

Jersey. "Hence it seems unlikely that any real stars follow such an orbit. On the other hand, the universe is a big place, so who knows?"

—DANA MACKENZIE

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## Random Packing Puts Mathematics in a Box

Anyone who's been on a crowded subway has unwillingly experienced random close packing. Mathematicians and physicists, on the other hand, relish the subject. For decades, they have been arguing about a simple version of the crammed subway car: How closely can you pack randomly arranged spheres into a box? Now a team of engineers appears to have settled the debate with a surprising answer: There is no single answer.

Visit any supermarket, and you'll see that the grocer already knows how to pack oranges or grapefruit—

or any other uniformly sized spherical object—in the most efficient way possible. The little pyramids of oranges are packed in the so-called face-centered cubic configuration, in which only about 26% of the pile is empty space. In 1611, Johannes Kepler wrote a booklet called *The Six-Cornered Snowflake*, in which he guessed that this was

the tightest packed configuration possible. Two years ago, Michigan mathematician Thomas Hales proved Kepler's conjecture: It's impossible to pack spheres so that the "packing fraction" is more than about 74% (*Science*, 28 August 1998, p. 1267).

Kepler could rest easy, but mathematicians and physicists kept arguing about a related problem: How tightly can you pack spheres if you dump them randomly into a box? Beginning in the 1960s, experimenters put ball bearings and other spheres in rubber balloons, shook them into boxes, and simulated them on computers. Their conclusion: The maximum packing fraction was about 64%. This "maximally packed" state was dubbed random close packing. Yet scientists couldn't agree on exactly what that state was. "If you look in the literature, people ended up getting different values," says Salvatore Torquato, a materials scientist at Princeton University. Most recently, in 1997, researchers at the École Polytechnique in France showed that they could get packings as high as 67% by shaking their apparatus in different ways. However, de-

spite these differences, most people in the field still assumed that there was a universal constant, a maximum random close packing fraction.

Using computer simulations of spheres being compressed in a box at different speeds, Torquato and his colleagues show that there is no such constant. "What we found was that you can go way beyond what we thought was the maximum," says team member Pablo Debenedetti, a chemical engineer at Princeton. In the experiment, described in *Physical Review Letters*, the team got higher and higher packing fractions by compressing the spheres ever more gently, finally approaching the ultimate limit set by Kepler.

"What we conclude is that you can always pack things more and more densely, but you get more and more order," says Debenedetti. That is, "random" and "close packed" are not independent concepts; looking for the maximally close-packed random collection makes no more sense than searching for the tallest short guy in the world. "The fact that there's a maximal value turns out to be ridiculous," says Torquato. "It's not

mathematically well defined."

"The assumption had been that there was a unique random closest packing number, but I think Torquato and his collaborators have unequivocally demonstrated that this is not the case," says Frank Stillinger, an engineer at Lucent Technologies in New Jersey. Even though the lab experiments and simulations got values of roughly 64%, it was due to the laboratory conditions rather than to any universal rule—which explained why the experimenters never could quite agree on the true value.

Torquato and colleagues suggest a more precise way of approaching the problem. Instead of looking at "close packing," they investigate "jammed" states, where no spheres are free to rattle around if you shake the box they are in. Not only might there be a jammed state that is maximally random—the analogous, but more precise, concept to a random closest packed state—but there might also be some jammed structures that have a very low packing fraction. "They would be jammed but have an enormous amount of open space," says Stillinger. Straphangers, take heart. —CHARLES SEIFE

