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with the Kolmogorov-Smirnov (K-S) test [W. H. Press et al., Numerical Recipes in FORTRAN: The Art of Scientific Computing (Cambridge Univ. Press, New York, ed. 2, 1992), pp. 617–622]. Low probability values for the K-S statistic suggest that two cumulative distributions are significantly different. K-S probabilities >0.9 (Table 2) for all summation cross sections show that random fractal velocity fluctuations produce log-normal distributions of PHA.

- R. R. Youngs et al., Bull. Seismol. Soc. Am. 85, 1161 (1995).
- 33. K. W. Campbell, Seismol. Res. Lett. 68, 154 (1997).
- 34. Direct S waves, which dominate PHVs and PHAs recorded from larger magnitude earthquakes, traverse increasingly larger cross sections of the crust associated with larger fault dimensions. The summation cross sections in Table 2 are a proxy for this process, but these calculations show that PHA dispersion decreases as direct S waves are summed over increasing propagation cross sections.
- 35. R. K. McGuire, Bull. Seismol. Soc. Am. 85, 1275

(1995); I. G. Wong and J. C. Stepp, Probabilistic Seismic Hazard Analyses for Fault Displacement and Vibratory Ground Motion at Yucca Mountain, Nevada (U.S. Department of Energy, Washington, DC, 1998).

- A. Frankel, Bull. Seismol. Soc. Am. 83, 1020 (1993);
   R. W. Graves, *ibid.*, p. 1042; K. B. Olsen, R. J. Archuleta, J. R. Matarese, Science 270, 1628 (1995); K. B. Olsen and R. J. Archuleta, Bull. Seismol. Soc. Am. 86, 575 (1996).
- C. M. Rubin, S. C. Lindvall, T. K. Rockwell, *Science* 281, 398 (1998).
- D. D. Oglesby, R. J. Archuleta, S. B. Nielsen, *ibid.* 280, 1055 (1998).
- T. Mikumo and T. Miyatake, in Seismic Strong Motion Synthetics, B. A. Bolt, Ed. (Academic Press, New York, 1987), pp. 91–151; Geophys. J. Int. **112**, 481 (1993);
   E. Fukuyama and T. Mikumo, J. Geophys. Res. **98**, 6529 (1993).
- A. Herrero and P. Bernard, Bull. Seismol. Soc. Am. 84, 1216 (1994).
- Support for this research was provided by the Dam Safety Program of the U.S. Bureau of Reclamation. L.

## Unconditional Security of Quantum Key Distribution over Arbitrarily Long Distances

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Quantum key distribution is widely thought to offer unconditional security in communication between two users. Unfortunately, a widely accepted proof of its security in the presence of source, device, and channel noises has been missing. This long-standing problem is solved here by showing that, given fault-tolerant quantum computers, quantum key distribution over an arbitrarily long distance of a realistic noisy channel can be made unconditionally secure. The proof is reduced from a noisy quantum scheme to a noiseless quantum scheme and then from a noiseless quantum scheme to a noiseless classical scheme, which can then be tackled by classical probability theory.

The art of secure communication—cryptography—has a long history. Before two parties can communicate securely, they often must share a secret random string of numbers (a key) for encryption and decryption. The secrecy of the message depends on the secrecy of the key. A problem in conventional cryptography is the key distribution problem: In classical physics, there is nothing to prevent an eavesdropper from monitoring the key distribution channel passively, without being caught by the legitimate users.

Quantum key distribution (QKD) (1-5) has been proposed as a solution to the problem. The quantum no-cloning theorem states that it is impossible to make an exact copy of an unknown quantum state ( $\delta$ ). Thus, it is generally thought that eavesdropping on a quantum channel will almost surely produce detectable disturbances. The two users can, therefore, use part of their quantum signals to test for eavesdropping. Only when the error rates are acceptable will they use the quantum signals to generate a key. Thus, the two users (commonly called Alice and Bob) have the confidence that if an eavesdropper (commonly called Eve) has a nonnegligible amount of information on the final key, she will almost surely be caught, even if she has infinite computing power and access to a quantum computer. Incidentally, several recent experiments have demonstrated the feasibility of QKD over tens of kilometers (7).

"The most important question in quantum cryptography is to determine how secure it really is" (8, p. 16). QKD is widely claimed to provide perfect security. However, this viewpoint has been under renewed scrutiny for two reasons. First, contrary to well-known claims of unconditional security (9), a class of other quantum cryptographic schemes, including so-called quantum bit commitment and quantum one-out-of-two oblivious transfer, has recently been shown to be insecure (10). Cheaters can defeat these schemes by a subtle application of the well-known Einstein-Podolsky-Rosen (EPR) paradox (11)

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and by delaying their measurements. These "no-go" theorems not only shattered the longstanding belief in the security of those schemes, but they also undermined the confidence in QKD itself. Second, a convincing and rigorous proof of the security of QKD has been missing despite extensive investigations (12-15). Thus, the foundation of quantum cryptography has been shaky. Here, we solve this long-standing problem by proving that, given quantum computers, QKD can be made unconditionally secure over arbitrarily long distances.

A rigorous proof of the security of a QKD scheme requires the explicit construction of a procedure such that, whenever Eve's strategy has a nonnegligible probability of passing the verification test by Alice and Bob, her information on the final key will be exponentially small (16-17). This procedure must be secure and efficient, even when Alice and Bob use imperfect sources and devices and share a noisy quantum channel.

Most analyses of the security of QKD have dealt with single-particle eavesdropping strategies (12), with immediate or delayed measurements, as well as the so-called collective attacks (13), in which Eve brings each signal particle into interaction with a separate probe system but then, after hearing the public discussion between Alice and Bob, measures all probes together. Security against the most general type of attack, the so-called joint attack, has been investigated by Deutsch et al. and also by Mayers. The discussion by Deutsch et al. was restricted to the special case of perfect devices (14). It introduced the concept of quantum privacy amplification, based on a process called entanglement purification, which was studied by Bennett, DiVincenzo, Smolin, and Wootters (BDSW) (18). Earlier versions of Mayers's proof (15) have not been accepted as definitive. His most recent version of the proof is more detailed and complex (19). He proposes a proof of security of the Bennett and Brassard (BB84) (2) scheme against joint attacks in the presence

