BOOKS: MATHEMATICS

Connections Through a Matrix

Herbert S. Wilf

Proofs and Confirmations is one of the most brilliant examples of mathematical exposition that I have encountered, in many years of reading the same.

Proofs and Confirmations
The Story of the Alternating Sign Matrix Conjecture
by David M. Bressoud

Cambridge University Press, New York, and Mathematical Association of America, Washington, DC, 1999. 290 pp. $74.95. ISBN 0-521-66170-6. Paper, $29.95. ISBN 0-521-66646-5.

In the early 1980s, William Mills, Howard Rumsey, and David Robbins put forth the alternating sign matrix (ASM) conjecture. It offered a formula for the number of alternating sign matrices (objects that generalize permutation matrices) of size $n \times n$. These are square matrices of 1s, −1s, and 0s for which (i) the sum of the entries in each row and in each column is 1 and (ii) the nonzero entries of each row and of each column alternate in sign. One such matrix is

$$\begin{pmatrix} 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 1 & -1 & 0 & 0 & 1 \\ 0 & 1 & -1 & 1 & 0 \\ 0 & 0 & 1 & 0 & 0 \end{pmatrix}$$

By playing around with a computer, you could easily convince yourself that there are 429 different 5×5 ASMs; 7436 different 6×6 ASMs; 218,348 7×7 different ASMs; and so forth. You might then wish for a general formula that tells how many $n \times n$ ASMs exist. Mills, Robbins, and Rumsey found a formula that fit all of the data their computer had produced: the number of different $n \times n$ ASMs is exactly

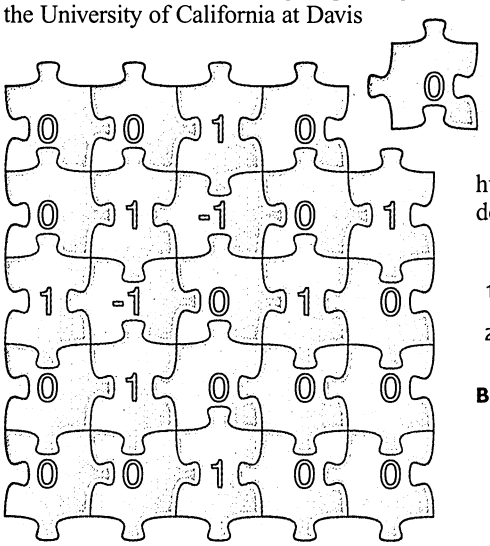
$$\prod_{j=0}^{n-1} \frac{(3j+1)!}{(n+j)!}$$

So, for example, the number of 5×5 ASMs is

$$\frac{1!}{5!}\frac{4!}{6!}\frac{7!}{7!}\frac{10!}{8!}\frac{13!}{9!}$$

which does work out to be 429. But they did not prove that their formula is valid for all n; they only conjectured that it was.

The problem remained tantalizingly unsolved until the mid-1990s, when Doron Zeilberger of Temple University proved the conjecture completely. His 1995 proof (*1*) is about 84 pages long. Scores of people worldwide refereed the paper; each was responsible for one node of an organizational tree of the paper that Zeilberger had prepared. In 1996, Greg Kuperberg of the University of California at Davis found another proof of the ASM conjecture. His proof (*2*) uses results from the theory of "square ice" (a two-dimensional arrangement of water molecules) developed by physicists working in statistical mechanics.



Explaining the solution. The numbers in this picture puzzle form an alternating sign matrix. And there are 429 ways to arrange pieces with the green or blue shapes in a 5 x 5 tray that has notches on the sides and knobs on the top and bottom.

The search for the proofs involved material drawn from a wide range of mathematical topics including the theory of partitions, lattice paths, plane partitions, symmetric functions, the Macdonald conjecture, and an algorithm for evaluating determinants discovered by Charles Dodgson (better known as Lewis Carroll). Bressoud has tackled the unenviable task of explaining this entire cluster of ideas to the widest possible audience. He has succeeded. The book could form the basis for an advanced undergraduate or graduate course. For students who have previously learned the basics of combinatorial mathematics, the course could be more or less self-contained. For those to whom the notion of, say, generating functions, is foreign, the instructor would need to intersperse such background material at several points.