## GEOLOGIC MODELING

# Seeing a World in Grains of Sand

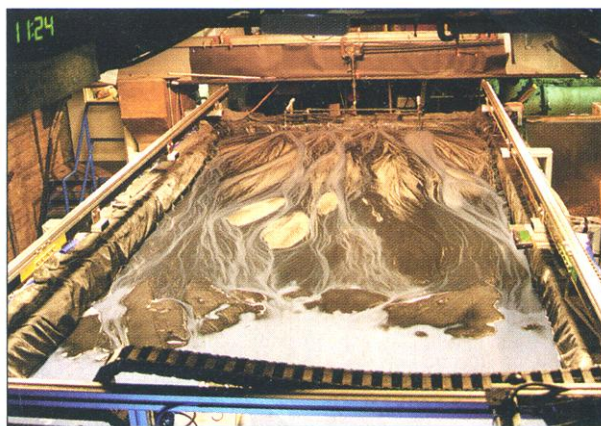
Sophisticated physical models of how sediment flows through rivers into the sea are offering high-tech views into the genesis of complex stratigraphy

Housed in a cavernous laboratory on the banks of the Mississippi is one of the most expensive sandboxes in the world. The rectangular tank is half the size of a tennis court, can hold some 200 tons of sand, and cost about half a million dollars to build. Dubbed "Jurassic Tank," the apparatus is on the leading edge of a new generation of physical models that can simulate the rise and fall of sea level, the effects of swings in climate, and the sinking of tectonic plates.

In initial runs with this new device, which has just been completed, sedimentary geologist Chris Paola, civil engineer Gary Parker, and their team at St. Anthony Falls Laboratory in Minneapolis are creating scaled-down versions of complex geology to figure out how intricate patterns of sediment layers are deposited by rivers and ocean currents. "You could think of the stratigraphic record as an old

violin: You can examine it, take it apart, analyze it, model it, but even with all that, you still aren't entirely sure how it was made," Paola says. "Now, just imagine you could watch Stradivari at work."

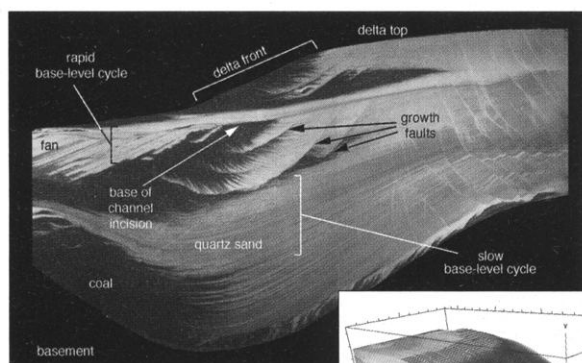
Although by no means a perfect simulation of the real world, the sandboxes are the first attempt to reproduce entire sedimentary basins in a quantitative way. The effort



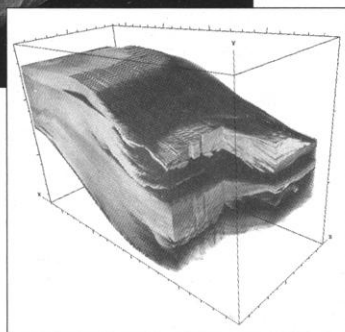
**Small world.** After a few days, miniature rivers build up realistic-looking layers of sediment up to 1.3 meters thick.

CREDITS: (LEFT TO RIGHT) CORBIS; ST. ANTHONY FALLS LABORATORY





**Look inside.** Digitized cross sections from the tank can be combined into 3D views of the strata.



is decidedly high-tech. The Minnesota scientists map the evolving landscape with a laser and, as they painstakingly dissect the fresh terrain with a meter-long blade akin to a giant cheese slicer, capture digital images that can be re-assembled into a three-dimensional (3D) computer mock-up. Researchers can then fly through the virtual underground strata and examine any type of sediment layer, such as potentially oil-rich sands.

What makes Jurassic Tank truly unique, though, is a computer-controlled floor that sinks or deforms with a precision of 100 micrometers. By warping this flexible surface, the team can reproduce the effects of rising plumes of subsurface salt, a gently subsiding continental margin, or almost any other sort of tectonic activity. This ability will help researchers figure out how sea level and basin tectonics conspire to build different kinds of stratigraphy. "That's a major question," says Rudy Slingerland, a sedimentary geologist at Pennsylvania State University, University Park. "This is a perfect methodology to answer it."

