. Therefore, we may designate our modern biology as a continuity-biology.

Thus, as the Russian Alexeieff has pointed out, after the continuity world-scheme had captured the fundamental natural sciences, geometry, mechanics, astronomy, physics, chemistry, had intrenched itself in them and dowered them with generality, uniformity, universality, it went over gradually with scientific investigators by habit so to say into flesh and blood, and began to penetrate and dominate in physiology, in psychology, in sociology, in biology.

Darwin's attempt to found the law of the evolutionary origin of species is an outcome of the continuity world-scheme, permeated, saturated with its basal idea, continuity.

Just so strengthens itself more and more the persuasion of the continuous growth and continuous perfection of all the elements of human society in its natural advance.

The evolutionary development of social life permeates always more and more the view of the historian. Many writers are so habituated to this continuity world-scheme, that, without sufficiently critical consideration, they apply it where it is essentially inapplicable and inappropriate.

So we have the doctrine of a fatalist causality, denial of efficient freedom of the will, belittling of the idealistic endeavor of mankind, hence the pessimistic attitude toward the whole of human existence.

Paraphrasing a Russian poet:

Nature thus speaks to man:
"Thou mayst be head of creation,
But who gives thee any crown?
Dost thou believe, poor fool, in blind delusion,
That I am slave to thee, and thou my lord and master?
Of the thick veil lift I a corner tip
And Pygmy, then presumst thou
All through me that thou seest?
Seeing thine own small law and plan, art then deluded
Into the holy of holies to have pushed?
Oh fool! I do but nod and wretchedly thou'llt shudder,
Cower like timid dog on the sod. The earth
I shake and suddenly is dust
Thy pride and might, the greatest of thy cities.
War I send and pestilence its sister,
The blooming fields transform I into deserts,
The sea I drink up and the sun shroud I in darkness,
And thou, brute-like, wilt howl with pain, with anguish.
What you strive for and hope, to me that is indifferent.
Pity know I none, and my law of the number
Knows neither weal nor woe, knows neither praise nor blaming.
To unknown lands I stride in war, in whirlwind.
I know no aim, no end and no beginning.
I beget and I destroy, not prating, never angry,
The elephant and the worm, the wise man and the foolish.
So live as all live. Float out on the flood eternal
One instant brief, and vanish then forever.
Presume not stupid-bold with me to wage a contest,
With me eternal mother of all living and all dead."
So thunders nature with a million voices
In hail, in surge, in storm-wind and the lightning.

So much for the continuity world-scheme in biology.

But the latest advances in mathematics have rendered unnecessary for biology the wearing of this misfit garment.

The new mathematics gives now a standpoint for the explanation and treatment of natural phenomena from which the individuality of the biologic elements need not be suppressed.

It has triumphed for its own domain in cases where the continuity methods were wholly inapplicable, where arithmology, discrete mathematics, was called for and victorious.

Such are the problems which relate to the properties of whole numbers, solved so brilliantly in number-theory.

Such again are the questions relating to the enumeration of the geometric forms with n parameters which satisfy n given conditions. These even in the simplest cases showed themselves insoluble until finally between 1860 and 1870 the French mathematicians created special discrete methods. Thence sprang a wholly new branch of mathematics, enumerative geometry.

A third, an epoch-making universe of discrete mathematics, is the wonderful invariant theory of the great Sylvester and his brother-in-arms Cayley, two men whose loss left the English-speaking world without a single mathematician of first rank, of the rank of Hilbert and Poincaré.

In chemistry this discrete mathematics has shown itself of such use and power that we may assuredly say chemistry owes its present standpoint almost wholly to two lines of advance, both discrete, the atomic structure theory of Kekulé and Mendelieev's periodic system of the chemical elements.

The brilliant and rapid advances in chemistry have come not from suppressing but from stressing the individuality of the elements. Its mathematics has been essentially discrete.

The arithmologic scheme of chemical research, the atomic structure theory of Kekulé, coincides completely with the scheme of the symbolic invariant theory, though both were worked out independently.

