

## DISCUSSION AND CORRESPONDENCE.

## THE LENGTH OF A CURVED LINE.

I SHOULD be very sorry to have anyone interpret my remarks in a recent number of SCIENCE (see page 533) as imputing ignorance of fundamental principles to so distinguished a geometer as Prof. Halsted. In saying that Prof. Halsted 'appears to believe' that he has given a logically complete discussion, my meaning was that he so appears to the unassisted reader of his 'Elements of Geometry.' My criticism was directed at the book rather than at the man. Further, as he says in his reply on page 656 of SCIENCE, the criticism is not applicable to his more recent work, 'Elementary Synthetic Geometry.'

In my opinion, it is not possible to discuss, in an elementary manner, propositions relating to the magnitude of curved lines until after the introduction of the following postulate: *The magnitude of a curved line is the limit toward which a broken line made up of consecutive chords of that curved line approaches, when the number of chords is increased in such a manner that the chords are all diminished without limit.* After the introduction of this postulate it is possible to compare the magnitude of a curved line with that of a straight line.

To turn again to Prof. Halsted's 'Elements of Geometry,' not only was it an error of logic to attempt to demonstrate without this postulate, or its equivalent, that a straight line is the shortest line joining two fixed points; but it was an error of the same sort to introduce, on pages 162-165 of that work, propositions relating to isoperimetric figures, which from their very nature depend on a comparison of non-congruent lines.

It seems worth while to insist upon the points made in this note and in my preceding note, because they relate to subjects treated in almost every American text-book of geometry; but in none, so far at least as the writer is aware, has a thoroughly satisfactory treatment been given.

In the very recent text-book of Beman and Smith, of which the writer has expressed a high opinion (See SCIENCE, this volume, page 203), the following appears on page 187:

"POSTULATE OF LIMITS. The circle and its

circumference are the respective limits which the inscribed and circumscribed regular polygons and their perimeters approach if the number of their sides increases indefinitely.

"This statement is so evident that a proof is not considered necessary. Like valid proofs of many fundamental principles, it is too difficult for an elementary text-book."

The statement consists of two parts, one relating to superficial magnitude, the other to linear magnitude. The former is capable of simple proof. The circle is greater than any inscribed polygon, and any circumscribed polygon is greater than the circle; by the axiom, *the whole is greater than any of its parts*. Proofs based upon these considerations are older than the text of Euclid. The second part of the statement is a 'postulate' in a strict sense. It cannot be proved at all except from equivalent assumptions.

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## ON CRITICISMS OF ORGANIC SELECTION.

A LONG absence in Europe has prevented my seeing several criticisms of my papers in this JOURNAL, until very recently; and although the issues may now be forgotten by the critics as well as by the readers of SCIENCE, I venture to write a few lines, if only to express my thanks for the kindly words which have aided me to see where the articles were not clear.

First, I may say that I have published, in the *American Naturalist* (June and July, 1896), a paper of some length under the title 'A New Factor in Evolution,' gathering the positions of the SCIENCE articles into a single sketch, thus carrying out, to a degree, the suggestion made by Prof. Wesley Mills in SCIENCE, May 22 (a suggestion which, however, I did not see until my return in September). Condensed summaries of the two main positions involved in the doctrine of Organic Selection (which I ventured to call a 'new factor') were quoted in this JOURNAL for July 31, p. 139, and I need not stop to requote them.

I am glad to know, both from Prof. Mills' article in SCIENCE, May 22d, and also from a personal letter from him, that he accepts the class of facts which I have emphasized, and admits their importance (having himself before