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of detector and channel noise but with an ideal trusted single-photon source. Our current work and work by Mayers (19) are contemporaneous and independent. They differ greatly in their premises, methods, and consequences. (i) Mayers's work deals with the standard BB84 QKD protocol (2) for preparation, transmission, and measurements of nonorthogonal states. His approach does not require Alice and Bob to have a quantum computer, although Eve may have one. In contrast, our proof applies to a new OKD protocol, involving fault-tolerant sharing and purification of so-called EPR pairs, and requires that Alice and Bob have quantum computers. (ii) Mayers's work (19) assumes an ideal single-photon or EPR-pair source, thus disallowing a beam-splitter attack. [A testing procedure for an allegedly ideal EPR-pair source from an untrusted vendor has recently been suggested by Mayers and Yao (20).] In contrast, our work allows the reception of untrusted imperfect quantum signals from the channel (21). (iii) Our proof and protocol allow QKD to be securely extended over arbitrarily large distances through a chain of insecure relay stations. A similar extension of BB84 in Mayers's proposed proof would require secure relay stations, to which Eve does not have access. (iv) Our proof is conceptually simpler. (v) Our techniques have widespread applications outside QKD.

Why is a proof of security of QKD so difficult? In a joint attack, Eve treats the whole sequence of quantum signals as a single entity. She couples this entity with her probe and then unitarily evolves the combined system. She forwards a subsystem to Bob and keeps the remaining subsystem for eavesdropping purposes. Eve can use any unitary transformation she likes, and yet, a secure QKD scheme must defeat all of them. Moreover, Eve may attempt to mask her presence by attributing the errors caused by her eavesdropping attack to normal transmission noise. Furthermore, because the particles are now generally entangled with each other, a naïve application of classical probability theory may lead to fallacies [see the EPR paradox (11)].

Despite these apparent difficulties, we show that it is possible to distinguish a malicious Eve from noise. Moreover, it is possible to use classical probability theory to establish the security of QKD.

**Techniques and importance of results.** Assuming that users have access to quantum computers, we show the security of QKD by a reduction in two steps. The central theme of the first step is to reduce the noisy quantum scheme (imperfect devices, noisy channels, storage errors, and so forth) to a noiseless quantum scheme. We do this by combining the ideas of "quantum repeaters" (22, 23) and fault-tolerant quantum computation (FTQC) (24, 25). Although these are existing ideas in the field, we make the nontrivial observation that they can be combined and applied to QKD to distinguish noise from a malicious Eve. In particular, we note that knowing the error syndrome does not help an eavesdropper. Therefore, we can give an eavesdropper full control of the quantum repeater stations without compromising security.

Even in a noiseless quantum scheme, Alice and Bob are required to verify that the particles are untampered by Eve. Things will be easy if one can apply classical arguments to solve this quantum problem at hand. However, as illustrated by the EPR paradox, naïve classical arguments often lead to fallacies. The most important technical contribution of this paper is our second theme—reducing the noiseless quantum verification scheme to a classical one. Finally, we establish the security of the classical verification scheme by classical probability theory. The security of the quantum scheme then follows.

The use of classical arguments in our quantum problem allows us to simplify our discussion greatly. We emphasize that the validity of this usage is highly paradoxical. Classical arguments work in our quantum problem because all of the observables  $O_i$ 's under consideration are diagonal with respect to a single basis B. In more detail, let us consider the observable M, which represents a complete von Neumann measurement along the same basis  $\mathfrak{B}$ . Because all of the  $O_i$ 's under consideration are diagonal with respect to  $\mathfrak{B}$ , *M* commutes with all the observables  $O_i$ 's. Therefore, the measurement M along basis B will in no way change the outcome of the subsequent  $O_i$ 's. Without any loss of generality, one can imagine that such a complete von Neumann measurement M is always performed before the measurement of subsequent  $O_i$ 's. In other words, the initial state of the quantum system is simply a classical mixture of eigenstates of M, and hence, classical arguments carry over to the quantum case. We remark that the  $O_i$ 's that we consider are coarse-grained observables (observables with degenerate eigenvalues) rather than fine-grained observables (observables with nondegenerate eigenvalues).

Quantum-computational protocols. The execution of our secure QKD scheme requires large-scale quantum computers for both error correction and verification. Building such computers is a technological feat that is far beyond our current technology. However, all existing QKD security analyses require some idealization also. In an actual experimental implementation of polarization-coding BB84 (a standard "prepareand-measure" P/M scheme) over a substantial distance (say 40 km) of a lossy quantum channel using coherent states. Eve may, in principle, break the system completely by a generalized beam-splitting attack (26). This is so even when the bit error rate of the quantum signals is strictly zero.

Quantum-computational protocols like ours are worthy of analysis for several reasons. First, unlike the usual P/M schemes, they extend the range of secure QKD to arbitrarily long distances even with insecure quantum repeaters. Second, when implemented over a noisy channel without repeaters, it is conceivable that they can tolerate a higher noise level than a standard P/M scheme. Third, a proof of security and the tradeoff between noise and key rate are much easier than those for P/M schemes. Indeed, our scheme provides a conceptually simple and rigorous proof of the security of QKD without the full complexity of a P/M scheme.

**EPR pairs.** Before we report our QKD scheme in detail, let us first recapitulate the usefulness of an EPR pair, that is, a singlet state  $(1/\sqrt{2})(|\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle)$  of a pair of quantum bits (qubits) (27) in QKD. If two members of an EPR pair are measured along any common axis, each member will give a random outcome, and yet, the outcomes of the two members will always be antiparallel. This spooky action at a distance defies any simple classical explanation and is at the core of EPR paradox (11).

Now, suppose two distant users share REPR pairs. Then, the random outcome of measurement along a common axis generates an *R*-bit key between them. The laws of quantum physics assure that the key is truly random and that Eve cannot have any information on its value. Indeed, the two lemmas in supplementary material (available at www.sciencemag.org/ feature/data/984035.shl) show that, to generate an almost perfectly secure R-bit key, Alice and Bob only need to share R EPR pairs of almost perfect fidelity (28). Therefore, all we need for secure QKD is a way for Alice and Bob to share EPR pairs and to verify that, indeed, they are EPR pairs. We focus on these EPR distribution and verification problems. There are two issues that Alice and Bob have to address: noise and Eve

**Reduction to a noiseless scheme.** One can classify errors in an EPR-pair distribution process into four types. First, the quantum communication channel between Alice and Bob is generally noisy. Second, the EPR source may be imperfect in itself. Third, errors may occur during the storage of quantum information. Fourth, errors may also occur during computation; because elementary gates and measuring devices for quantum computation are generally imperfect, gate errors and measurement errors may arise.

