view, but there are too many assumptions," says Anton Zeilinger, a physicist at the University of Innsbruck in Austria. For example, no one is sure that a moving piece of paper is, in fact, as good as a moving detector.

The experimenters also assumed, as most physicists do, that the photon chooses its quantum state at the moment it strikes a detector. In some formulations of quantum mechanics, however, the photon makes its choice at other points in the experiment—even as late as the time when a conscious being finally looks at the data on the computer. Zeilinger hopes to narrow the possibilities, perhaps by inserting rapid, randomly activated switches into the experimental setup.

However they interpret the results, scientists agree that the Geneva experiments are a technological feat. "This is, in a certain sense, a new line in experimental work," says Suarez. "You are putting quantum mechanics in a relativistic frame."

—CHARLES SEIFE

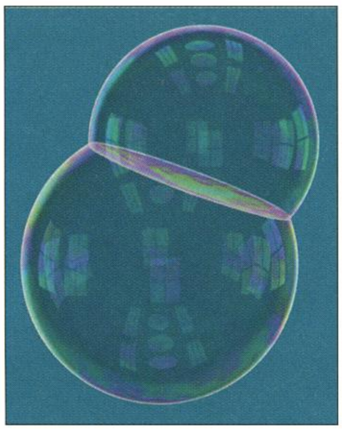
MATHEMATICS

Rounding Out Solutions to Three Conjectures

Three long-standing puzzles involving spherical bodies—the configuration of double bubbles, stable orbits of three stars, and random packing of spheres in a box—have all been solved

Why Double Bubbles Form the Way They Do

Need to entertain a child? Try blowing soap bubbles. Need to keep a mathematician busy? Just ask why bubbles take the shapes they do. Individual soap bubbles, of course, are spherical, and for a very simple reason: Among all surfaces that enclose a given volume, the sphere has the least area (and in the grand scheme of things, nature inclines toward such minima). On the other hand, when two soap bubbles come together, they form a "double bubble," a simple complex of three partial spheres: two on the outside, with the third serving as a wall between the two compartments. Scientists have long considered it obvious that double bubbles behave this way for the same minimum-seeking reason—because no other shape encloses two given volumes with less total surface area. But mathematicians have countered with their usual vexing question: Where's the proof?



Soap solution. Mathematicians prove that nature's way of forming double bubbles is best.

Now they have it. An international team of four mathematicians has announced a proof of the double bubble conjecture. By honing a new technique for analyzing the stability of competing shapes, Michael Hutchings of Stanford University, Frank Morgan of Williams College in Williamstown, Massachusetts, and Manuel Ritoré and Antonio Ros at the University of Granada have shown that only the standard shape is truly minimal—any other, supposedly area-minimizing shape can be ever so slightly twisted into a shape with even less area, a contradiction which rules out these other candidates.

What other shape could two bubbles possibly take? One candidate—or class of candidates—has one bubble wrapped around the other like an inner tube. But it could be even worse: Mathematically, there's no objection to splitting a volume into two separate pieces, so it's possible that siphoning off a bit of the central volume and reinstalling it as a "belt" around the inner tube would actually reduce the total surface area. And conceivably, then, siphoning a bit of the inner tube and placing it as a band around the belt would lead to smaller area yet, and so forth. There's not even any obvious reason that the true, area-minimizing double bubble can't have "empty chambers"—enclosed regions that don't belong to either volume.

Just about the only thing that's (relatively) easy to prove is that the solution must have an axis of symmetry—in other words, it can't have lopsided bulges. Hutchings took the first big step toward ruling out the more bizarre possibilities in the early 1990s. He ruled out empty chambers and showed that the larger volume must be a single piece. Besides the standard double bubble, his results limited the possible solutions to ones consisting of a large inner tube around a small central region, perhaps with a set of one or more belts circling the outside. Hutchings also found formulas that provide bounds on the number of belts, as a function of the ratio of the two volumes. In particular, if the two volumes are equal, or even nearly equal, there can be no belts, so the only alternative is a single inner tube around a central region.

Based on Hutchings's work, in 1995 Joel Hass of the University of California (UC), Davis, and Roger Schlafly, now at UC Santa Cruz, proved the double bubble conjecture for the equal-volume case. Their proof used computer calculations to show that any inner tube arrangement can be replaced by another with smaller area. "Ours was a comparison method," Hass explains. He and Schlafly found they could extend their results for volume ratios up to around 7:1, but beyond that the possible configurations to be ruled out became too complicated.

