which, instead of being a synonym of *Uromys*, should take the former's place at 52.

One change which I myself pointed out in 1895, but forgot in 1896, has escaped the lynx eyes of Mr. Palmer, namely, that *Pygeretmus* Gloger * (1841) antedates and supersedes *Platycercomys* Brandt (1844), No. 117 of the list.

In what has been called the real part of the paper, I doubt if Mr. Palmer's criticisms on the suppression of the *Lophiomyidæ* and the separation of the *Spalacidæ* and *Bathyergidæ* would have been made had he ever compared the teeth—practically identical—of *Lophiomys* and *Cricetus cricetus*, or realized to what an extent similar fossorial habits may mask real differences by a superficial resemblance, so that the two families referred to, really incomparably more different in essentials than the Americen *Geomyidæ* and *Heteromyidæ*, have yet become so alike externally that zoologists of an earlier generation naturally thought them to be nearly allied.

But on these and other points further criticism is much to be desired, and I can only repeat how fortunate it is that my mistakes and omissions in the nomenclatural part of the paper should have had the advantage of revision by such an authority on the subject as Mr. Palmer.

OLDFIELD THOMAS.

* Naturgesch., p. 106.

MARRIAGE BY CAPTURE IN ARABIA.

Antar is a Bedouin romance reputed to have been written by Asmai, one of the learned men of the court of Haroun-al-Raschid, shortly before the beginning of the ninth century.† From the translation by Terrick Hamilton (London, 8vo., 1820), Vol. IV., pp. 388–9, the following description of an early Arabian marriage custom is quoted. The custom is a well known one. Asmai's explanation of it is new to me.

"Now, there was a certain curious custom current among the Arabs at that period. The night on which a bridegroom should wed his wife they brought a quantity of camel pack-saddles and heaped them one upon another, decorating them with magnificent garments. Here they conducted the bride, and having seated her on high, they said to the bridegroom, "Come on, now, for thy bride!" And the bridegroom rushed forward to carry her off, whilst the youths of the tribe, drawn up in line, right and left, with staves and stones in their hands, as soon as the bridegroom rushed forward, began beating and pelting him and doing their utmost to prevent his reaching his wife. If a rib or so were broken in the affair it was well for him; if he were killed it was his destiny.

"But should he reach his wife in safety, the people quitted him and no one attempted to approach him. ('I inquired about this circumstance,' says Asmai, 'and what it was they were about.' 'Asmai,' they answered, 'the meaning of this is to exhibit the bride to the warriors, that should her husband die, anyone else might take a fancy to her and take her off.')"

So far as my reading goes, the explanation of marriage by simulated capture, which is given in the last sentence, is entirely novel.

EDWARD S. HOLDEN.

LICK OBSERVATORY,
August 15, 1897.

† It is, in fact, a compilation of the XIIth century.

SCIENTIFIC LITERATURE.

The Foundations of Geometry. By B. A. W. RUSSELL. Cambridge: The University Press. 1897. Pp. xvi + 201.

Here is a book especially opportune, on a subject of transcendent interest. The author's mathematical equipment is refreshingly sound, and his metaphysical results are delightfully suggestive, even where the mathematician may feel constrained to return as verdict 'not proven.' So much the more to be regretted is it that the Chapter I., 'A Short History of Metageometry,' should open with a glaring error, as follows: "The liquefaction of Euclidean orthodoxy is the axiom of parallels, and it was by the refusal to admit this axiom without proof that Metageometry began. The *first* effort in this direction, *that of Legendre*, was inspired by the hope of deducing this axiom from the others."

Mr. Russell cites Halsted's Bibliography of Hyper-Space and Non-Euclidean Geometry (1878), but can evidently never have seen it, since its first page speaks of 'The enormous number of unsatisfactory attempts to prove this postulate,' and states that Sohncke gives a list of 92 authors on the subject before 1837, and that Perronet Thompson gave in English an account of like attempts before 1833, the very year our author cites for Legendre.

Mr. Russell goes on to say :

"Parallels are defined by Legendre as lines in the same plane, such that, if a third line cut them, it makes the sum of the interior and opposite angles equal to two right angles. He proves without difficulty that such lines would not meet."

But so had every school boy in the subject, since this is part of Euclid, Book I, Prop. 28:

"Similarly he can prove that the sum of the angles of a triangle cannot exceed two right angles, and that if any one triangle has a sum equal to two right angles all triangles have the same sum."

But these very demonstrations were published just a century before Mr. Russell's '*first* effort,' in 1733, by Saccheri.

Mr. Russell proceeds to speak of 'The originator of the whole system, Gauss,' and then says: "In 1799, writing to W. Bolyai, Gauss enunciates the important theorem that in hyperbolic geometry there is a maximum to the area of a triangle."