The last three types of errors can be fixed by recently developed quantum error correction (29) and fault-tolerant quantum computation (FTQC) (24, 25) techniques. In particular, there is a "threshold result" in FTQC: Assuming an independent noise model and that the error rates for each primitive computational gate and for each time step of storage are smaller than some positive threshold values, one can perform arbitrarily long quantum computations with an arbitrarily high fidelity (25). The essence of FTQC is to defeat errors by encoding quantum states in quantum error-correcting codes (QECC) and then performing quantum computation on the encoded states.

**Quantum repeaters.** We must consider the first type of noise—channel noise. If the quantum communication channel is very noisy (for example, it is very long), we cannot apply the threshold result in FTQC to combat quantum communication errors. Fortunately, the idea of quantum repeaters has been proposed as a much more efficient way of correcting quantum communication errors (23). This idea is summarized as follows.

Given impure EPR pairs shared between two distant observers, they can apply local operations and classical communication to distill a smaller number of higher fidelity EPR pairs in a procedure known as entanglement purification (18). However, for distances much longer than the coherence length of a noisy quantum communication channel, the probability that a quantum state will remain error-free is exponentially small. Therefore, the fidelity of transmission is so low that standard purification methods are not applicable.

Quantum repeaters are essentially simple quantum computers installed throughout a quantum communication channel. They are used to divide the channel into shorter segments, which are then purified separately, before they are connected. The number and locations of quantum repeaters are chosen so that it is possible to create EPR pairs with sufficiently high fidelity between the two ends of each segment. After creating EPR pairs that are shared between the two ends, one applies entanglement purificaton by using quantum repeaters. This will, at the cost of discarding some pairs, increase the fidelity of the remaining pairs. Afterwards, EPR pairs shared between various segments are connected together by "quantum teleportation" (30). Indeed, a highly efficient procedure involving a sequence of entanglement purification and teleportation has been devised that allows the reliable sharing of EPR pairs between two arbitrarily distant locations (23).

Three important remarks are in order. First, with two-way classical communication, quantum repeaters can greatly improve the yield of distillation (18, 23) over the standard fault-tolerant circuits. Second, even highly imperfect quantum repeaters can do the job very well: It has been argued (23) that an error rate in the percent level is readily tolerable. Third, a strength of our approach is that, assuming perfectly reliable local quantum operations by Alice and Bob, one can actually calculate the threshold value for tolerable noise between two adjacent quantum repeat-

ers. For example, in the case of a depolarizing channel, it is known that a fidelity of 1/2 is the threshold value (18).

With quantum repeaters and FTQC, the usual threshold result can be extended to distributed quantum computation over a realistic noisy quantum channel (31). In other words, any distributed quantum algorithm, including the EPR-pairs distribution process, that works in the noiseless case can always be extended to the noisy case.

Error syndrome contains no useful information for an eavesdropper. We make the most generous assumption that Eve completely controls the quantum repeaters and the quantum communication channel. Alice and Bob need only trust their own quantum computers and authenticated classical messages from each other. There is a subtlety for us to address. If Eve follows the correct procedure, she will not be caught. However, she does learn about the error syndrome (that is, the pattern of measurement results) generated during a FTQC, which allows the error-correction apparatus to correct a corrupted state to a former value. So, the question is Can the error syndrome tell her anything useful about the state? The answer is "no" because of the following. Mathematically, each of Alice and Bob's state can be written as a tensor product of the logical qubits (which actually contain the quantum information) and the ancillary qubits (which contain the error syndromes) (32); that is, the wave function  $|\Psi\rangle$ can be written as  $\sum_{i,j} c_{ij} |a_i\rangle_L \otimes |e_j\rangle_A$ , where subscripts *L*, *A*, *i*, and *j* represent the logical qubits, the ancillary qubits, the logical state (in some orthonormal basis), and the error syndrome, respectively, and  $c_{ii}$ 's are some complex coefficients. (In reality, Alice and Bob's system is generally entangled with Eve's probe. However, this does not change the essential point of our argument.) Because of FTQC, although the state of the ancillary qubits evolves unpredictably over time, the state of the logical qubits will, with a very high fidelity  $[1 - O(e^{-\ell})]$  for some arbitrarily chosen  $\ell > 0$  (33)], follow the desired computation and remain unaffected by the errors. As long as the gate error rate and storage error rate are sufficiently small, the subsequent verification and key generation steps can be thought of as being performed solely on the logical qubits. In other words, the ancillary qubits decouple from the verification step. Accordingly, we shall ignore the ancillary qubits and focus only on the logical qubits.

If there is no appreciable eavesdropping, the logical qubits will represent the desired state. Of course, an honest Eve can learn as much as she likes about the error syndrome. The general theory of QECC tells us that the error syndrome contains absolutely no information about the encoded quantum state (29).

In summary, we have reduced the proof of security of our noisy QKD (or EPR delivery) scheme to that of a noiseless one. Now, we focus on the noiseless scheme.

The goal of verification. To make sure that there is no substantial eavesdropping, Alice and Bob must verify that the state of the logical qubits is, indeed, that of N EPR pairs. Without any loss of generality, we can allow Eve to not merely act on the N EPR pairs while they are being shared but to actually prepare them in an arbitrary state of her choosing and then give them to Alice and Bob (14). She claims that they are perfect EPR pairs. Alice and Bob will be happy to sacrifice a small number *m* of those pairs to verify Eve's claim. If any one of the m tested pairs fails the test, then all the N pairs are discarded. However, if all the m pairs pass the test, the remaining N - m pairs will be accepted as singlets and used to generate the key. The goal of the verification is for Alice and Bob to make sure that Eve has a very small probability of cheating successfully. By cheating successfully, we mean that the m tested pairs pass the verification test and yet the remaining N - mpairs, if given a yes or no test of being N - msinglets, will give "no" as an answer (34).

The security of our quantum verification scheme will automatically guarantee the security of the corresponding QKD scheme [refer to (28) for an explicit bound on Eve's information]. We will now consider the security of our quantum verification scheme.

Essentially, what Alice and Bob are trying to do is to distinguish singlets from triplets. Although there is no way for Alice and Bob to do so with certainty using only local operations and classical communication, they can do so with a very high probability.