Surprisingly, the general proof requires no computers, just pencil and paper. The key idea consists of finding an "axis of instability" for each inner tube arrangement. Twisting the two volumes around this axis—with a motion rather like wringing out a washcloth—leads to a decrease in surface area, contradicting the shape's ostensible minimality. "We always thought that these remaining cases were unstable," Morgan says. The proof confirms their suspicions, although it leaves open the possibility that some nonminimizing configuration could also be stable. The twisting argument is new and a bit subtle, Morgan notes. The hardest part is figuring out where to position the axis of instability so that the twisting procedure wouldn't change the volumes of the two regions as well as the surface area. "For a while, it was hard to frame the right questions, especially in Spanish."

Although the proof is only now being announced, the main results were established last spring, when Morgan visited Granada during a sabbatical. Since then, a group of undergraduates in a summer research program at Williams College has extended the results to analogs of the double bubble conjecture in higher dimensions. (The two-dimensional double bubble conjecture was proved by an earlier group of undergraduates in 1990.) Ben Reichardt of Stanford, Yuan Lai of the Massachusetts Institute of Technology, and Cory Heilmann and Anita Spielman of Williams College have shown that an axis

CREDIT: J.M. SULLIVAN/UNIVERSITY OF ILLINOIS of instability always exists for nonstandard shapes in the four-dimensional case, and also in higher dimensions under the mild assumption that the larger volume consists of a single, connected region.

What about triple bubbles? Once again, nature provides a relatively simple and obvious answer, but, Hass notes, "we don't know how to get started" proving it. The triple bubble problem is even open in two dimensions, with equal-sized area (for example, what's the least amount of fencing required to create three acre-sized pens, to separate, say, sheep from goats from hippopotami?) And it gets less certain from there, Hass says. "Once you get up to 20 or 30 regions, we don't even have a conjecture." **–BARRY CIPRA**

Triple Star Systems May Do Crazy Eights

The ancient Greeks spoke of the "music of the spheres," a mystical harmony supposedly produced by the stars circling Earth. This theory did not survive the Copernican revolution, but mathematicians have now produced a modern counterpart: the dance of the stars. They have proved that three stars can chase each other forever in a figure-eight pattern, with each one passing between the other two in turn—and that this orbit is stable. Somewhere in the universe, as yet unnoticed by Earth-based astronomers, a trio of stars could be dancing a Scottish reel.

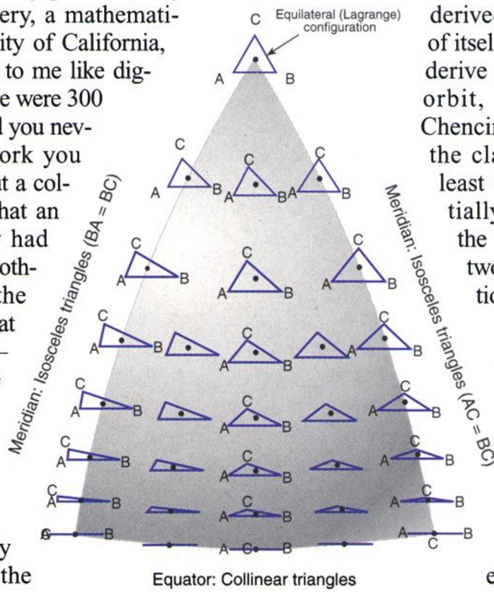
The orbits of multiple stars have long puzzled mathematicians and astronomers. Isaac Newton's theory of gravitation explained well enough why binary stars orbit each other in ellipses. But for 300 years, the only kinds of stable, repeating orbits known for groups of three or more stars have been minor variations on the Newtonian theme. For example, in Alpha Centauri—the nearest star cluster to us—a small third star makes large elliptical loops around a stalking pair of sun-sized giants.

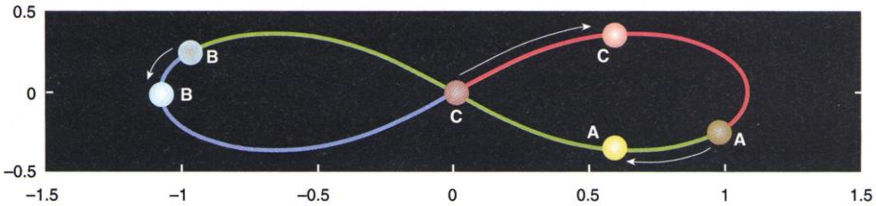
The problem of computing the motion of three objects, interacting solely according to Newton's inverse-square law of gravitation, is known as the "three-body problem." Scientists find approximate solutions every day with computers, but finding and proving exact solutions is notoriously difficult. For this reason the class of proven periodic solutions, in which the objects return exactly to their initial starting places after a certain amount of time, has remained embarrassingly small. One such example occurs when three bodies form an equilateral triangle, an arrangement known as the Lagrange configuration. Such orbits have been used for satellites and are seen in the moons of Saturn. Another class of periodic orbits, in which one of the three objects is extremely small compared to the other two, was discovered about a century ago by the French mathematician Henri Poincaré. These take the form of slightly perturbed Newtonian ellipses.

