BOOKS: MATHEMATICS AND ART

Algorithms of Boundless Beauty

Gregory Buck

Arabesques
Decorative Art in Morocco
by Jean-Marc Castéra

ACR [Art, Création, Réalisation], Paris, 1999. 480 pp. 680 FF, EUR 103.67. ISBN 2-86770-124-4.

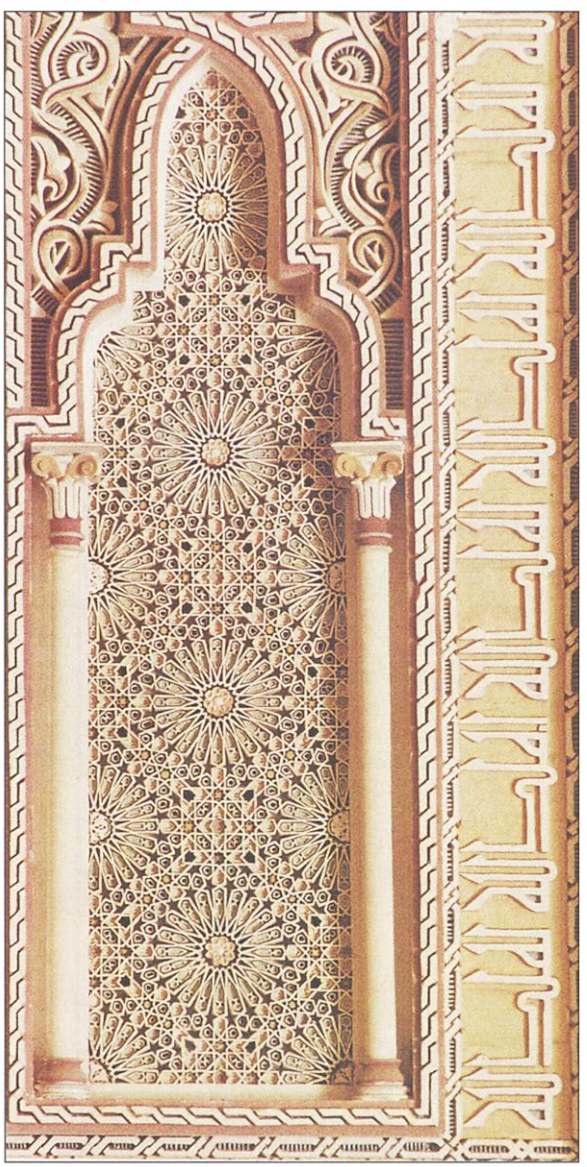
Most readers of *Science* are primarily interested in the truth, as supported by the scientific method. Me too. But all truths are not equal. I recall well when I first grokked Newton's arguments giving the special properties of the inverse square law. I was so moved by the elegance of the constructions, I found myself wiping away tears. Now why should this be? Why should aesthetic appeal have anything to do with the truth? The question is particularly interesting to mathematicians. We make many of our working decisions on aesthetic principles—accepting ugly proofs like an obedient child takes brussel sprouts, but reaching for pretty results like they are a plate of cookies. Answer the question and you'll get a chair in philosophy at Harvard.

The artists and craftsmen of Morocco have never found any conceptual difficulty here. They saw the beauty of geometrical and topological algorithms a thousand years before Escher and Mandlebrot. The French artist and mathematician Jean-Marc Castéra has produced a book worthy of their efforts, a nearly impossible feat. *Arabesques* comprises nearly five hundred full-color pages of perfectly composed photographs by Françoise Peuriot and Philippe Ploquin that present decorative and architectural works of eye-numbing beauty. This extensive sampling of designs from Moroccan mosques, palaces, and cities is accompanied by Castéra's knowledgeable and insightful analysis.

Every bit of the featured work is mathematical. I have only space and time to mention rudiments. These craftsmen understand planar tilings the way Kepler understood trajectories. They are happy to swim in the complex mathematics, as adept with symmetry groups as a salesman is with a cell phone. The designers evidently understand the fundamental facets of mathematical knot theory (which we now apply to DNA). They comprehend the symmetry properties some knots have (and others lack), and they clearly have command of the concept of alternating knots—if you trace any strand you will find that it travels under one crossing strand then over the next, and so on. This understanding is shared with another magnificent tradition of decorative topology, Celtic knotting (see especially the illuminated manuscripts of the Lindesfarne Gospels and the Book of Kells). Castéra does a fine job explaining some of the myriad approaches taken with the geometry. He also provides a pretty solution to one of the basic problems of such work: how does one even get started? The designs are so complex, the symmetries so demanding, that a novice could easily be flummoxed from the start.