Now to biology and sociology, having to do with single individuals differing from one another, in biology cells, in sociology human personalities, the continuity mathematics with its universalism is so ill adapted by its nature that the discrete way of thinking must here soon take the chief rôle, giving as it does large and free play to the individual peculiarities of the elements to be studied.

The continuity thought-way strives to reduce all phenomena of nature to a general mechanism with fate-determined movement. Just contrary to this then is the view that living nature is a rationally-correlated realm, in which everything is harmonic, shows adaptation, strives toward perfection.

Are not the mechanical form-phenomena of the living organism only its most elementary properties, upon which are built others higher, psychic? Now the psychic properties of a living organism can not be studied by observation and comparison of the accompanying mechanical properties unless they flow from these mechanical properties. If these accompaniments be unessential, the psychic properties can not be concluded from them. Here is even yet the battleground.

Biologists are at present emphasizing the statistical method, but upon this modern mathematics has for them another message. They rely upon the method of least squares and mean value. But Chebyshev has demonstrated that not the great number, but the independence of the metric phenomena plays the chief part in the application of the theory of mean value. This independence is the essential requisite, and it is the very thing whose unwarranted assumption vitiates much biologic research.

An illustration may be drawn from fire insurance. From the records of past conflagrations of single houses, if the burning of each one is independent of that of every other, the theory of mean value can get a number which can be counted upon to recur with slight variation from year to year, and upon it can be based the charges for insurance.

To realize how completely this essential requirement may be lacking, we have only to remember the Chicago fire, or the Baltimore fire.

Biologists have treated their combinations as if they were simple summations of independent elements.

More likely are the combinations composed of interdependent factors whose symbolization must be at the simplest a product.

A tremendous illustration of variation under change of stimuli is given by Japan.

For centuries environment and potential variability were in static balance; variation was zero.

Then came Commodore Perry, humiliations to the inordinate pride of a hermit nation, defeats, contempt, a tremendous response to the changes in stimuli, and to-day dark pagan Japan is easily defeating the largest European Christian white nation: variability unchanged, variation the greatest recorded in human history.

According to Quetelet's celebrated law of variability published some years after Darwin's 'Origin of Species,' it is subject to the law of probability, and according to this law the occurrence of variations, their frequency and their degree of variation can be calculated and predicted in the same way as the chance of death, of murders, of fires.

But such applications did not fit actual evolution, since the law is to deal with different degrees of the same qualities, giving a continuity production of species, while as de Vries has so stressed, the origin may be by abrupt jumps, by sports, by mutations.

De Vries has said that a thorough study of Quetelet's law would no doubt at once have revealed the weak point in Darwin's conception of the process of evolution. It would have shown that the phenomena which are ruled by this law and which are bound to such narrow limits, can not be a basis for the explanation of the origin of species.

It rules the degrees and amounts of qualities, but not the qualities themselves.

Species, however, as de Vries says, are not in the main distinguished from their allies by quantities, nor by degrees; the very qualities differ.

How such differences of qualitative character have been created is the burning question. They have not been explained by continuous accretion of individual variations.

The attitude of the new mathematics strongly favors attempts like the mutation theory, based on the abrupt, explosive changes, wholly discrete, which under the name of 'sports' had long been observed and known in horticulture and animal breeding, and of which De Vries has found a whole fusilade being shot off by 'Lamarck's evening primrose.'

Here he says there is no gradual, no continuous change or modification, nor even a common change of all the individuals. On the contrary, he says, the main group remains wholly unaffected by the production of new species. After eighteen years it is absolutely the same as at the beginning. It is not changed in the slightest degree. Yet it produces in the same locality, and at the same time, from the same group of plants, a number of new species diverging in different ways.

The vastly vaunted natural selection, then, can only destroy new species, never create them.

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THE ADMISSION OF STUDENTS TO COLLEGE BY CERTIFICATE.[1]

WHICH is better: the western plan of admitting students to colleges and universities by certificates from duly inspected secondary schools, or the eastern method of admitting only by examinations conducted by representative boards or otherwise?

The question assigned me as a topic is a pressing one at this moment in the history of American education. Within a few years it may be determined which plan,

[1] Paper read before the Department of Higher Education of the N. E. A., Asbury Park, N. J., Friday morning, July 7, 1905.