### That sinking feeling

Jurassic Tank has a long pedigree. Generations of geologists have studied faults by squeezing boxes of sand and clay, recreated colliding plates with wax-filled trays, modeled subduction zones by pouring together fluids of various densities, and built miniature river systems to shed light on the secrets of meanders and flooding.

Yet stratigraphic modeling lacked a key factor: subsidence. For sediment to accumulate without being eroded by waves or wind, either the basin bottom must sink or the water level rise. High sea levels tend to fall again, though, and then erosion wipes away marine deposits. The most reliable way to keep sediment intact for millions of years is for crustal plates to sink. "Subsidence is the absolute sine qua non for creating and preserving stratigraphy," Slingerland says.

Geologists can glimpse how subsidence operated in the field, and they can mimic it with computers. Most computer models, however, offer only cross-sectional views into stratigraphy. Physical models provide a 3D perspective of an entire basin. And compared with algorithms, they are comfortably solid, as wet and dirty as the real world they seek to simulate.

A sandbox renaissance began in the early 1990s. For example, when Shell Research & Technology Services of Rijswijk, the Netherlands, wanted to verify and calibrate a 2D mathematical model of sediment transport, the company approached George Postma, a sedimentary geologist at Utrecht University, and asked him to build a tank that would track sand migration as sea level fluctuated. The 8-meter-by-4-meter basin was kept top secret for 2 years. A team at Colorado State University, Fort Collins, was already doing experiments on the effects of changing sea level on valley formation, while researchers at the University of Leeds, United Kingdom, also ran experiments simulating the deposits of rivers.

In 1994, Paola was developing computer models of stratigraphy and experimenting with river deposits. Working at St. Anthony Falls Laboratory, an arm of the University of Minnesota that specializes in physical experiments about hydraulics, he and Parker had an idea: build a large basin whose bottom could drop, mimicking subsidence and creating stratigraphy. "We sat around for 3 to 4 months and brainstormed about how to do it," Paola recalls.

Then a team member from the Midwest, Jim Mullin, was inspired by agricultural silos. Taking a cue from the way a pile of grain collapses in a hopper as seeds fall out from below, he developed a honeycomb of gravel-filled funnels. A computer controls the flow out of each funnel, and the dimples from adjacent funnels merge to make any desired landform. A year later, they had a prototype with 10 subsidence funnels that could evolve a natural-looking basin floor.

### Where's the beach?

In two preliminary runs, the tank has shown that it can realistically reproduce a number of geologic processes. Within its geological

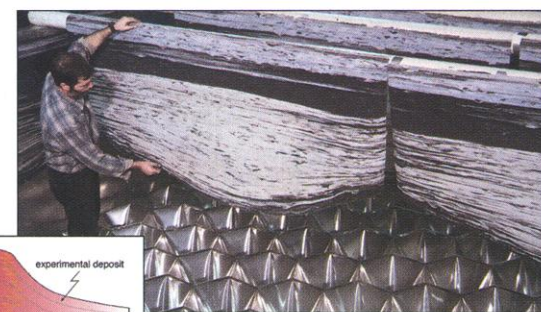
microcosm, networks of so-called braided streams whisk sand to deltas that mirror those found in the Gulf of Mexico and elsewhere. Shorelines and river systems shift and swerve in ways that geologists in the field would recognize—if they could watch landscapes evolving over millions of years.

The first experiment, in fact, gave field geologists some welcome reassurance about the interplay between shoreline location and sea level. Shorelines are a key feature of paleogeography, and they yield clues about a smorgasbord of geologic questions, including the volume of oceanic spreading centers, the ebb and flow of ice sheets, and the size and quality of potential petroleum reservoirs.

