PERSPECTIVES: COSMOLOGY

Is the Universe Fractal?

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One of the fundamental issues in modern cosmology is the question of whether the spatial distribution of galaxies is homogeneous at a given scale. The cosmological principle, formulated originally by Einstein, states that the large-scale universe is spatially homogeneous and isotropic. It is this principle, together with Einstein's general relativity, that provides the theoretical framework on which the standard hot big bang model for the origin of the universe is based. However, the principle is an assumption and needs to be verified by observations.

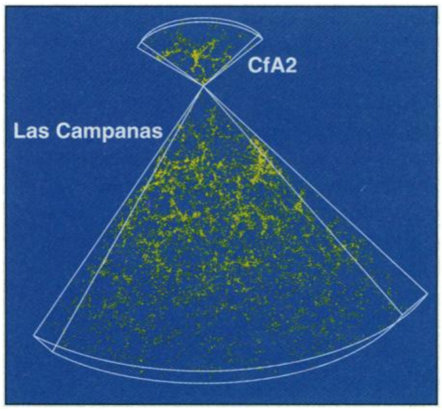
The majority of astrophysicists accept the validity of the cosmological principle. Others follow the ideas envisaged by Charlier (*1*) and de Vaucouleurs (*2*) of an unbounded clustering hierarchy in which stars group into galaxies, galaxies into clusters, clusters into superclusters, and so on. This hierarchical clustering view of the universe was recently taken up by authors arguing for a self-similar or fractal distribution of galaxies (*3*, *4*).

In recent years, the controversy over whether the universe is smooth on large scales or has an unbounded fractal hierarchy has received increasing attention (*5*), because analyses of recent galaxy redshift surveys have reached different conclusions.

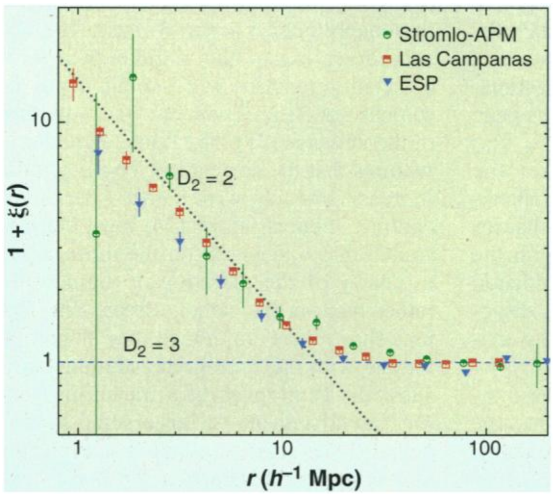
During the past two decades, catalogs of galaxies mapping the universe in three dimensions have been compiled (*6*). These surveys list not only the position on the celestial sphere of each galaxy but also its redshift. By the Hubble law, the latter is proportional to the distance of the galaxy. Comparison of the galaxy positions in the southern slices of the Las Campanas catalog (*7*) with the first slice of the Center of Astrophysics second survey (CfA2) (*6*) (see top figure) shows "the beginning of the end" (*8*): Although we can see the same structures (walls, filaments, and voids) in the Las Campanas slice as in the CfA2 catalog, we do not see similar structures of larger size than those in the CfA2 sample. In a fractal pattern, the size of these structures should be larger for the deeper slice. This diagram would thus suggest that homogeneity is being reached at larger scales.

The most popular tool for statistical analysis of redshift galaxy surveys is the two-point correlation function, $\xi(r)$ (*9*), which measures the clustering in excess [$\xi(r) > 0$] or in defect [$\xi(r) < 0$] compared with a Poisson distribution, for which $\xi(r) = 0$. In contrast, the correlation integral $C(r)$ (*10*) measures the average number of galaxies within a sphere of radius r of any given galaxy. In a fractal set, this function is proportional to r^{D_2}, where D_2 is the correlation dimension, one of the most common "fractal" dimensions used in the literature. For a uniform distribution, $C(r)$ is proportional to the volume of the sphere, and therefore $D_2 = 3$. If, instead of taking an average, we look at the number of neighbors included in a sphere of radius r centered on Earth, $M(r)$, we can define the "fractal dimension" D_M as the exponent of the relation $M(r) \propto r^{D_M}$ (mass-radius relation). This relation is less accurate than $C(r)$, which considers all galaxies in the sample as possible centers but has the advantage that the measure of the dimension can be extended to much larger scales, because the redshift surveys are typically centered at the observer on Earth.

It is established that $\xi(r)$ follows a well-defined power law at small separations, $\xi(r) \approx (r/r_0)^{-1.8}$, where the correlation length $r_0 \approx 5h^{-1}$ Mpc (h is the Hubble constant in units of 100 km s^{-1} Mpc^{-1}; Mpc = 3.26 × 10^6 light-years) is the distance at which the density of galaxies is on average twice the mean number density. Given the power-law behavior of $\xi(r)$, in the range where $\xi(r) \gg 1$, the correlation integral provides a value of $D_2 = 1.2$. This result, together with the fact that the correlation function of clusters of galaxies, $\xi_{cc}(r)$, was originally fitted to a power law with the same exponent [$\xi_{cc}(r) \propto r^{-1.8}$], has led several authors (*11*) to model the universe's large-scale structure as a bounded fractal with dimension $D_2 = 1.2$.



A matter of scale. The galaxy distribution for the southern slices of the Las Campanas redshift survey together with the first slice of the CfA2 catalog at the Northern Hemisphere. Although the depth of the Las Campanas slices is four times (in redshift) the depth of the CfA2 slice, the size of the structures is the same in both samples, contrary to what is expected for an unbounded fractal.