The goal of a quantum verification scheme is to verify that the state of the Npairs is, in fact, N singlet. A direct testing of a random subset of EPR pairs requires an exponential amount of resources in terms of the security parameter k, where the probability for Eve to cheat successfully is, at most,  $e^{-k}$ . Direct testing of a random subset is, therefore, not an efficient verification scheme. To understand this point, suppose Eve cheats by inserting a single nonsinglet among the N pairs and only m random pairs are tested by Alice and Bob. There is a probability (N - m)/N for this nonsinglet to remain untested. Consequently, Eve has at least a probability (N - m)/N of cheating successfully. To prevent this from happening, it is necessary for m to be of order N. Even when m equals N - 1, the probability for Eve to cheat successfully is still at least 1/N. For this to be exponentially small in k, the number of photons transmitted N, must be exponentially large in k.

A much more efficient way of verifying a quantum state exists. It is due to the randomhashing idea by BDSW (18). (BDSW proposed it for error correction, but here we use it for verification.) It is in the same spirit of a classical random-hashing scheme which we will now describe.

A classical verification scheme. Imagine a game in which Eve locks an N-bit string x in a box and Alice and Bob are allowed to ask a small number m < N of "fair" questions about it, which Eve must answer truthfully. A fair question is a yes or no question whose answer is "yes" for exactly half of all N-bit strings. Thus, Is the first bit 1? and Are the first and third bits equal? are fair questions, but Are all the bits 1's? is unfair. If all of Eve's answers are consistent with the assumption that the string x is all 1's. Alice and Bob must "accept" the string. Otherwise, Alice and Bob must "reject" the string. Finally, Eve opens the box and shows Alice and Bob the string to prove that she has answered faithfully. Eve wins if the string is not all 1's and yet Alice and Bob have accepted the string. [Alice and Bob win if the string is not all 1's and they have rejected the string. If the string is, in fact, all 1's, the game is a draw].

If Alice and Bob ask only single-digit questions of the form Is the kth bit a 1? then Eve has a good chance of winning by choosing a string with a single 0 at a random location. However, if Alice and Bob instead ask Eve about the parities of random subsets of the bits, they quite likely catch any string that is not all 1s.

For example, if the unknown string is x =1101 and Alice and Bob choose a subset consisting of the second and third bits (this can be represented conveniently by an index string s = 0110), the parity  $x \cdot s$  is 1. This test reveals that x is not all 1's, because an all-1's four-bit string would have had parity 0 on this subset. More generally, the parity of a subset s of the bits in a string x is the inner product, or modulo-2 sum of the bit-wise AND of strings x and s, and it is denoted by  $x \cdot s$ . [In this example,  $x \cdot s = 1 \cdot 0 + 1 \cdot 1 + 0 \cdot 1 + 1$  $1 \cdot 0 = 1 \pmod{2}$  The probability that two different strings give the same answer for m iterations of random parity check is no more than  $2^{-m}$  (18). Thus, by checking only a few subset parities (say 20), Alice and Bob can reduce their chance of accepting an x that is not all 1s to less than one in a million.

Eve must not know the index strings beforehand. Otherwise, she could always cheat successfully, in a similar way as a smuggler who knows beforehand which of the several bags a custom inspector will open in an airport. Indeed, because the string has N bits and there are only m constraints (generated by m rounds of parity verification), there are clearly exponentially many (namely,  $2^{N-m}$ ), strings that will pass the test. However, because Eve does not know the index strings beforehand and because the index strings are chosen randomly, Eve effectively has to put her bet on a single string without prior knowledge. We see from the last paragraph that any string  $x \neq 11 \cdots 1$  chosen by Eve has only an exponentially small probability  $(2^{-m})$  of passing the verification test.

Our quantum verification scheme. Now, we construct an efficient quantum verification scheme that is similar to the classical verification scheme that we have just described. Consider the so-called Bell basis,  $\Psi^{\pm}$  and  $\Phi^{\pm}$ , where

$$\Psi^{\pm} = \frac{1}{\sqrt{2}} \left( \left| \uparrow \downarrow \right\rangle \pm \left| \downarrow \uparrow \right\rangle \right) \quad (1)$$

and

$$\Phi^{\pm} = \frac{1}{\sqrt{2}} \left( \left| \Uparrow \right\rangle \pm \left| \downarrow \downarrow \right\rangle \right) \quad (2)$$

With the convention in (18), Bell basis vectors are represented by two classical bits

$$\Phi^{+} = \tilde{0}\tilde{0}$$

$$\Psi^{+} = \tilde{0}\tilde{1}$$

$$\Phi^{-} = \tilde{1}\tilde{0}$$

$$\Psi^{-} = \tilde{1}\tilde{1}$$
(3)

(Because Bell basis vectors are maximally entangled, one should never think of them as direct product states.) A complete basis for *N*-ordered pairs of qubits (what we shall call *N*-bell basis) consists of products of Bell basis vectors, each of which is described by a 2N-bit string. In the absence of an eavesdropper, Alice and Bob share *N* singlets, whose state is described by a 2N-bit string of  $\tilde{1}$ 's,  $|\tilde{1}\tilde{1}\cdots\tilde{1}\rangle$ .

What happens when there is an eavesdropper? Recall that we allow Eve to not merely act on the *N* EPR pairs while they are being shared but to actually prepare them in an arbitrary state of her choosing and then give them to Alice and Bob. The pairs may be entangled among themselves as well as with a probe in Eve's hands (14). A system described by any mixed state can be equivalently described by a pure state of a larger system consisting of the original system and an ancilla (10, 14). As discussed by Deutsch *et al.* (14), by considering the larger system instead, we shall, without a loss of generality, consider that Eve prepares a pure state

$$|u\rangle = \sum_{i_1, i_2, \cdots, i_N \mid j} \sum_{j \mid i_1, i_2, \cdots, i_N, j} \alpha_{i_1, i_2, \cdots, i_N, j}$$

$$\times |i_1, i_2, \cdots, i_N\rangle \otimes |j\rangle \qquad (4)$$

**Fig. 1.** A sample oneway hashing protocol used to determine  $s_1 \cdot x_1$ and  $s_2 \cdot x_2$  for an unknown three-Bell state. Following the convention of (18),  $B_x$  and  $B_y$ denote bilateral rotation of  $\pi/2$  along the *x* and *y* axis, respectively;  $\sigma_x$  denotes a unilateral rotation of  $\pi$  along the *x* axis; and the symbol —  $\oplus$  denotes a bilateral controlled NOT oper-



ation. M denotes a bilateral measurement.

which runs from 00 to 11,  $\alpha_{i_1i_2...,i_Nj}$ 's are some complex coefficients, and the  $|j\rangle$  values form an orthonormal basis for the ancilla. Each state  $|u\rangle$  represents a particular cheating strategy chosen by Eve.

where  $i_k$  denotes the state of the kth pair,

The goal of a quantum verification scheme is to verify that the string describing the state of the N pairs is, in fact, all  $\tilde{1}$ 's. We now construct an efficient quantum verification scheme based on the quantum random-hashing idea by BDSW (18), BDSW showed that one can compute the parity of any subset of the 2N-bit string by using local operations and classical communication only (35). The parity is "collected" into a single destination pair; it is determined by the outcomes of measurements performed on that pair, which has to be discarded afterward. More specifically, the parity is found by noting whether the measurement outcomes on the two members of the destination pair are parallel or antiparallel.

