our national capital, might be a good place for experimentation.

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ASTIGMATIC IMAGES OF THE BOTTOM OF A POOL OF WATER.

IF light radiate from a point below the surface of water, it can pass out through the surface only within a circle forming the base of a right cone whose semi-angle is the critical angle.

Consider such rays lying in a vertical plane passing through the radiant point. The rays which have passed out into the air, if produced below the surface, are tangent to a virtual caustic. This caustic is a portion of the evolute of an ellipse, one cusp of which is in a vertical through the radiant point, and at a depth $\frac{d}{n}$, where d is the depth of the radiant point, and n is the index of refraction. The branches of the caustic are tangent to the surface in the circle determining the critical angle. Successive sets of consecutive rays having an increasing angle of incidence do not intersect at a common point, but they intersect at consecutive points on the caustic. If the vertical plane be rotated slightly in azimuth, the rays from the same radiant point will intersect in the caustic in its new position. This caustic from the same radiant point will always lie on a surface of revolution, formed by revolving the caustic in any vertical plane about the vertical line through the radiant point.

If the radiant point be viewed by an eye placed at a fixed point, the pupil of the eye may be conceived divided into vertical zonal elements. Rays from the radiant point in these various elements will intersect in a definite area upon the surface of revolution. The point would, therefore, appear as a hazy patch upon the caustic surface. The text-books all represent the apparent position of a coin seen through a water surface, as being lifted up and towards the eye of the observer, upon the caustic surface.

It is, however, evident that if the rays diverging from the radiant point in all azimuths, and at a fixed angle of incidence, be produced backwards after passing out into the air, they will all intersect in a common point upon the vertical line through the radiant point. If, therefore, the pupil of the eye be divided into horizontal zonal elements, all the rays entering the eye will have a virtual intersection on this vertical line. The focus of the upper zonal elements of the eye will be slightly below those of the lower. Nevertheless, the intersection of all rays entering the eye from the radiant point will be upon a line, instead of being spread out over an area as in the other case. The fact is that a plumb line deeply piercing still water appears straight throughout. The image upon the vertical line is much more distinct than that formed upon the caustic surface. The latter image imparts a haziness to the appearance of the body viewed, but the apparent position is determined by lines which intersect in a common point, rather than by those which do not.

With this view of the matter the writer in May, 1881, presented to the Academy of Science of St. Louis a discussion of the apparent form of the flat bottom of a pool as seen through the surface.* The appearance was found to be represented by a conchoid, which was related in a simple way to the conchoid of Nicomedes. The equations of both curves were deduced, and several other cases were discussed.

In a recent number of *Annalen der Physik*,† Mattheissen has deduced the equations of these two conchoids and has pointed out that the surface produced an astigmatic effect. He likewise deduces the equation for the nebulous image due to intersection upon the caustic. The minimum of this surface and that of the conchoid are coincident and tangent to each other, and they have the water surface as a common asymptote.

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NOTES ON INORGANIC CHEMISTRY.

THE earliest determinations of the density o sulfur vapor were by Dumas and Mitscherlich, and gave figures which pointed to the molecule S_6, and this has passed current until quite recent times. In 1860 Deville and Troost found

* *Trans. Acad. of Sc. of St. Louis*, Vol. IV., No. 2, p. 325.

† No. 10, 1901, S. 347.