In the decades since then, no fundamentally new periodic orbits have been found. In fact, mathematicians had moved in the opposite direction, discovering a wide variety of inherently unpredictable, chaotic orbits (especially as more bodies are added to the problem). Searching for periodic orbits began to look old-fashioned.

"As a graduate student, I never wanted to work on the three-body problem," says Richard Montgomery, a mathematician at the University of California, Santa Cruz. "It felt to me like digging in graves. There were 300 years of history, and you never knew whose work you were repeating." But a colleague suggested that an idea Montgomery had used to work on another old chestnut—the problem of how a cat lands on its feet—could apply to the three-body problem as well.

The idea was not to study the three motions separately, but to study how the shape of the triangle formed by all three stars evolves. Each shape can be represented by a single point on the surface of a sphere, called a "configuration space" (see figure). The sphere's north and south poles correspond to the two possible equilateral triangles, points at the equator correspond to arrangements in which all three stars lie on a single line, isosceles triangles run along six of the meridians, and all the intermediate triangles are ranged in between. As the stars move in an orbit, the triangle they form may change, and so the corresponding point in configuration space may move around on the sphere as well. The Lagrange configuration, because it is always an equilateral triangle, does not change its shape at all and thus is represented by a single point in configuration space (the north or south pole). A system of an eclipsing binary and a distant companion, like the Alpha Centauri system, makes only a small loop in configuration space—it passes through the equator twice, once for each time the three stars line up to form an eclipse. But Montgomery wondered whether any three-body system could ever trace a more complicated figure.





Three-body solution. Assigning points on a sphere to triangles formed by three-star systems (*top*) allowed mathematicians to find a stable figure-eight orbit.

After several false starts, Montgomery, together with Alain Chenciner of the Bureau des Longitudes in Paris, found a way to do it. Assuming that the three bodies all have equal mass, the orbit is symmetrical enough that the entire orbit can be derived from a small piece of itself—the first twelfth. To derive the first piece of the orbit, Montgomery and Chenciner used a version of the classical "principle of least action," which essentially reduced to finding the shortest path between two points in configuration space.

Although Montgomery's path in configuration space was relatively complicated, Chenciner soon realized that the individual stars in real space traced out simple figure eights. The two announced their find at a December conference in honor of the 60th birthday of Donald Saari, a leading expert on celestial mechanics. But shortly before the conference, they received a "birthday present" of their own, when Carles Simó of the University of Barcelona drew the first accurate computer rendition of the orbit and showed that it remained stable even if the bodies changed their mass or their initial position slightly. This was quite unexpected, because the original argument required equal masses. Moreover, it means that the solution could conceivably be observed in the universe. But the window of stability is very small. "It becomes unstable if the mass of any one of the bodies differs by more than one part in 100,000," says Joseph Gerver, a mathematician at Rutgers University in Camden, New

CREDIT: (BOTTOM) R. MONTGOMERY

Jersey. "Hence it seems unlikely that any real stars follow such an orbit. On the other hand, the universe is a big place, so who knows?"

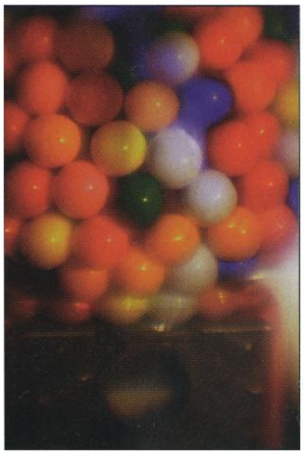
–DANA MACKENZIE

Dana Mackenzie is a writer in Santa Cruz, California.

Random Packing Puts Mathematics in a Box

Anyone who's been on a crowded subway has unwillingly experienced random close packing. Mathematicians and physicists, on the other hand, relish the subject. For decades, they have been arguing about a simple version of the crammed subway car: How closely can you pack randomly arranged spheres into a box? Now a team of engineers appears to have settled the debate with a surprising answer: There is no single answer.

Visit any supermarket, and you'll see that the grocer already knows how to pack oranges or grapefruit—or any other uniformly sized spherical object—in the most efficient way possible. The little pyramids of oranges are packed in the so-called face-centered cubic configuration, in which only about 26% of the pile is empty space. In 1611, Johannes Kepler wrote a booklet called *The Six-Cornered Snowflake*, in which he guessed that this was the tightest packed configuration possible. Two years ago, Michigan mathematician Thomas Hales proved Kepler's conjecture: It's impossible to pack spheres so that the "packing fraction" is more than about 74% (*Science*, 28 August 1998, p. 1267).