If Eve prepares a classical mixture of products of Bell states, it is not too difficult to show that classical arguments apply directly to the quantum verification problem and Eve's probability of cheating successfully is negligible [see (36) for details].

Why do classical arguments work for a quantum problem? However, instead of preparing a classical mixture of products of Bell states, in the most general eavesdropping strategy as shown in Eq. 4, Eve prepares a general state, which is entangled with her probe. The big question is Can Eve prepare a more general state to enhance her probability of cheating successfully? The crux of our paper is the following claim: If Eve prepares a general state to cheat in the BDSW random-hashing verification scheme, her probability of cheating successfully will be exactly the same as in the situation when she premeasures that state along the N-Bell basis before handing it over to Alice and Bob. In other words, a general state offers no advantage over a mixture of products of Bell states. With this quantum to classical reduction result, (36) applies to any eavesdropping strategy. This proves that Eve's probability of cheating successfully is negligible and our QKD scheme is secure against all possible attacks.

Proof of our claim. Consider the following observables on a state  $|u\rangle$  of N pairs of qubits shared between Alice and Bob. We define these observables by their actions on the  $2^{2N}$  N-Bell states, which form a complete basis. Let W, defined by  $W|w\rangle = w|w\rangle$ , be the observable that gives the 2N-bit string representing the state w in BDSW notation. For any index string s, let  $Q_s$ , defined by  $Q_s |w\rangle = (s \cdot w) |w\rangle$ , be the observable that gives the parity of the subset s of the bits. Finally, let  $R = |\tilde{1}\tilde{1}\cdots\tilde{1}\rangle\langle \tilde{1}\tilde{1}\cdots\tilde{1}|$  be the projector onto a state of N singlets. All the above operators refer to a single basis (namely, the N-Bell basis). Because all the observables  $(R, W, and O_{\perp})$  are simultaneously diagonalizable with respect to the N-Bell basis, R and all the  $Q_s$  values commute with W. Therefore, neither the value of R nor any of the  $Q_s$  values are affected by a prior measurement of W. In other words, for any state  $|u\rangle$ that Eve might have supplied, neither the sequence of subset parities measured in the verification stage nor the result of the final hypothetical measurement of R would have been affected if Eve had premeasured  $|u\rangle$  in Bell basis (that is, made a measurement of W) before handing the state to Alice and Bob. Incidentally, the fact that a premeasurement does not change the outcomes of some subsequent measurements is highly reminiscent of work by Griffiths and Niu (37).

**Subtleties in our proof.** The following example illustrates the computation of parities and the subtleties involved. Suppose Alice and Bob share three pairs of qubits. With the procedure specified in BDSW, the computation of the parity of the first subset (for example,  $s_1 = 001101$ ) can be done by the circuit diagram shown in Fig. 1A. The parity is collected into a single pair and is determined by whether that pair gives a parallel or antiparallel outcome when both members are measured along the *z* axis.

The computation itself, up to phases, performs a permutation on the space of all 2N-bit strings. After the computation, the measured pair is dropped from consideration, and only two pairs remain. The computation of the parity of the second subset (for example,  $s_2 = 1001$ ) by the BDSW procedure is shown in Fig. 1B. After the computation, another pair is measured and dropped from consideration. Therefore, only a single pair of the original three is left after the computation of the two parities (for  $s_1$ ) and  $s_2$ ). A simple unitary description (10) of the overall computation is that it maps, up to phases, the state  $|\tilde{1}\tilde{1}\tilde{1}\tilde{1}\tilde{1}\rangle$  to  $|\tilde{1}\tilde{0}\tilde{1}\tilde{1}\tilde{1}\tilde{1}\rangle$ . Suppose also that, on passing the verification test, Alice and Bob generate their secret key by measuring the remaining pair along the z axis, with an "up" for Alice's result meaning "0" and a "down" meaning "1."

A number of subtleties deserve careful discussions. First, as in the classical case, the choice of subsets can be announced only after

Alice and Bob receive all of their quantum particles. So long as Eve does not know the subsets beforehand, her probability of cheating successfully is exponentially small (see supplementary material, available at www.sciencemag.org/feature/data/984035.shl).

Second, during the computation of the parities of subsets, the state of the N pairs of qubits is transformed by a unitary transformation  $U_{s_1,s_2,\cdots,s_m}$ , which depends on the subsets  $s_i$ . But would that unitary transformation  $U_{s_1,s_2,\cdots,s_m}$  somehow spoil our reduction argument from a quantum to classical verification? Fortunately, the answer is "no". Despite the apparent complexity of the parity computation procedure, the bottom-line answer that Alice and Bob obtain is simply the parities (that is, the eigenvalues of the operators  $O_{\mu}$ 's of their choice). Therefore, the verification test proves that, for any general cheating strategy by Eve that passes the test with a probability of at least  $2^{-r}$ , the conditional fidelity of the state before the parity computation as N singlets,  $|\tilde{1}\tilde{1}\cdots\tilde{1}\rangle$ , is 1 - $O(2^{-(m-r)})$ . Consequently, the state after the parity computation will, with the same fidelity, be  $U_{s_1,s_2,\dots,s_m} | \tilde{1} \cdots \tilde{1} \rangle$ . Third, in our quantum verification proce-

Third, in our quantum verification procedure, Alice and Bob have to disclose all their measurement outcomes in a public channel. For each measured pair, there are four possible outcomes,  $\uparrow\downarrow$ ,  $\downarrow\uparrow$ ,  $\uparrow\uparrow$ , and  $\downarrow\downarrow$ , thus resulting in two bits of information. This is more than the one-bit (0 or 1) parity information. Now, the question is Can Eve somehow benefit from this additional information? The answer is "no" (this discussion is available at www.sciencemag.org/feature/data/984035.shl). Finally, the issue of a quantum Trojan horse attack is addressed in (21). This completes our proof of security of QKD.