The author provides very careful and stimulating explanations of the many connected topics and of the context in which the problem resides. He accompanies his discussions with a large and rich collection of exercises of varying difficulty. Thus, with study, one can gain the broad picture of the many threads that were pulled together by Zeilberger and by Kuperberg in their respective proofs, together with all of the devilish details.

This is not for the faint-hearted, nor is *Proofs and Confirmations* a book that can be read in an easy chair, like a novel; it demands active participation by the reader. But Bressoud rewards such readers with a panorama of combinatorics today and with renewed awe at the human ability to penetrate the deeply hidden mysteries of pure mathematics.

References and Notes
1. D. Zeilberger, *Electron. J. Combinatorics* **3**(2), R13 (1996). Available at www.combinatorics.org
2. G. Kuperberg, *Int. Math. Res. Notes* **1996**, 139 (1996).

The author is in the Department of Mathematics, University of Pennsylvania, Philadelphia, PA 19104–6395, USA. E-mail: wilf@math.upenn.edu

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BOOKS: ZOOLOGY

Fishing Stories

Axel Meyer

Just before Christmas 1938, Majorie Courtney-Latimer, the curator at a small museum in East London, South Africa, found a strange-looking fish at the local docks. She sent a brief description and rough sketch of it to the South African ichthyologist J. L. B. Smith. The bony head plates, heavy scales, and limb-like fins were sufficient for Smith to identify the new species, which, in honor of its discoverer, he named *Latimera chalumnae*, as a living coelacanth. The fish belonged to a group that was thought to have gone the way of the dinosaurs 70 million years ago. The oldest-known coelacanths appeared in the Devonian (before 375 million years ago), and some paleontologists saw the group as ancestral to the earliest terrestrial vertebrates. Study of this "living fossil" could facilitate our understanding of the vertebrate's

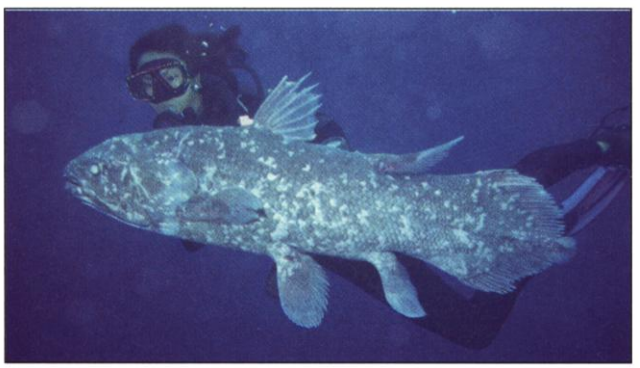
A Fish Caught in Time
The Search for the Coelacanth
by Samantha Weinberg

Fourth Estate, London, 1999. 239 pp. £13.99. ISBN 1-85702-906-2. Harper and Collins, New York, 2000. 240 pp. $24, $C36.50. ISBN 0-06-019495-2.

The author is in the Department of Biology, University of Konstanz, 78457 Konstanz, Germany. E-mail: axel.meyer@uni-konstanz.de

evolutionary transition to life on land. Thus, *Latimeria* was hailed as the zoological "discovery of the century." In *A Fish Caught in Time*, British writer Samantha Weinberg tells riveting stories of the protagonists in the search for the coelacanth and of their obsession.

It took Smith 14 years of intensive searches up and down the eastern coast of Africa to find a second specimen of *Latimeria*. That came from the French territory of Comoro Islands in the western Indian Ocean, where almost all of the 200 or so known specimens have since been found. When Smith arranged for the South African airforce to airlift this specimen from the Comoros in 1952, he caused a diplomatic stir between French and Anglo-Saxon researchers. The resulting export embargo of coelacanths to all countries other than France meant that for more than a decade only French scientists were permitted access to new specimens of the coelacanth. Events of the past couple of years threaten to revive this rift.



Sulawesi coelacanth. The specimen that would be described as the second species of *Latimeria*, photographed swimming along the reef edge after its capture.

The siren-like power of the coelacanth to cause sudden career shifts and to bring about intrigue and drama is still unbroken. It is attested to by the developments that followed the chance discovery in 1997 of a new species of coelacanth in an Indonesian fish market in Sulawesi, ten thousand kilometers from the Comoro Islands. This coelacanth was recognized, but unfortunately not kept, by Mark Erdmann, at the time a graduate student studying mantis shrimp at Berkeley who was on his honeymoon. After ten months of interviewing fishermen and studying fishing sites, Erdmann found a second specimen of the new coelacanth in July 1998. With the September 1998 report on its discovery (*1*), Erdmann's advisor Roy Caldwell instantly became a television expert on coelacanths—a new direction for someone who, like his student, had heretofore focused on invertebrate behavior and ecology.