There are many wonderful details in this book. One of my favorites is a photograph of a collection of some of the complex tiles the artists use. (Home renovators: don't bother look for these at Tile World.) Castéra shows that several of the designs consist of a central motif of one symmetry group, a border of another symmetry group, and a transition zone which must be appealing but cannot carry either symmetry exactly. Such transitions are a recurring problem in aesthetics. Consider an analogous situation in music, where opposing tonal demands ask us to divide the octave in different ways; the hard part is to make the whole sound good anyway. As the illustrations in Castéra's book demonstrate, the Moroccan artists make their whole designs look great.

One cannot help but be impressed by the magnitude of the artists' efforts. These design elements can take years to complete. Possibly the mathematical ambition is focused by the traditional Islamic prohibition on representations of humans or animals. Think of the song lyrics which might have been written if there were a ban on references to sex and drugs. (Well, maybe that analogy doesn't obtain: I'm not sure that Britney Spears would be singing about, say, non-euclidean geometry under any circumstances.) There is a sense of elation when one first realizes that these patterns are algorithmic. Castéra includes the proof in the book. His computer efforts produce designs that are theoretical equivalents of those created by the craftsmen, and even give them a pretty good run as images on the page. In one light, this is a near miracle. Perhaps we all can fashion this sort of beauty; perhaps such handmade beauty need not cost a lifetime of



Starry window. A chemmassiat at the Great Hassan II Mosque, Casablanca, in which 20-pointed stars are connected by 7-pointed stars. Chemmassiat (from the Arabic for sun) originally meant small arched windows created by carving completely through plaster panels; it is now applied to decorative false windows as well.

CREDIT: FRANÇOISE PEURIOT AND PHILIPPE PLOQUIN/COURTESY ACR EDITIONS, PARIS

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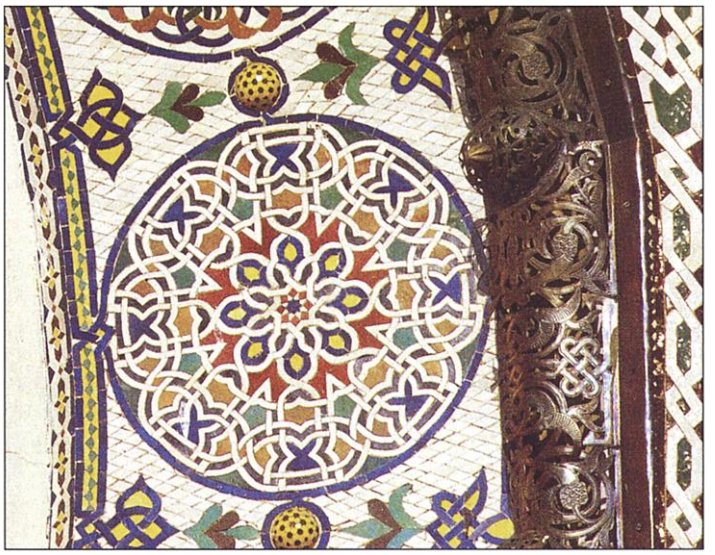
bent backs and dusty lungs. But for me this elation is matched by a tactile melancholy. Why do we have hands if not to use them? Castéra observes that the Moroccan craftsmen seem happy. It is easy to imagine that they would be. How would you feel if your job was creating works of transcendent beauty to grace the public places of your community?

It is trite but true to say that we are the most scientifically and technologically advanced society in the history of Earth. Yet when we look out the window, what do we see? I suppose it depends on your particular window, but my guess is that your view doesn't offer the aesthetic equal of these Moroccan works. If you see human artifice, you likely see linear blocks. I imagine this seems barbaric to someone from Morocco. After all, the mathematics of our art and architecture we teach to ten-year-olds; the mathematics of their decorations, we study in graduate school. It should come as no surprise that the great mathematical designer, M. C. Escher (the official artist of graduate student offices and mathematics departments), made his own trip to Morocco. Perhaps the reason Escher's work seems so surprising lies in our bias—he isn't of the western tradition. But by Moroccan lights his is the logical next step. If you want to understand Escher, call your travel agent or buy this book.