How utterly misleading, nay, fantastic, is this statement will appear on quoting the letter from 'Halsted's Science Absolute of Space,' 4th edition, Austin, 1896, which our author cites. Gauss says:

"I very much regret that I did not make use of our former proximity to find out *more* about your investigations in regard to the first grounds of geometry; I should certainly thereby have spared myself much vain labor, and would have become more restful than any one, such as I, can be, so long as on such a subject there yet remains so much to be wished for.

"In my own work thereon I myself have advanced far (though my other wholly heterogeneous employments leave me little time therefor), but *the* way, which I have hit upon, leads not so much to the goal which one wishes and which you assure me you have reached, as much more to making doubtful the truth of geometry.

"Indeed, I have come upon much, which with most no doubt would pass for a proof, but which in my eyes proves as good as NOTHING.

"For example, if one could prove that a rectilineal triangle is possible, whose content may be greater than any given surface, then I am in condition to prove with perfect rigor all geometry.

"Most would, indeed, let that pass as an axiom ; I not ; it might well be possible that, how far apart soever one took the three vertices of the triangle in space, yet the content was always under a given limit.

"I have more such theorems, but in none do I find anything satisfying."

From this letter it is perfectly clear that in 1799, so far from having the remotest idea of a hyperbolic geometry, or any non-Euclidean geometry, Gauss was still trying to prove that Euclid's is the only non-contradictory system of geometry and that it is the system of the external space of our physical experience. The first is false; the second can never be proved. But that both Gauss and W. Bolyai continued for the next five years to pound away in attempts to do the impossible, we have now obtained demonstrative evidence, in recovering a treatise finished and sent to Gauss by W. Bolyai in 1804.

In a great casket at Maros-Vásárhely all the unpublished papers of Bolyai János are preserved. All were placed freely at my disposal on my pilgrimage to this shrine of the non-Euclidean geometry. There, with extended researches anticipating the discoveries of Cayley and Klein in this subject, is an autobiography of János containing extracts from two letters written by Gauss to W. Bolyai (Farkas) and of transcendent importance as freeing János forever from the calumny again repeated by Mr. Russell where he says:

p. 12. "Gauss was, as we have seen, the inspirer of Wolfgang Bolyai. Wolfgang appears to have left to his son, Johann, the detailed working out of the hyperbolic system."

Nothing could be more false.

János, wholly unaided, discovered by himself the non-Euclidean geometry and taught it to Wolfgang, who transmitted it to Gauss. The two letters quoted by János are one before and one after this transmission.

This cry from the dead for tardy justice has since been shown exactly accurate by my friend, Fr. Schmidt, of Budapest, finding that the originals of these letters in the handwriting of Gauss still exist at Göttingen. The first is dated November 25, 1804, in answer to a letter from W. Bolyai of September 16, 1804, accompanied by a Latin treatise, *Theoria parallelarum.*

It read as follows: "Now * * * yet somewhat about your geometric communication. I have read through your treatise with great interest and attention, and am right delighted at its really profound keenness. But you do not wish my empty praise, which also might seem in a measure partial because your train of ideas has very great resemblance to the way I formerly sought the untieing of this Gordian knot and vainly seek till now. You wish only my candid, open judgment. And this is, that your procedure does not give me satisfaction. I will seek, with as much clearness as I can, to bring to light the stone of stumbling which I still find therein (and which also again pertains to the same *group* of rocks wheron my attempts have hitherto been wrecked).

"I have indeed yet ever the hope that those rocks some day, and even before my end, will grant a thoroughfare. Meanwhile I have now so much other business on hand that I at present cannot think thereon, and, believe me, it will heartily delight me if you precede me and attain to overmaster all obstacles. I would, then, with inmost joy, do all in my power to make your service current and put it in the light.

* * * * * * * * * *

Could you prove $dke = ekf = fkg$, etc., then would the thing be perfect. But this theorem is indeed true, only difficult to prove rigorously without already presupposing the theory of parallels. * *

You have my candid judgment. I have given it, and I repeat that it would genuinely delight me if you overcome all difficulties."

Here we see, with startling clearness, that in 1804 both Gauss and W. Bolyai (Bolyai Farkas) believe that Euclid's Parallel-Postulate can be proven, and indeed are racing to demonstrate it.

Before the next letter the unaided genius of the son, Bolyai János, has created the new universe, has found out all about it, mapped it, and proved Euclid's Postulate forever indemonstrable.