Kepler could rest easy, but mathematicians and physicists kept arguing about a related problem: How tightly can you pack spheres if you dump them randomly into a box? Beginning in the 1960s, experimenters put ball bearings and other spheres in rubber balloons, shook them into boxes, and simulated them on computers. Their conclusion: The maximum packing fraction was about 64%. This "maximally packed" state was dubbed random close packing. Yet scientists couldn't agree on exactly what that state was. "If you look in the literature, people ended up getting different values," says Salvatore Torquato, a materials scientist at Princeton University. Most recently, in 1997, researchers at the École Polytechnique in France showed that they could get packings as high as 67% by shaking their apparatus in different ways. However, despite these differences, most people in the field still assumed that there was a universal constant, a maximum random close packing fraction.

Using computer simulations of spheres being compressed in a box at different speeds, Torquato and his colleagues show that there is no such constant. "What we found was that you can go way beyond what we thought was the maximum," says team member Pablo Debenedetti, a chemical engineer at Princeton. In the experiment, described in *Physical Review Letters,* the team got higher and higher packing fractions by compressing the spheres ever more gently, finally approaching the ultimate limit set by Kepler.

"What we conclude is that you can always pack things more and more densely, but you get more and more order," says Debenedetti. That is, "random" and "close packed" are not independent concepts; looking for the maximally close-packed random collection makes no more sense than searching for the tallest short guy in the world. "The fact that there's a maximal value turns out to be ridiculous," says Torquato. "It's not mathematically well defined."

"The assumption had been that there was a unique random closest packing number, but I think Torquato and his collaborators have unequivocally demonstrated that this is not the case," says Frank Stillinger, an engineer at Lucent Technologies in New Jersey. Even though the lab experiments and simulations got values of roughly 64%, it was due to the laboratory conditions rather than to any universal rule—which explained why the experimenters never could quite agree on the true value.

Torquato and colleagues suggest a more precise way of approaching the problem. Instead of looking at "close packing," they investigate "jammed" states, where no spheres are free to rattle around if you shake the box they are in. Not only might there be a jammed state that is maximally random—the analogous, but more precise, concept to a random closest packed state—but there might also be some jammed structures that have a very low packing fraction. "They would be jammed but have an enormous amount of open space," says Stillinger. Straphangers, take heart.

–CHARLES SEIFE

GEOLOGIC MODELING

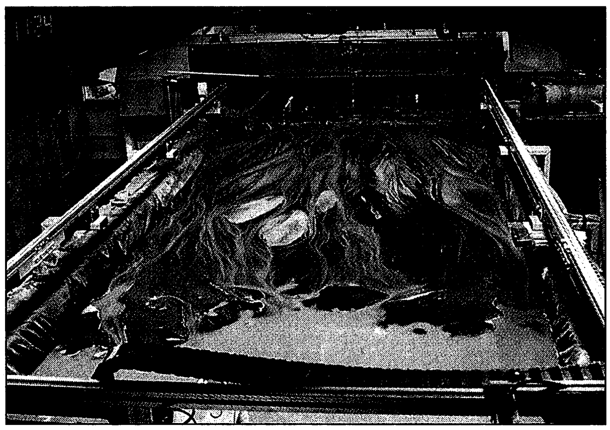
Seeing a World in Grains of Sand

Sophisticated physical models of how sediment flows through rivers into the sea are offering high-tech views into the genesis of complex stratigraphy

Housed in a cavernous laboratory on the banks of the Mississippi is one of the most expensive sandboxes in the world. The rectangular tank is half the size of a tennis court, can hold some 200 tons of sand, and cost about half a million dollars to build. Dubbed "Jurassic Tank," the apparatus is on the leading edge of a new generation of physical models that can simulate the rise and fall of sea level, the effects of swings in climate, and the sinking of tectonic plates.

In initial runs with this new device, which has just been completed, sedimentary geologist Chris Paola, civil engineer Gary Parker, and their team at St. Anthony Falls Laboratory in Minneapolis are creating scaled-down versions of complex geology to figure out how intricate patterns of sediment layers are deposited by rivers and ocean currents. "You could think of the stratigraphic record as an old violin: You can examine it, take it apart, analyze it, model it, but even with all that, you still aren't entirely sure how it was made," Paola says. "Now, just imagine you could watch Stradivari at work."

Although by no means a perfect simulation of the real world, the sandboxes are the first attempt to reproduce entire sedimentary basins in a quantitative way. The effort



Small world. After a few days, miniature rivers build up realistic-looking layers of sediment up to 1.3 meters thick.

CREDITS: (LEFT TO RIGHT) CORBIS; ST. ANTHONY FALLS LABORATORY