Flowery star. The zellij technique, mosaics of small pieces cut from enameled ceramic tiles, is usually used for geometric designs. The eight-pointed star in this zellij medallion (from an exterior door at the Royal Palace, Fes) incorporates curves with floral ends.

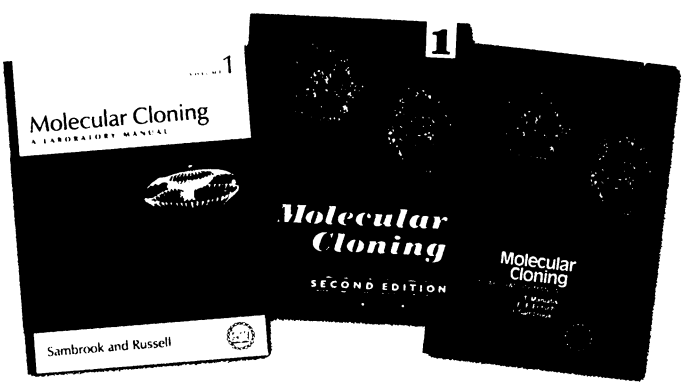
Let's look out the window again. We see our artifice, regular as crystals. We see nature in its glory, the flora and fauna, and the fauna we're most fond of—us. The Moroccan craftsmen cannot represent the human form. But if they tried, we would not have this art; on the large scale, we do not appear algorithmic. However, one of the great technological achievements of the last century is the exponential improvement in our acuity. We can now see the very small. And in the universe of the very small, in the molecules and atoms that are us, we see this same algorithmic magnificence we create for our own pleasure. Thus it seems that the Moroccan artists and craftsmen render us after all. Their work and this book make me wonder: perhaps what exists in our imagination is as wonderful and beautiful as anything nature can create. You pick *Arabesques* up fascinated; you put it down humbled. It is one of the most beautiful books I have ever seen.

NOTA BENE: MOLECULAR BIOLOGY

A Last Hurrah and New Directions

Molecular Cloning
A Laboratory Manual, 3rd ed.
by Joseph Sambrook and David W. Russell

Cold Spring Harbor Laboratory Press, Cold Spring Harbor, NY, 2001. 3 vols., 2288 pp. $295, £231. ISBN 0-87969-576-5. Paper, $195, £149. ISBN 0-87969-577-3.



Nearly twenty years ago, laboratory manuals that touted the words "molecular biology" and "cloning" seemed to hold exclusive keys to the magical world of DNA manipulation. Today, with the completion of the sequencing of the human genome, scientists worldwide are poised to understand the many intricacies of human biology, and never before have they been better equipped for the occasion. The automation of once dreary and repetitive techniques and the ready availability of commercial reagents and kits have changed the focus as well as the nature of research. The third edition of Cold Spring Harbor Laboratory Press's *Molecular Cloning: A Laboratory Manual*, assembled by Joseph Sambrook and David W. Russell, is testimony that accessing appropriate methodologies has evolved with the times. The authors' descriptions of essential techniques for biomolecular research are accompanied by explanations of why they work and how they were developed and have changed with time. The three-volume set, updated and expanded to include new selections on detecting protein-protein interactions and microarray technologies, now has an electronic partner: www.MolecularCloning.com. This fully searchable site, which currently houses abbreviated protocols and bioinformatics resources, is still in its infancy. But Cold Spring Harbor Laboratory promises to expand it to include the rich molecular details of the printed volumes, peer-reviewed updates, and an extensive collection of links to other electronic resources (such as key databases, sequence analysis programs, and reagent information) and sites of particular interest to molecular biologists. Because of the continual evolution of methods and materials, it is likely that this is the last time we will see this classic manual—and perhaps such collections of protocols in general—in printed form. In this postgenomic era of fast electronic access to information, it seems that the movement to the Web now underway was inevitable.

—LISA CHONG

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