The historical clash between French and English-speaking coelacanth researchers resurfaced when geneticist Laurent Pouyaud (of the French Institute for Development Research in Jakarta) and his collaborators at the Indonesian Institute of Science described the specimen collected by Erdmann as a new species, *Latimeria menadoensis* (*2*). They based their conclusion on the mitochondrial DNA sequences they reported (in which the Indonesian specimen differs by about 4% from Comoro samples) and on some morphological characteristics that make the Indonesian coelacanth look slightly different from its Comoro cousins. Pouyaud's analysis was published before one from Erdmann and his collaborators, who felt that they were unfairly scooped (*3*). The Berkeley and Texas group's more rigorous analysis of a larger data set (*4*) confirmed many of the findings of the French-Indonesian team.

Many of the current coelacanth researchers have been drawn into the fray by reading J. L. B. Smith's personal account of the discovery of the first two specimens (*5*). Weinberg serves an easy-to-digest, almost science-free, and often romantic account. She emphasizes the personalities of researchers whose careers have been made by their intense desires to study every aspect of *Latimeria*'s biology. The greatest value of her book lies in keeping alive the mystique of this unusual fish for yet another generation of biologists. Readers might dream of working outside of the laboratory on a fish that is, as yet, hardly known. The coelacanth still harbors many mysteries about the biological adaptations that permitted some ancestral fish to colonize land—and ultimately give rise to tetrapods, including ourselves.

References

1. M. V. Erdmann, R. L. Caldwell, M. K. Moosa, *Nature* **395**, 335 (1998).
2. L. Pouyaud *et al.*, *C. R. Acad. Sci. III Life Sci.* **322**, 261 (1999).
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5. J. L. B. Smith, *Old Fourlegs* (Longmans, London, 1956).

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BROWSINGS

The Governance of Science. Ideology and the Future of the Open Society. *Steve Fuller.* Open University Press, Buckingham, UK, 2000. 179 pp. $85, £50. ISBN 0-335-20235-7. Paper, $24.95, £15.99. ISBN 0-335-20234-9. Issues in Society.

Even as science has become tangled in political and economic issues, it remains an activity more widely trusted than understood. Examining science policy from the perspective of democratic social and political theory, Fuller argues for a "republican" approach in which scientists have a "right to be wrong." Among the topics he discusses are science's interdependence with the university and the legacy of the science policy from the U.S. New Deal.

Life and Death on Mt. Everest. Sherpas and Himalayan Mountaineering. *Sherry B. Ortner.* Princeton University Press, Princeton, NJ, 1999. 390 pp. $26.95, £16.95. ISBN 0-691-00689-X.

Few climbers have reached the tops of the Himalayas without the labor of the Sherpas, a small, mostly Buddhist ethnic group that lives in the high valleys of Nepal. Weaving together the results from her own fieldwork and the extensive literature on the region, Ortner discusses the complex and still-changing relations between the Sherpas and the "sahibs" who employ them. Her ethnography provides a fascinating account of how mountaineering has affected the lives and culture of the Sherpas.

Minds Behind the Brain. A History of the Pioneers and Their Discoveries. *Stanley Finger.* Oxford University Press, New York, 2000. 378 pp. $35, £24.99. ISBN 0-19-508571-X.

Stanley Finger sketches the development of our understanding of the brain through a series of profiles of the lives and findings of major figures. Abundantly illustrated, his account ranges from the study of "the marrow of the skull" in ancient Egypt to Roger Sperry and his Nobel Prize–winning research on split brains.

The Origins of Music. *Nils L. Wallin, Björn Merker, Steven Brown,* Eds. MIT Press, Cambridge, MA, 2000. 512 pp. $60, £37.50. ISBN 0-262-23206-5.

Why is music present in all human cultures? What universal features of music underlie the great diversity of musical systems? The contributors, who include musicologists, anthropologists, and neuroscientists, address such questions from an evolutionary perspective in their considerations of vocal communication in animals, the relations between music and language, and possible roles of selective forces and cognitive mechanisms.

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