epileptic seizure it was .50 of a second for touch, and .37 for sound. In another patient the re-action times were .35 of a second for touch and .30 for hearing three hours after an attack, as against .21 of a second and .16 normally. A third patient, whose normal re-action times were .28 of a second (touch) and .34 of a second (sound), two hours after a seizure, re-acted in .40 of a second to touch and .37 of a second to a sound. The same patient, seventy-two hours after the last of fifteen successive attacks, required 1.11 seconds to re-act to touch, and 1.25 seconds to re-act to a sound. In an independent research, M. Féré had shown that in the average of twenty cases the dynamometric power was reduced to 45 per cent of its normal value immediately after a seizure, to 33 per cent after one-quarter of an hour, to 25 per cent after an interval of one half-hour, and to 17 per cent after an interval of three-quarters of an hour. Apart from special relations of the nature of the seizure to the diminution in muscular power, the general thesis of M. Féré is well borne out by these facts.

In normal individuals the same relations can be demonstrated, though the contrasts are not as sharp. Fatigue diminishes muscular force, and increases the times of re-action. Intelligent persons, speaking generally, have a short re-action time and a high dynamometric pressure. In order to study in closer detail the relation of re action time and motor power in special motor groups, M. Féré had constructed a dynamometer in which the pressure of each finger was recorded separately. With this apparatus M. Féré was able to establish that the movements of flexion were from three to ten times as powerful as those of extension; that the power of different fingers varies with different individuals, and stands in relation to the profession of the individual, the third and fourth fingers being especially strong in piano-players; and that intellectual persons have an especially strong thumb, an essentially human movement.

	Flexion.		Extension.	
	Dynamometer.	Re-action Time. Seconds.	Dynamometer.	Re-action Time. Seconds.
Thumb	4.2	.163	1 2	.190
Forefinger......	4.0	.191	1.0	.261
Middle finger...	3 5	.193	.9	.280
Third finger....	2.0	.201	.6	.299
Little finger ..	1.9	.203	.4	.310
Thumb........	2.7	.230	1.0	.335
Forefinger......	3.3	.160	1.1	.260
Middle finger...	2.2	.180	.4	.277
Third finger...	2.0	.195	.35	296
Little finger ..	1 8	.246	.3	.309
Thumb	4.1	.170	1.1	.220
Forefinger......	3.0	.191	.6	.210
Middle finger ..	3.2	.182	.7	.190
Third finger ...	2.2	.181	.7	.183
Little finger..	5.1	.171	.5	.142
Thumb..... ...	2.8	.282	.6	.340
Forefinger......	2 6	.359	.4	.516
Middle finger..	2.5	.346	.3	.515
Third finger ...	1.7	.436	.1	.639
Little finger....	1.4	.515	.2	.517

The first three records were obtained from officials of the hospital, and exhibited very fairly the points in discussion, while the third subject is also a pianist, and shows a remarkable power of flexion of the little finger as well as a quick re-action time for both flexion and extension of this finger. The fourth record is of an intelligent epileptic patient. We see, that, while the dynamometer shows movements of flexion far superior to those of extension, the re-action times show only a slight superiority, and that exercise seems to increase not only the power of flexion, but the speed of extension. If we make separate observations on the right and left hands, we will find that the preferred hand presses more strongly and re-acts more quickly than the other hand.

The same method can be applied to the movement of other organs. The energy of extension of the tongue has been measured, and varies in normal subjects from 500 to 850 grams. In deaf-mutes and patients afflicted with aphasia it may be as low as 100 grams. That the energy of this movement is related to the re-action time is shown in the following results: F (a normal subject) moves the tongue with a force of 850 grams, and performs this motion in .13 of a second; L (also normal), 400 grams and .15 of a second; J (partially aphasic), 300 grams and .30 of a second; F (a stammerer), 200 grams and .33 of a second.

