PERSPECTIVES: QUANTUM COMPUTING

Computation from Geometry

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"Let no one ignorant of geometry enter here," read the inscription on Plato's Academy, and for more than 2000 years, knowledge of geometry was the sine qua non of the educated person. Only in the past 50 years has geometry been replaced by computation as the branch of mathematics most needed to succeed in intellectual life. Now, however, it appears that geometry is making a comeback. On page 1695 of this issue, Duan *et al.* show that geometry can be exploited to perform any desired quantum computation (*1*).

Quantum computers are devices that process information at the scale of individual atoms and photons (*2–5*). They operate by mapping the quanta of information—the bits—onto quanta of energy and angular momentum. The information is processed by transforming the physical quanta. Quantum mechanics is famously weird: Quantum systems such as photons or electrons are perfectly comfortable occupying several places at once, and quantum bits can therefore "register" 0 and 1 at the same time. Quantum computers exploit this quantum weirdness to perform many computations at the same time. This phenomenon, known as quantum parallelism, can be used to speed up a variety of computations, including factoring large numbers (a problem of interest in cryptography), searching databases, and simulating quantum systems.

Several operating quantum computers have been constructed, and simple quantum algorithms have been performed (*5*). Quantum computation is usually performed by taking quantum systems such as atoms or molecules and addressing them with electromagnetic waves (*2*). Zapping quantum systems with lasers or microwaves is a highly effective way of performing elementary quantum logic operations. In other words, electromagnetic resonance can be thought of as "nature's machine language."

This "conventional" method of performing quantum computation through optical or magnetic resonance does not obviously have anything to do with geometry. How then can geometry be used to implement logic? Is geometry not the study of static, timeless systems? Plato certainly thought so, but a more contemporary view describes geometry in terms of transformation.

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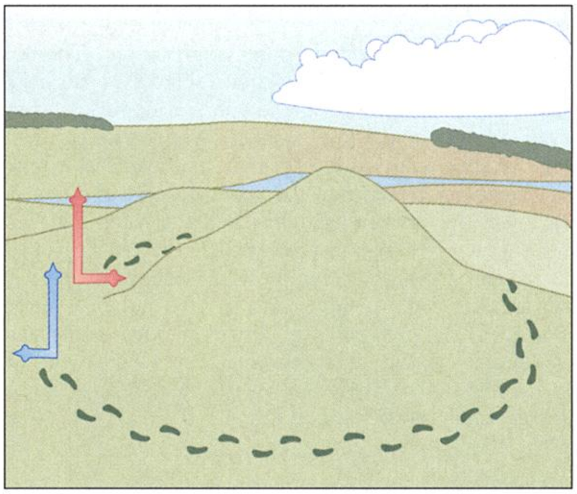
To see how geometry can induce transformation, imagine yourself walking over a gently curving landscape (see the figure). You twist and turn, following the shape of the hills. Eventually you wind up back where you started. But to your surprise, you are now facing the opposite direction. The intrinsic transformation induced by following paths through a geometric landscape is called a holonomy. Holonomies are well known in quantum mechanics: The most famous example is Berry's phase, the intrinsically geometric quantum phase acquired by transforming a quantum system through a loop in parameter space (*6*).



Logic from geometry. Two quantum bits are taken for a "walk" along a potential landscape. The bits could, for example, be stored in ions in an ion trap, and the walk could be induced by shining lasers on the ions to alter their electromagnetic potential. At the end of the walk, the bits have returned to their original position but are transformed according to a quantum logical operation. In this case, one of the bits has been flipped if and only if the other bit is a 1.

Holonomies suggest a way of performing quantum logic. Store a quantum bit on a quantum system such as a nuclear spin, so that 0 and 1 are represented, for example, by two hyperfine levels in an atomic nucleus. Now take that bit on a "quantum walk" over the landscape given by the atom's electromagnetic potential. This quantum walk can be effected by turning lasers on and off at different frequencies related to the resonant frequencies of the atom. When all the lasers have been turned off, the quantum bit has returned to its original place in the energy landscape, but now it is facing in the opposite direction: It has flipped. By taking the quantum bit for a walk, one has performed a NOT operation. Geometry induces logic.

AND, OR, and COPY logic operations can be performed in a similar way by starting with two quantum bits, stored on hyperfine levels in two interacting atoms. Applying lasers to take the two qubits for walks through potential space allows the application of two-bit quantum logic operations.

The idea of using holonomies to perform quantum computation was proposed previously (*7–10*), and individual holonomic quantum logic operations have been performed (*7*). Duan *et al.* show theoretically that by storing quantum bits on ions in an ion trap and applying the proper sequence of laser pulses, arbitrary quantum computations can be built up out of one- and two-qubit holonomic operations. In this proposal, quantum computation can be reduced completely to geometry.

Holonomic quantum computation provides other advantages over conventional quantum computation. It is flexible in the way a computation is implemented (many different paths are possible), and it has an enhanced resistance to noise and errors. Whether exploiting geometry is as experimentally accessible as Duan *et al.* suggest remains to be seen. But even if ion traps do not prove the ideal geometric computational system in practice, there are many other systems—such as nuclear spins, electrons in solid state, and superconducting quantum bits—to which holonomic methods can be tailored. Any system in which quantum bits can be taken on a nontrivial walk through potential space is a good candidate for holonomic quantum logic. Where there is geometry, there is computation.

References

1. L.-M. Duan, J. I. Cirac, P. Zoller, *Science* **292**, 1695 (2001).
2. S. Lloyd, *Science* **261**, 1569 (1993).
3. P. Shor, in *Proceedings of the 35th Annual Symposium on Foundations of Computer Science*, S. Goldwasser, Ed. (IEEE Computer Society, Los Alamitos, CA, 1994), pp. 124–134.
4. D. Divincenzo, *Science* **270**, 255 (1995).
5. M. A. Nielsen, I. L. Chuang, *Quantum Computation and Quantum Information* (Cambridge Univ. Press, Cambridge, 2000).
6. M. V. Berry, *Proc. R. Soc. London A* **392**, 45 (1984).
7. J. A. Jones, V. Vedral, A. Ekert, G. Castagnoli, *Nature* **403**, 869 (1999).
8. P. Zanardi, M. Rasetti, *Phys. Lett. A* **264**, 94 (1999).
9. J. Pachos, P. Zanardi, M. Rasetti, *Phys. Rev. A* **61**, 010305(R) (2000).
10. J. Pachos, S. Chountasis, *Phys. Rev. A* **62**, 052318 (2000).