In transmitting in print to Gauss the immortal treatise of his son János, the most marvellous two dozen pages in the whole history of human thought, the father, Farkas, writes on June 20, 1831:

"My son is already First Lieutenant in the Engigineering Corps, and will soon be Captain, a handsome youth, a virtuoso on the violin, a fine fencer and brave, but has often dueled, and is still altogether too wild a soldier—but also very refined—light in darkness and darkness in light, and an impassioned mathematician with very rare gifts of mind. At present he is in the garrison at Lemberg—a great admirer of you—capable of understanding and appreciating you. At his desire, I send you this little work of his. Have the goodness to judge it with your sharp, penetrating eye, and to write your high judgment unsparingly in your answer, which I ardently await. It is the first beginning of my work, which is under the press. I would gladly send with this the first volume, but it is not yet out.

"According to my view, is in the work of my son, u (namely, where a first does not cut the b) geometrically constructed; whence, however, is not determined how great u is, from O on up to R (that excluded, this included).

"Yet everything in geometry is either dependent on u or not; (*e. g.*) spherical trigonometry is in § 26 settled as independent of it. * * *

"At the end he also shows that if u not $= R$, then the circle can be squared."

Thus we see that the treatise sent to Gauss on June 20, 1831, was the immortal Appendix just as published. The Gordian knot, at which Gauss himself had for years tried in vain, was here forever gloriously untied.

After waiting six months the anxious father tries again, and on January 16, 1832, once more sends Gauss the work of János, saying in the accompanying letter:

"My son was not present when his little work was printed. He had printed the errata (which follow); in order to be less burdensome to you, I have corrected the most with a pen.

"He writes from Lemberg that he has since simplified and made more elegant many things, and has proven the impossibility of determining *a priori* whether Axiom XI be true or not."

To this, on March 6, 1832, comes from Gauss at length an answer as follows:

* * * "Now somewhat about the work of your son:

"If I thus begin 'that I dare not praise it' you will a moment wonder; but I cannot otherwise; to praise it would mean to praise myself. For the whole content of the book, the way your son has hit out and the results to which he is led, are identical almost throughout with my own meditations, made in part already 30–35 years ago. In fact, I am thereby extremely delighted. My intention was to let nothing be known during my lifetime of my own work, of which moreover until now little has been put on paper. Most men have not at all the right sense for what is here in question, and I have found only few people who received with special interest that which I communicated to them. In order for that one must first have felt right keenly what really is lacking, and about that most men are wholly indistinct.

"On the other hand, my intention was, with time, to put all on paper, so that it at least would not hereafter perish with me.

"Therefore am I greatly pleased that this trouble can now be spared me, and it is most highly delightful to me that the son of just my old friend is he who in so remarkable a way has anticipated me.

"I find his notations very pregnant and abridging. Yet I believe it would be good to establish for many chief ideas not merely symbols or letters, but definite names, and I have already since long thought of some such names.

"So long as one thinks through the thing only in immediate intuition, one needs no name or symbol; these are first necessary, if one wishes to be comprehensible to others. So, for example, the surface which your son calls F could be called a Parasphere, the line L a Paracykle: they are, in fact, spheres or circles of infinite radius. Hypercykle could be named the complex of all points which have like distance from a straight with which they lie in a plane; even so Hypersphere. Yet those are all only unimportant incidents; the main thing is the matter, not the form.

* * * * * *

"Just exactly in the impossibility to decide *a priori* between Σ and S lies the clearest proof that Kant was wrong to maintain, Space is *only Form* of our intuition."

About the other independent discoverer of the non-Euclidean geometry, Lobachévski, Gauss writes to Schumacher, November 28, 1846, without a word of reference to Bolyai, as follows:

"I have lately had occasion to reread the opuscule of Lobatschewsky, intitled: *Geometrische Untersuchungen zur Theorie der Parallellinien.* This opuscle contains the elements of the geometry which would exist and development of which would form a rigorous chain, if the *Euclidean* geometry were not true. * * * You know that since fifty-four years (since 1792) I share the same convictions, without speaking here of certain developments which my ideas on this subject have since received. Therefore, I have not found in the work of Lobachevski any fact new to me; but the exposition is wholly different from that which I had projected, and the author has treated the matter with a master hand and with the veritable geometric spirit."

How reconcile these letters with that of 1804? And since one says that Bolyai's exposition is identical with that of Gauss, while the other declares Lobachévski's wholly different from that, how reconcile them with the statement of Mr. Russell, p. 11: "Very similar [to Lobachévski's] is the system of *Johann Bolyai*, so similar, indeed, as to make the independence of the two works, though a well-authenticated fact, seem all but incredible?"

This letter of 1846 shows no hint of that other sort of non-Euclidean geometry which Riemann gave in his wonderful Probevorlesung, 'Ueber die Hypothesen welche der Geometrie zu Grunde liegen,' June 10, 1854.