That nutritive processes play an important part in these movements is more than likely. Cold retards and heat accelerates the re-action times. The following table shows the effect of warming upon the re-action time in movements of flexion and extension of the five fingers: —

	Flexion.		Extension.		
	Before Warming.	After Warming.	Before Warming.	After Warming.	The movements of extension, and especially those ordinarily the slowest, seem to be most benefited by this additional warmth.
Thumb	.346	.233	.362	.194	
Forefinger........... ..	.269	.234	.270	.186	
Middle finger......... .	.266	.261	.280	.201	
Third finger............	.255	.239	.320	.250	
Little finger...........	.283	.237	.312	220	

This research, though incomplete, and founded upon rather few experiments with each subject, yet admirably suggests the close relations that exist between the motor, sensory, and nutritive functions of the psycho-physical organism. As our knowledge of this relation becomes more and more exact, the possibilities of utilizing such knowledge for making the elementary processes of knowledge and action easier and quicker, become more and more real.

RAPIDITY OF MOVEMENTS. — A pianist, in playing a presto of Mendelssohn, played 5,595 notes in four minutes and three seconds. The striking of each of these notes, it has been estimated, involved two movements of the finger, and possibly more. Again, the movements of the wrists, elbows, and arms can scarcely be less than one movement for each note. As twenty-four notes were played each second, and each involves three movements, we would have seventy-two voluntary movements per second. Again, the place, the force, the time, and the duration of each of these movements, was controlled. All these motor re-actions were conditioned upon a knowledge of the position of each finger of each hand before it was moved, while moving it, as well as of the auditory effect in force and pitch, all of which involves at least equally rapid sensory transmissions. If we add to this the work of the memory in placing the notes in their proper position, as well as the fact that the performer at the same time participates in the emotions the selection describes, and feels the strength and weaknesses of the performance, we arrive at a truly bewildering network of afferent and efferent impulses, coursing along at inconceivably rapid rates. Such estimates show, too, that we are capable of doing many things at once. The mind is not a unit, but is composed of higher and lower centres, the available fund of attention being distributable among them.

BOOK-REVIEWS.

A Treatise on Linear Differential Equations. By THOMAS CRAIG. New York, Wiley. 8°.

THE theory of differential equations has undergone within the last thirty years a most fundamental change. The object of the older theory was to integrate a given differential equation "in finite form;" that is to say, by means of the elementary functions of analysis. But though the importance of this problem for practical purposes must be acknowledged, the problem itself, understood in this form, is in general an impossible one.

The modern theory, inaugurated by Briot and Bouquet's and Fuchs's discoveries, has reversed the whole problem. It considers the differential equation (together with a proper number of initial conditions) as defining a function, and proposes to derive directly from the differential equation the characteristic properties of its integrals, true to the general principle of the theory of functions, that the essential thing about a function is not its form, which usually may be varied in many ways, but the totality of its characteristic properties.

It is in particular the theory of linear differential equations that has been very fully considered from this standpoint; and there is scarcely any branch of mathematical science that has attracted a more general attention in our day, and in which more important discoveries have been made, than the theory of linear differential equations. Still every one who wished to become familiar with it, and who had to work his way through the vast and difficult literature on the subject, has keenly felt the want of a systematic exposition uniting the numerous researches scattered in the different mathematical journals and publications of learned societies.

To meet this want, and to give an account of the theory as it stands to-day, is the object of the "Treatise on Linear Differential Equations," by Professor Thomas Craig of Johns Hopkins University. The first volume, which is to be followed by a second one, is entitled "Equations with Uniform Co-efficients," and deals principally with Fuchs's theory and the investigations immediately connected with it. The rich material has been carefully sifted, and is presented in a clear and intelligible language in the most natural order of ideas.

An introductory chapter gives the general properties of a system of linear differential equations of a more formal character, among others the well-known theorems on systems of independent particular integrals.

Next follows an elegant exposition of the theory of linear differential equations with constant co-efficients, where the reader will find, besides Euler's solution, an account of various ingenious methods due to Cauchy, Hermite, and others.

After these preparations, we are led, in Chapter III., into the very centre of the modern theory; viz., the determination of the form of the integrals in the region of a critical point. It is first shown, that, if the differential equation be written in the form

$$\frac{d^n y}{dx^n} + p_1 \frac{d^{n-1} y}{dx^{n-1}} + \quad . \quad . \quad . \quad + p_n y = o,$$

the critical points of any one of its integrals are always found among the critical points of the system of co-efficients, $p_1, p_2 \ldots p_n$. Then Fuchs's theorems concerning the form of the integrals in the region of a critical point are developed with all the details about "groups of integrals" added by Hamburger, Floquet, and others.