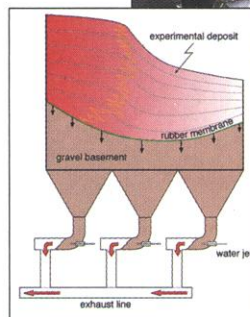
Ancient shorelines can be readily though crudely tracked in outcrops. A cliff where deposits of beach sand are topped by deep-sea clays tells a geologist that water got deeper, either because sea level rose or because the land sank. Whichever it was, the shoreline must have moved inland. If ancient shorelines in basins around the world show the same sorts of changes at the same time, geologists can conclude that they have found a global signal—a literal sea change.

Or can they? In 1978, Walter Pitman of the Lamont-Doherty Earth Observatory in Palisades, New York, proposed that the commonsense scenario might be wrong: Over the eons the rise and fall of the oceans might redistribute sediment on the coast so that shorelines wind up misbehaving, even moving seaward when oceans rise. Long-held interpretations of sea level might be all wet.

Field geologists were stymied. Pitman's theory was a thorn in their side, but they could



**Chute 'em down.** Gravel falls through an array of funnels, lowering the bottom of the modeled basin.



not remove it. They could not date deposits precisely enough to tell whether changes in the shoreline were subtly out of synch with sea level. Nor was anyone sure exactly how long, for typical continental margins, the oddball behavior would take to appear—whether tens of thousands or millions of years. In some cases, the knowledge of ancient sea levels was shaky, too.



The St. Anthony Falls team set out to test Pitman's idea. For about a year, they honed the subsidence mechanism, wrote software to deform the floor precisely, and built a prototype basin. Then it took just a few days to carefully load the funnels with gravel, smooth out the floor, and fill hoppers with sand. After the 54-hour run was finished and the tank drained, researchers spent about 2 months slicing the 1.3-meter-long stack of sand layers like salami to produce a "Visible Basin" akin to the online "Visible Human Project."

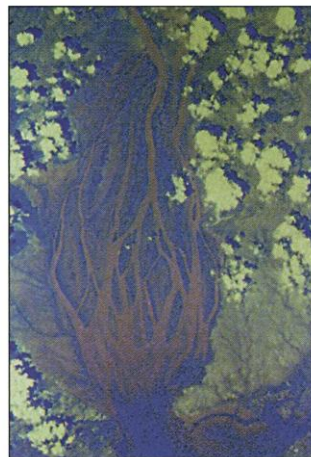
The results, which will appear next month in *Sedimentology*, should reassure geologists trying to figure out how global sea levels changed in the past, Paola says. Over simulated eons, as the modeled ocean rose and fell, the shoreline stayed in synch. But some Pitman-like oddities did crop up. When rapidly rising sea level began to slow down, for example, the shoreline stopped moving inland and began to migrate toward the sea. According to observations of the tank as well as a mathematical model developed by team members John Swenson and Vaughan Voller, the decelerating sea level upset the subtle balance among subsidence, sea-level rise, and sediment supply that maintains the shoreline. Once the rate of sea-level rise had slowed down enough, sand pouring out of the rivers was able to force the beach outward. For the most part, however, the shoreline behaved just as common sense said it should.

The postmortem showed real-world features underground as well. Once the stratigraphy was sectioned, the team noticed a series of dramatic faults. "It was completely unexpected," Paola says. "For most of the experiment there was no indication whatsoever on the sediment surface of the presence of the faults." These so-called growth faults represent slow, continuous subsurface deformation as sediment piles up. Not only are growth faults a hazard for cities like Houston, which is built on a thick accumulation of sediments, but they also strongly influence the distribution of sand reservoirs that host hydrocarbons. Oil companies would like to know how the faults might shift sediment layers, perhaps isolating sand bodies.