Discussion. An important idea behind our quantum to classical reduction is that a quantum mechanical experiment has a classical interpretation whenever observables that refer to only one basis (the N-Bell basis in our case) are considered. The fine-grained measurement operators by Alice and Bob along the three random bases do not commute with the Bell-basis projection operators. However, Alice and Bob base their decision on whether to accept the alleged singlets not on those fine-grained measurement results but on the coarse-grained (parallel or antiparallel) ones. Those coarse-grained operators all commute with a complete von Neumann measurement along the Bell basis (38).

Our quantum to classical reduction technique is a powerful tool of widespread applications. It guarantees that one can apply standard results in the classical world (such as probability theory and statistics theory) to the original quantum problem without leading to fallacies. In effect, this means the extension of classical statistical theory to quantum mechanics, resulting in a quantum statistical theory. To illustrate this point, we give two other examples of applications of our quantum to classical reduction result. (i) Suppose two distant observers share N pairs of qubits, and estimate the number of singlets in those N pairs. By the number of singlets, we mean the expected number of "yes" answers if a singlet or triplet measurement was made on each pair individually. (ii) Under the assumption that signal carriers are perfect single photons, put a probabilistic bound on an eavesdropper's information in BB84 as a function of the error rates of the sampled photons. (These examples are discussed at www.sciencemag. org/feature/data/984035.shl.)

The second example gives us a quantitative statement on the trade-off between information gain and disturbance (39). This is a strong result to a notoriously difficult problem because (i) the bound applies not merely to a strategy in which Eve couples a probe to each signal particle but to any information extraction strategy that is consistent with quantum mechanics and (ii) the bound can be derived by a random sampling of a small subset. In other words, a concrete experimental random-sampling procedure (rather than an abstract mathematical equation with little physical meaning) is presented here (40).

Finally, let us return to QKD itself. Although we have focused on the case when Alice and Bob receive allegedly good EPR pairs from Eve, our proof of security of QKD also applies to the case when Alice sends qubits (rather than halves of EPR pairs) to Bob. Consider the following situation. Alice prepares N EPR pairs in her laboratory. She then chooses the subsets for parity determination beforehand and performs all the computations and measurements on her halves of the N EPR pairs in her own laboratory before sending out the other halves to Bob. After Alice's measurements, the subsystem that she sends to Bob is in a pure state; that is, gubits rather than halves of EPR pairs are sent to Bob. However, because Alice's operation is local, it must commute with Eve's eavesdropping operator. Therefore, this qubitbased scheme must be as secure as the original EPR-based scheme. (Just as in the EPR-based case, it is of the utmost importance for Alice to withhold information on the choice of subsets for the parity determination until Bob acknowledges the receipt of quantum transmission. Otherwise, Eve can cheat easily.)

#### **References and Notes**

- The idea of quantum cryptography was first proposed by S. Wiesner around 1970 but remained unpublished until 1983 [S. Wiesner, SIGACT News 15 (no. 1), 78 (1983)]. Wiesner proposed quantum money and multiplexing channel (essentially one-out-of-two oblivious transfer) but not QKD per se.
- C. H. Bennett and G. Brassard, in Proceedings of the IEEE International Conference on Computers, Systems, and Signal Processing (IEEE Press, New York, 1984), pp. 175–179.

- 3. A. K. Ekert, Phys. Rev. Lett. 67, 661 (1991).
- 4. H.-K. Lo, S. Popescu, T. Spiller, Eds., Introduction to Quantum Computation and Information (World Scientific, Singapore, 1998).
- For a review on quantum cryptography, see, for example, H.-K. Lo, in (4), pp. 76–119.
   W. K. Wootters and W. Zurek, *Nature* 299, 802
- (1982); D. Dieks, Phys. Lett. A 92, 271 (1982)
- 7. P. D. Townsend, Electron. Lett. 30, 809 (1994); A. Muller, H. Zbinden, N. Gisin, Europhys. Lett. 33, 335 (1996); R. J. Hughes, G. G. Luther, G. L. Morgan, C. G. Peterson, C. Simmons, in Advances in Cryptology: Proceedings of CRYPTO '96, vol. 1109 of Lecture Notes in Computer Science, N. Koblitz, Ed. (Springer-Verlag, Berlin, 1996), pp. 329-342. For a review, see, for example, H. Zbinden, in (4), pp. 120–142.
  8. G. Brassard and C. Crépeau, *SIGACT News* 27 (no. 3),
- 13 (1996).
- 9. G. Brassard, C. Crépeau, R. Jozsa, D. Langlois, in Proceedings of the 34th annual IEEE Symposium on the Foundation of Computer Science (IEEE Computer Society Press, Los Alamitos, CA, 1993), pp. 362-371, and references cited therein.
- 10. For the impossibility of bit commitment, see the following: D. Mayers, Los Alamos e-Print archive (available at http://xxx.lanl.gov/abs/quant-ph/9603015); H.-K. Lo and H. F. Chau, Phys. Rev. Lett. 78, 3410 (1997); D. Mayers, ibid., p. 3414; H.-K. Lo and H. F. Chau, Physica D 120, 177 (1998). For the impossibility of other schemes, including one-out-of-two oblivious transfer, see H.-K. Lo, Phys. Rev. A 56, 1154 (1997). For a review, see, for example, H. F. Chau and H.-K. Lo, Fortschr. Phys. 46, 507 (1998): (5)
- 11. J. S. Bell, *Physics* **1**, 195 (1964); [reprinted in *Quantum* Theory and Measurement, J. A. Wheeler and W. H. Zurek, Eds. (Princeton Univ. Press, Princeton, 1983), pp. 403–408]; A. Einstein, B. Podolsky, N. Rosen, Phys. Rev. 47, 777 (1935) [reprinted in Quantum Theory and Measurement, J. A. Wheeler and W. H. Zurek, Eds. (Princeton Univ. Press, Princeton, 1983), pp. 138–141]. 12. C. H. Bennett, F. Bessette, G. Brassard, L. Salvail, J.
- C. H. Bernett, F. Bessette, G. Brassard, L. Salvan, J. Smolin, J. Cryptol. 5, 3 (1992); N. Lütkenhaus, Phys. Rev. A 54, 97 (1996); C. H. Bennett, T. Mor, J. A. Smolin, *ibid.*, p. 2675; C. A. Fuchs, N. Gisin, R. B. Griffiths, C.-S. Niu, A. Peres, *ibid.* 56, 1163 (1997); R. B. Griffiths and C.-S. Niu, *ibid.*, p. 1173; N. Lütkenhaus and S. M. Barnett, Los Alamos e-Print archive (available at http://xxx.lanl.gov/abs/quant-ph/9711033).
- 13. E. Biham and T. Mor, Phys. Rev. Lett. 78, 2256 (1997); E. Biham, M. Boyer, G. Brassard, J. van de Graaf, T. Mor, Los Alamos e-Print archive (available at http:// xxx.lanl.gov/abs/quant-ph/9801022).
- 14. D. Deutsch et al., Phys. Rev. Lett. 77, 2818 (1996); D. Deutsch et al., ibid. 80, 2022 (1998).
- 15. D. Mayers, in Advances in Cryptology: Proceedings of CRYPTO '95, vol. 963 of Lecture Notes in Computer Science, D. Coppersmith, Ed. (Springer-Verlag, Berlin, 1995), pp. 124–135; in Advances in Cryptology: Proceedings of CRYPTO '96, vol. 1109 of Lecture Notes in Computer Science, N. Koblitz, Ed. (Springer-Verlag, Berlin, 1996), pp. 343-357.
- 16. In some applications, a nonnegligible amount of information leakage to Eve can be disastrous. The following example is due to J. Smolin (41). If a key is used in a so-called one-time pad to encrypt a president's order that ends with the password for launching a nuclear missile, an adversary who is aware of the structure of the message will, in principle, be able to steal the password.
- 17. The goal of making Eve's expected information small, conditional only on passing the test, is generally unattainable. One must include the following proviso: for any eavesdropping strategy with a nonnegligible chance of success.
- 18. C. H. Bennett, D. P. DiVincenzo, J. A. Smolin, W. K. Wootters, Phys. Rev. A 54, 3824 (1996).
- 19. D. Mayers, Los Alamos e-Print archive (available at http://xxx.lanl.gov/abs/quant-ph/9802025), version 4.
- 20. D. Mayers and A. Yao, in Proceedings of 39th Annual Symposium on Foundations of Computer Science (IEEE Computer Society Press, Los Alamitos, CA, 1998), pp. 509-515 [also available at Los Alamos e-Print archive (http://xxx.lanl.gov/abs/quant-ph/ 9809039)].
- 21. A big worry in cryptography is the Trojan horse attack.

