BOOKS: MATHEMATICS

# Representing the World in a Computer

**Brian Sleeman**

To grasp the essence of mathematical modeling one must begin with a problem or an observation. The problem can be a well-defined physical question requiring a mathematical solution. For example, one might wish to determine the flow of water around a sphere placed in a stream of finite depth. In a sense the modeling aspect of such problems is minimal. Fluid mechanics provides the governing equations. After one specifies the correct flow conditions around the sphere, on the stream bed, and at the free surface, one's mathematical skills are applied to solve the equations and thus determine the flow characteristics of the water. If one is lucky, an analytical solution may be obtained. But this will likely be a formidable task, and even if a solution is found it may not be easily interpreted in physical terms. An alternative approach would be to use a computationally efficient numerical method to obtain a graphical solution. Or one might model the flow with a cellular automata, which simulates the behavior of a set of connected parcels of fluid. These various approaches are among the techniques that Neil Gershenfeld, head of the physics and media group at MIT's Media Lab, describes concisely in *The Nature of Mathematical Modeling*.

**The Nature of Mathematical Modeling**
*by Neil Gershenfeld*
Cambridge University Press, Cambridge, 1999. 356 pp. $39.95, £24.95. ISBN 0-521-57095-6.

What Gershenfeld really has in mind are not the formalized problems described above, but more intriguing and open-ended puzzles: "How would you describe the flickering of a flame?...Highway traffic during a rush hour?...The flight of a paper airplane?"

Problems such as these are simultaneously imprecise and challenging, and they can be approached from several directions. To describe the flight of a paper airplane, for example, should one first study the aerodynamics of flight and then apply it to the paper glider? Here there are questions of lift, drag, aspect ratios, wing geometry, and wind conditions. These and other considerations produce a frightening degree of complexity. Another approach would be to try to copy the flight characteristics without recourse to the equations of aerodynamics. In other words, one might develop a sort of mathematical flight simulator. But what mathematics to use? Alternatively, one could attempt to assess the important observable characteristics of paper airplane flight and thus arrive at an "approximate" (reasonably accurate) mathematical description. Again, what mathematics should be used?

In a compact but accessible manner, Gershenfeld offers a wide-ranging overview of mathematical ideas and techniques that provide a number of effective approaches to problem solving. These include traditional concepts such as the analytical use of differential equations and variational principles (which state problems in terms of an unknown function that makes an integral take on an extremum) and classical numerical models including Runge-Kutta approximations and finite element methods. In a more modern vogue, Gershenfeld provides a taste of Fourier transforms, wavelets (families of orthogonal transformations that generalize Fourier transforms), and methods such as simulated annealing and genetic algorithms for parameter optimization.

Although these techniques have traditionally been, and continue to be, of great importance in mathematical modeling, they are not always adequate for coping with problems of current scientific interest. Over the last two decades, we have realized that we do not inhabit a comfortable deterministic world, one in which events and predictions are based on secure initial states and conditions with well-defined outcomes. More commonly, phenomena depend on imprecise "noisy" data and outcomes are far from predictable. This is clearly the case in predicting weather or the behavior of stock markets, but even the bouncing of a tennis ball or the cueing of a billiard ball are not really deterministic: over time, they display chaotic and random outcomes.

Another important aspect that Gershenfeld addresses is the decision between discrete and continuum approaches to modeling events. For example, in a biological model when should one shift the consideration from individual cells to cell densities? His discussions of the purposes and capabilities of techniques such as time series analysis, cellular automata, lattice gases, and state-space reconstructions will help readers model complex realities.

Gershenfeld presents his overview of modeling through a series of problems in which he describes methods and how each should be applied. To keep his text concise, he focuses on the underlying concepts and leaves refinements to a challenging series of problem sets. He hopes that readers will work through these sets because he believes "the study of modeling is inseparable from the practice of modeling." The included solutions focus on developing and extending the various models. Gershenfeld also provides an excellent series of references for readers wishing to develop a deeper understanding of the topics covered.

*The Nature of Mathematical Modeling* is a great compendium of techniques. It should be kept within easy reach of anyone who wants to build computer models to help understand the world around us.

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BOOKS: NEUROSCIENCE

# Themes of Thought and Thinking

**C. R. Gallistel**

For his compilation, Robert Sternberg persuaded a number of cognitive psychologists to write essays on enduring conceptual themes in the study of cognition. He has organized the contributions into five sections: philosophical and psychological foundations, representation and process, methodology, kinds of cognition, and group and individual differences. The volume is intended for the advanced undergraduate and the graduate student, but many essays in it can be profitably read by the scientifically literate reader. A not always successful attempt is made to give individual chapters and the book as a whole a dialectical structure. The chapters are essays, not reviews. Their coherence and personal flavor make the book much more suitable for students than a collection of reviews of the recent literature on standard topics.

**The Nature of Cognition**
*Robert J. Sternberg, Ed.*
MIT Press, Cambridge, MA, 1999. 746 pp. $60. ISBN 0-262-19405-8. Paper, $35. ISBN 0-262-69212-0.

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