Gradual transition to smoothness. The correlation function $1 + \xi(r)$ for the Stromlo-APM, the Las Campanas, and the ESP redshift surveys. For the first and the last surveys, the calculation has been performed over volume-limited subsamples. Two straight lines have been plotted for reference, corresponding to a fractal with correlation dimension $D_2 = 2$ and to a homogeneous distribution with $D_2 = 3$.

Alternatively, one can try to fit $1 + \xi(r)$, or the correlation integral $C(r)$, directly to a power law. This is particularly important in ranges where $\xi(r) \gg 1$ does not hold. When this was done with the CfA1 redshift survey, the value obtained for the exponent was slightly larger (*12*), $D_2 \approx 1.3$ to 1.5. At larger scales and for the Perseus-Pisces redshift survey, Guzzo *et al.* (*13*) found a value $D_2 \approx 2.2$. Since then, Pietronero and co-workers (*14*) have analyzed all available redshift surveys. They found that the large-scale clustering of galaxies is well described by a fractal pattern with dimension $D_2 \approx 2$ up to scales of at least $150h^{-1}$ Mpc, without a transition to homogeneity. Using the mass-radius relation, these authors extend the fractal range to up to 10^3h^{-1} Mpc with the same dimension $D_M \approx 2$. A transition to

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homogeneity would require an increasing correlation dimension with scale.

Other authors (*15*) have found that the distribution of galaxies in the currently available catalogs shows evidence for a transition to homogeneity at large scales. Scaramella *et al.* (*16*) have found that the mass-radius relation provides a dimensionality of $D_M = 3$ for the European Southern Observatory (ESO) slice project redshift survey (ESP) and for the ACO (Abell-Corwin-Olowin) catalog of galaxy clusters.

How did this controversy arise? The most important criticism made by Pietronero and co-workers of the standard analysis of the galaxy catalogs is that the estimators of the correlation function are usually based on the implicit assumption that the galaxy distribution is a homogeneous and isotropic point process. This assumption affects the way that the estimators are corrected for boundary effects. The estimators are all based on counting the number of neighbors at a given distance. When a galaxy lies close to the boundary of the sample, the count is underestimated. This can be corrected in different ways, leading to different estimators of the correlation function (*17*). Pietronero and co-workers propose the use of a so-called minus estimator, which omits as centers for counting neighbors at a given scale r those galaxies that are closer to the boundary than r. This solution was anticipated by Hauser and Peebles (*18*) but has not been used much in cosmology, mainly because it eliminates some of the information contained in the data. Moreover, at large distances, only a small fraction of galaxies are considered as centers, increasing the variance of the estimator. If one wants to make full use of the data contained in a catalog, an edge correction has to be applied, such as the Hamilton estimator (*19*). In the lower figure, previous page I show the correlation function $1 + \xi(r)$ calculated with this estimator for the Stromlo-APM (*20*) and the Las Campanas redshift surveys (*21*) together with that for the ESP survey (*22*) calculated with the standard Davis and Peebles (*23*) estimator. The fractal behavior at small scales disappears at larger distances, providing evidence for the transition to homogeneity.

If we are prepared to believe these results, then the universe is not fractal at large scales and the validity of the cosmological principle remains plausible. But are the results conclusive? The defenders of the fractal picture of the universe raise the following arguments against them:

1) The results could be spurious because the estimator used for $\xi(r)$ could introduce artificial homogeneity. This is probably the crucial point of the controversy, which reflects the different methodologies adopted by each side. The minus estimator can only be applied up to the radius of the largest sphere that can be enclosed within the sample boundaries, and therefore Pietronero and co-workers are overestimating the scale up to which fractal correlations are found (*24*). For cluster point processes, the difference between the minus estimator and the Hamilton estimator up to the distance where the first can be applied is negligible (*17*); this implies that the edge correction is not distorting the correlations, but again it is not clear whether the same is true at larger distances.

2) Although the samples analyzed in the figure are presently the best available deep redshift surveys in the optical band of the spectrum (*5*), several problems in their construction could affect the validity of their statistical analysis. Stromlo-APM is a sparse sample (*20*); only one galaxy in 20 has a measured redshift. The complicated boundaries of the Las Campanas survey (*7*), which consists of six separated slices, each 1.5° wide, makes a reliable global statistical analysis at large scales very difficult.

Because of these problems, the strongest observational evidence supporting the cosmological principle is not based on the redshift surveys but on the isotropy of the projected deep catalogs including the Infrared Astronomical Satellite survey (*5*) and radio sources (*9*) and on the analysis of the x-ray and cosmic microwave backgrounds (*9, 15*). Assuming the validity of the principle, it is remarkable that angular fluctuations in the temperature of the cosmic background radiation are consistent with a universe in which galaxies are reasonably good tracers of mass. The observed scaling of the angular two-point correlation function with sample depth also does not fit well with the fractal picture of the universe (*25*). The fractal hypothesis requires that the correlation length r_0 must increase linearly with sample depth. In contrast, Benoist *et al.* (*26*) have demonstrated that r_0 depends on the intrinsic luminosity of the galaxies in the sample rather than on the sample depth. The figures shown here—although they should be viewed with the appropriate caution—also show the fingerprint of a transition from the fractal regime to large-scale homogeneity. The scale at which $1 + \xi(r)$ flattens is about the same for the three samples analyzed here, strengthening the case for this interpretation.

The next generation of wide and deep redshift surveys (SLOAN and 2df) will likely provide a more conclusive answer to the question of the large-scale structure of the universe. In the meantime, the two sides should agree on the statistical quantities that have to be measured, the most appropriate estimators, the cosmological corrections to be applied to the data, and the scale at which a given statistical analysis can give meaningful results.

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