Any untrusted material received from an open channel poses serious security risks. As J. Smolin has often remarked (41), it is even conceivable that a robot is hidden in the received material and that it pops out to find and disclose secrets to adversaries. It is not just that the Trojan horse might leak information once it is in Bob or Alice's laboratory. One might think that this problem could be eliminated by simply shielding the laboratory very well or that such shielding is, in fact, assumed anyway in cryptographic protocols. The real problem is that the Trojan horse pretends to be real EPR pairs when Alice and Bob do their testing but behaves differently when they generate key, thus causing them to leak the information themselves. This worry is not unfounded because it is notoriously difficult to prepare almost perfect EPR pairs. [See (20) for a related discussion.] Real quantum systems often contain other degrees of freedom that are ignored in quantum computation. One might wonder if Eve could perform a quantum Trojan horse attack by hiding robots in (the hidden Hilbert space dimensions of) the quantum systems received by Alice and Bob. This would certainly make a rigorous proof of security of QKD based on imperfect sources impossible. Our answer is the following proposition. Proposition 1. As long as there is no security risk for Alice and Bob to receive untrusted classical messages, quantum Trojan horse attack can be foiled. Remark. Before we present our proof, notice that the assumption that there is no security risk in receiving classical messages is most reasonable because Eve can always send classical messages to Alice and Bob in a "man-in-the-middle" attack during a classical authentication process. If Alice and Bob could not afford to receive any untrusted classical message, the whole enterprise of cryptography would be hopeless. Proof. Instead of receiving any untrusted quantum system directly from an open guantum channel, a user (say Bob) demands that the state of the system must be converted into classical messages through teleportation (30) right at his doorstep. More concretely, Bob prepares trusted EPR pairs in his laboratory and sends one member of each pair to his untrusted representative Robert, who is working in an insecure area just outside his laboratory, when the untrusted quantum data (potentially a Trojan horse) is waiting. Robert teleports the nominal state of the untrusted system (that is, the state in its nonclandestine variables) into Bob's laboratory. In other words, Bob conveys the untrusted quantum state into his laboratory by means of trusted EPR pairs and untrusted classical messages. Now, assuming that there is no security risk in receiving classical messages, Bob can safely receive those classical messages and use them to reconstruct the original quantum state. Teleportation provides an exact counting of the effective dimensions of the Hilbert space because each qubit requires two classical bits to teleport. Therefore, there is no hidden Hilbert space to worry about in the reconstructed quantum system. This conclusion is valid even if the original EPR pairs prepared by Bob do contain hidden dimensions.

- 22. S. J. van Enk, J. I. Cirac, P. Zoller, Phys. Rev. Lett. 78, 4293 (1997); J. Preskill, Proc. R. Soc. London A 454, 385 (1998); J. I. Cirac, A. Ekert, S. F. Huelga, C. Macchiavello, Los Alamos e-Print archive (available at http://xxx.lanl.gov/abs/quant-ph/9803017).
- 23. H.-J. Briegel, W. Dür, S. J. van Enk, J. I. Cirac, P. Zoller, Philos. Trans. R. Soc. London Ser. A 356, 1713 (1998); H.-J. Briegel, W. Dür, J. I. Cirac, P. Zoller, Phys. Rev. Lett. 81, 5932 (1998); W. Dür, H.-J. Briegel, J. I. Cirac, P. Zoller, Phys. Rev. A. 59, 169 (1999).
- 24. P. W. Shor, in Proceedings of the 37th Symposium on Foundations of Computer Science (IEEE Computer Society Press, Los Alamitos, CA, 1996), pp. 56-65.
- 25. A. Yu. Kitaev, Russ. Math. Surv. 52, 1191 (1997); D. Aharonov and M. Ben-Or, in Proceedings of the 29th Annual ACM Symposium on the Theory of Computing, (ACM Press, New York, 1998), pp. 176-188; E. Knill, R. Laflamme, W. H. Zurek, Science 279, 342 (1998); for a review, see, for example, J. Preskill, in (4), pp. 213-269.
- 26. For instance, in the study of standard P/M schemes such as BB84 (2), one often assumes that the signal carriers are perfect single photons. Unfortunately, producing almost perfect single-photon pulses is beyond current technology, and dim coherent light pulses with a Poisson distribution in the number of photons are often

used instead. The attenuation of an optical fiber is also large (say 0.35 dB/km), and detector efficiencies are far from perfect. Therefore, rather surprisingly, in an actual experimental implementation of polarization-coding BB84 over a significant distance (say 40 km), Eve may, in principle, break the system by a generalized beam-splitting attack. The key point is that many of the signals contain more than one photons and as such Eve is allowed to make copies (details are available at www.sciencemag.org/feature/data/984035.shl). For short-distance applications, the relevance of such an attack remains an important subject for future investigations. In summary, standard theoretical security analyses on BB84 do not apply to most real-life experimental systems to date.