A particular integral is said to be regular in a critical point a, if it remains finite for $x=a$ after multiplication by some proper power of $x - a$; and, in order that all the integrals may be regular in a, it is necessary and sufficient that $(x-a)^\alpha p^\alpha$ $(\alpha=1, 2 \ldots n)$ be holomorphic in a. Chapter IV. contains an account of Frobenius's elegant treatment of this case, and gives a simple criterion for the non-appearance of logarithms.

The next chapters are devoted to that important class of differential equations (called regular equations) all of whose integrals are regular in all the critical points; and the fertility of the general methods is abundantly shown in the application to the equation of the second order, in particular that with three critical points, which, on account of its high importance, is very fully treated, with many interesting results concerning Riemann's P-function, spherical harmonics, Bessell's functions, etc.

The differential equation of the hypergeometric series, to which the above equation can always be reduced, takes such a central place in recent mathematical researches that it well deserves to be considered with all detail, as is done in Chapter VII., which contains a reproduction of Goursat's "Thesis on the Hypergeometric Series."

The theory of irregular integrals is still in a very imperfect state. Chapter IX. gives an account of Frobenius's and Thomé's researches, and the same subject is treated in Chapter X. by the elegant method of decomposition of a differential quantic into symbolic prime factors. Interesting special classes of irregular equations will be found in the chapters on Halphen's equations, and on equations with doubly periodic co-efficients.

The two remaining chapters might, it seems to us, as well have been reserved for the second volume, where the same subjects will be more fully dwelt upon. Still the two conceptions of group and of invariant of a differential equation which they develop are of so fundamental importance that they can scarcely be introduced too soon.

If the co efficients of a linear differential equation are uniform functions of x, any system of n independent particular integrals submit to a homogeneous linear substitution when the variable point x describes any closed path in its plane. The entire system of substitutions obtained in this way forms a group, called the "group of the differential equation."

The notion of "invariant" of a linear differential equation, on the other hand, arises when the given equation is transformed into another of the same form by the introduction of two new variables, and its definition is analogous to that of an invariant of an algebraic quantic.

We must confiné ourselves to these few indications, and refer the reader to the book itself for further information. Only then will he obtain an adequate idea of the thoroughness and completeness with which the subject has been treated. As far as we are able to judge, no investigation of any importance has been omitted, and the justice and conscientiousness with which continually reference to the original papers is given are a characteristic feature of this most valuable book, which, we are sure, will contribute a great deal to spread the knowledge of this important discipline.

We look forward with much interest to the appearance of the second volume, which will contain, among other things, an exposition of the theory of linear differential equations with algebraic integrals, and of Poincaré's theory of Fuchsian groups and Fuchsian functions.

AMONG THE PUBLISHERS.

THE Bulletin of the Ohio Agricultural Experiment Station for October, 1889, is Vol. I, No. 1 of a technical series, and contains three articles by Clarence M. Weed, — "Preparatory Stages of the 20-Spotted Ladybird," "Studies in Pond Life," and "A Partial Bibliography of Insects affecting Clover."

— The opening article in the December number of *Outing*, "Wabun Anung," by F. Houghton, is a clear description of a tour in the region of the Great Lakes. Another article is the "Merits and Defects of the National Guard," by Lieut. W. R. Hamilton. We note further the "Game of Curling," by James Hedley; "Wheeling through the Land of Evangeline;" "Game Protection;" "Instantaneous Photography," by W. I. Lincoln Adams; "Women and their Guns;" "The Yale Stroke;" "Alligator Shooting in Florida;" and "Na-ma-go-os," a fishing sketch.

— John Wiley & Sons have just published "A Hand-Book for Sugar Manufacturers and their Chemists," by Guilford L. Spencer of the United States Department of Agriculture. The volume contains practical instruction in sugar-house control, the diffusion process, selected methods of analysis, reference tables, etc. The essential requirements of a thorough chemical control and superintendence of a sugar-factory are carefully described, and only such analytical processes are given as relate to sugar-house products and the waste residues when necessary to a complete control. Technical chemical terms have as far as possible been avoided. The little book ought to stimulate our sugar-manufacturers and their chemists to more extensive investigations and more thorough work.

— Messrs. Ginn & Co. announce for publication early in December the first volume of a serial entitled "Harvard Studies in Classical Philology," edited by a committee of the classical instructors of Harvard University. It is the expectation that one volume,