During the shakedown run, the team used the 10-funnel baby Jurassic Tank. Last April, the team finished constructing a full-size basin, which is now 6.5 meters by 13 meters and has 432 subsidence funnels. In the first test of the completed tank, the researchers turned from shorelines to rivers. They created a 4-meter-long river system in a valley that was sinking faster downstream, like a hinged trap door. The shifts in terrain

were subtle. Even at the fast end, the basin was dropping only about a centimeter per hour, too slow for anyone to notice. Instead, most observers were mesmerized by the constant shifting of the river channels. The spell was broken every minute or so as gravel was shot out of the funnels to lower the river valley. "It sounds like money falling out of a slot machine," Paola says. The 218-hour experiment featured a light show too: A laser beam scanned the surface every 40 minutes to map the topography.

Team members are still slicing the deposits. They hope to find new ways in which stream-borne sands that harden into rock can help them figure out how a basin has subsided over time. "If there was a way you could walk up to an outcrop and say it was a period of high or low subsidence, that would be awfully convenient," Paola says. So far, no clear-cut stratigraphic giveaways have turned up.



**Twisted sisters.** Modeled channels (left), 2.8 meters across, resemble the braided streams of the Betsiboka delta, Madagascar, seen from space.

However, the model has supported one well-established pattern—but with a twist.

River channels should be drawn to the area of highest subsidence. But according to initial results to be presented next month at the annual meeting of the American Association of Petroleum Geologists by team members Ben Sheets and Tom Hickson, channel sands seem most concentrated somewhere else—a region between the center of the basin and the axis of greatest subsidence. Apparently the rivers are torn between attraction to the fastest subsidence and the urge to shoot straight down the center of the basin, the shortest route to sea. More findings should appear soon.

#### Get real

Compelling as it may be to watch sand grains bouncing down a Lilliputian river, Paola and his colleagues acknowledge that Jurassic Tank will never be anything but a simplification of the real world. The tank contains no clay (it would just float through without settling), and so the sediment can't stick together. Bays and

groundwater can't precipitate carbonate sediments. No plants can sprout and stabilize the model riverbanks. As a result, no meandering rivers can form, only braided streams. There is no single scale for translating model time into geologic time. This means experimental models are a kind of lens, Paola says. "They give us a glimpse—distorted and imperfect, certainly, but nonetheless formed of real grains and real water—of processes that in nature occur on time scales as far removed from human experience as the depths of space."

But the tank is getting more realistic. The St. Anthony Falls team will add waves and currents this summer, to better simulate the continental shelf. The group is also trying to figure out how to make a kind of undersea avalanche called a turbidity current form spontaneously in the tank. And Utrecht's Postma has just received funding to build a new 6.5-meter-by-14-meter tank that will be able to simulate subsidence patterns.

Whatever features may be added in the future, for geologists the biggest attraction will always be what they cannot get in the field: comprehensive knowledge and absolute control. "One of the beauties of this experiment is that you have a chance to see the entire stratigraphy. With rocks, it's more like seeing the trailer of a movie—you get only glimpses," Paola says.

That kind of certainty could be a big relief for oil company geologists, who have to predict the caliber of potential reservoirs, often based on fuzzy seismic images and a few core samples. As drilling wells becomes increasingly expensive the farther companies prospect offshore, questions about stratigraphy turn into high-stakes gambles. A physical model would provide welcome confirmation of a proposed geologic scenario. "When you're talking about a billion dollars ... [Jurassic Tank] is a cheap test of your ideas," Slingerland says.

Companies are already using initial findings to teach young geologists about petroleum reservoirs. Computer images from the experiments reveal the geometry of sand deposits, while sediment cross sections offer vistas unmatched in the field. In the university classroom, instructors ask students to interpret the geologic forces that created them—and then reveal the right answer. The exercises shed some interesting light on how geologists think, Paola says: Even a professional, interpreting the evidence, will almost always make the geologic history of a model out to be much more complex than it really was. "If there's one broad theme, it's a better appreciation of how a simple cause can create complicated stratigraphy." —ERIK STOKSTAD