But this dissertation was not published until 1867, so that the waters of oblivion seemed to close over it as over the works of Bolyai and Lobachévski.

Mr. Russell should not have omitted in his text all mention of Hoüel, for Hoüel it was who resurrected the non-Euclidean geometry, beginning with his own essay on the fundamental principles of geometry, published in 1863 at Greifswald. (See his life in the *Amer. Math. Monthly*, April, 1897.)

But not to give too much space to actual slips in history we must jump to the second of Mr. Russell's four chapters, 'Critical account of some previous philosophical theories of geometry:'

"The importance of geometry in the theories of knowledge which have arisen in the past can scarcely be exaggerated."

The author believes that the usual forms of non-Euclidean geometry, the hyperbolic, the double elliptic and the single elliptic are the only logically self-consistent systems, and so says: "I shall contend that those axioms, which Euclid and Metageometry have in common, coincide with those properties of any form of externality which are deducible, by the principle of contradiction, from the possibility of experience of an external world."

We see at once that pure projective geometry must be of supreme weight for him.

It is a treat to see our author overwhelm the apparent subordination of the non-Euclidean spaces by the introduction of different measures of distance. This was the painful mistake of

Emory McClintock in his article 'On the non-Euclidean geometry' in the Bulletin of the N. Y. (Amer.) Math. Soc., Vol. II., pp. 21–33, which reached the pitiful conclusion (p. 32): "The chief lesson to be obtained from all non-euclidian diversions (sic) is that the distinguishing mark of euclidian geometry is fixity of distance—measurement."

Mr. Russell, with equal deftness, puts in pillory the gross blunder made by Andrew W. Phillips and Irving Fisher, professors in Yale University, in the note on p. 23 of their Elements of Geometry, where they say: "Lobatchewsky in 1829 proved that we can never get rid of the parallel axiom without assuming the space in which we live to be very different from *what we know it to be through experience.*"

By experience, of course, we can never know or prove our space to be other than a non-Euclidean space with a comparatively large constant. How unexpected, then, the error of Professor H. Schubert, of Hamburg, in the *Monist*, Vol. VI., No. 2, p. 295, where he says:

"Let me recall the controversy which has been waged in this century regarding the eleventh axiom of Euclid, that only one line can be drawn through a point parallel to another straight line. The discussion merely touched the question whether the axiom was capable of demonstration solely by means of the other propositions, or whether it was not a special property, *apprehensible only by sense-experience*, of that space of three dimensions in which the organic world has been produced."

After 20 years' study of writers on the non-Euclidean geometry, the present reviewer cannot recall even one who was ever silly enough to think that the exact equality of the angle-sum of a rectilineal triangle to two right angles was apprehensible by sense-experience, or could ever be known through experience.

This new Yale geometry also makes the old *petitio principii* of defining a straight line as the *shortest* distance between two points. This our author treats in his third chapter, p. 167:

"We are accustomed to the definition of the straight line as the *shortest* distance between two points. * * * Unless we presuppose the straight line, we have no means of comparing the lengths of different curves and can, therefore, never discover the applicability of our definition."

In projective geometry any two points uniquely determine a line, *the straight.* But any two points and their straight are, in pure projective geometry, utterly indistinguishable from any other point-pair and their straight. It is of the essence of metric geometry that two points shall completely determine a spatial quantity, *the sect.* If our author had used for this fundamental spatial magnitude this name, introduced in 1881, his exposition would have gained wonderfully in clearness.

Both the accepted popular and the accepted mathematical definition of 'distance' make it always a number, as, *e. g.*, the Cayley-Klein definition: "The distance between two points is equal to a constant times the logarithm of the cross ratio in which the line joining the two points is divided by the fundamental quadric."

It is the misfortune of our author to use the already overworked and often misused word 'distance' as a confounding and confusing designation for a sect itself and also the measures of that sect, whether by superposition, ordinary ratio, indeterminate as depending on the choice of a unit, or projective metrics, indeterminate as depending on the fixing of the two points to be taken as constant in the varying cross ratios.

This whole book might be cited as an overwhelming vindication of the only American treatise on Projective Geometry against the attack on it made by a critic in SCIENCE, because, forsooth, it was founded and developed as *pure* projective geometry, without any quantitative ideas whatever.

Into the fourth and last chapter, 'Philosophical Consequences,' we will not here go. Suffice it to say that Projective and Metric Geometry, though eternally separate in essence, and each unable ever to absorb the other, are happily wedded, and expand joyfully ever after.

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Sight: An Exposition of the Principles of Monocular and Binocular Vision. JOSEPH LE CONTE. New York, D. Appleton & Co. 1897. Second edition, revised and enlarged. Pp. xvi + 318. $1.50.

A revised and enlarged edition of Professor