- 27. A qubit is simply a two-level quantum system. It plays the role of a fundamental unit of quantum information, just like a bit in classical information.
- 28. Alice and Bob share R EPR pairs, and they generate a key by measuring these pairs along any common axis. If the fidelity of their pairs is high (that is,  $\langle R \text{ singlets} | \rho | R \text{ singlets} \rangle > 1 - 2^{-k}$  for a sufficiently large  $\vec{k}$ ), then Eve's information on the final key will be bounded by  $2^{-c} + O(2^{-2k})$ , where c = k - c $\log_2[2R + k + (1/\log_e 2)]$ . In other words, Eve's information (more precisely, mutual information with the final key) is exponentially small as a function of k. This result follows directly from two lemmas (see discussion, available at www.sciencemag.org/feature/ data/984035.shl).
- P. W. Shor, *Phys. Rev. A* 52, 2493 (1995); A. M. Steane, *Phys. Rev. Lett.* 77, 793 (1996).
- 30. C. H. Bennett et al., Phys. Rev. Lett. 70, 1895 (1993). 31. Here, we assume that the error rate per unit length varies smoothly along the channel. For example, the errors for different parts of the channel are almost independent
- 32. The decomposition of the quantum state into the tensor product of the logical qubits and ancillary gubits is a mathematical one. In the actual physical system, the state of the local gubits is delocalized among all physical qubits. Such a delocalization is necessary for both error correction and fault-tolerant computation. See, for example, A. Peres, Los Alamos e-Print archive (available at http://xxx.lanl.gov/abs/ quant-ph/9609015).
- 33. Efficient quantum error correcting schemes exist for reducing the error rate to an exponentially small amount (see discussion, available at www. sciencemag.org/feature/data/984035.shl).
- 34. Such an "(N m)-singlets-or-not" measurement can be performed if Alice and Bob bring the two halves of each EPR pair together to perform a measurement along a Bell basis. This is a very subtle point because such a Bell measurement is not actually performed and, indeed, could not be performed without bringing the two halves together. Successful cheating thus means that the actual verification test is passed, but a hypothetical second test of bringing the remaining pairs back into the same laboratory and measuring them in a Bell basis would fail (that is, some of the remaining N - m pairs are shown to be nonsinglets upon Bell measurements).
- 35. This is a surprising result because Bell basis vectors are highly entangled and yet only local operations and classical communication are allowed here. The local operations needed are simply single-qubit operations and bilocal exclusive OR. More specifically, the three types of operations used are (i) unilateral rotations by  $\pi$ rad, corresponding to  $\sigma_{z'}$ ,  $\sigma_{y'}$ , and  $\sigma_{z'}$  (ii) bilateral rotations by  $\pi/2$  rad; and (iii) bilateral application of the two-bit quantum exclusive OR (or controlled NOT). These basic operations plus local measurements and classical communication allow Alice and Bob to correct quantum errors using the one-way random-hashing scheme by BDSW. See (18) for details.
- 36. We can safely use classical probability theory to derive an explicit bound on Eve's information for any eavesdropping strategy that passes the verification test with a probability of at least  $2^{-r}$  for some parameter r > 0. See also (17). Here, we work in the approximation of reliable local quantum operations by Alice and Bob. However, this assumption can be relaxed without changing our essential conclusion. If all the original N pairs are singlets, the remaining

N - m pairs must be singlets. Instead of computing the fidelity for the remaining N - m pairs to be N m singlets, let us compute the fidelity for the original N pairs to be N singlets. This will give us a good enough bound on the fidelity. With any cheating strategy against the quantum verification scheme by Eve, let  $p_1$  be the total probability for the state of the N pairs to be N singlets under the measurement along the Bell basis. The case of N singlets, which happens with a probability  $p_1$ , will automatically pass the verification test. This case is perfectly fine and secure. What about the other case? Upon a randomhashing verification scheme that sacrifices m pairs, the other case (which happens with probability 1  $p_1$ ) will pass an *m*-round random-hashing verification test with a conditional probability of, at most, 2-Therefore, the probability that a strategy passes the verification test is given by

$$P(\text{passing}) \le \rho_1 + 2^{-m}(1 - \rho_1) \le \rho_1 + 2^{-m}$$
 (5)

Eve would be most interested in a cheating strategy that passes the test with a nonnegligible probability (say at least  $2^{-r}$ , where we assume that 0 < r < m). Therefore, we demand that the probability

$$P(\text{passing}) \ge 2^{-r}$$

Combining Eqs. 5 and 6, we find that

 $p_1 + 2^{-m} \ge 2^{-r}$ 

$$\rho_1 \ge 2^{-r} [1 - 2^{-(m-r)}] \tag{7}$$

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Conditional on passing the verification test, the fidelity of the N pairs as singlets is given by

$$F' \geq \frac{\rho_1}{\rho_1 + 2^{-m}} \geq \frac{2^{-r} [1 - 2^{-(m-r)}]}{2^{-r} [1 - 2^{-(m-r)}] + 2^{-m}}$$
$$= [1 - 2^{-(m-r)}]$$
(8)

where Eq. 7 and the fact that  $\rho_1/(\rho_1 + 2^{-m})$  is an increasing function of  $\rho_1$  have been used. By choosing a value of *m* that is substantially larger than *r*, the conditional fidelity can be made very close to 1. Therefore, given any parameter *r*, one can increase the conditional fidelity in Eq. 8 by increasing the number *m* of random parities computed. In summary, consider any eavesdropping strategy that passes an *m*-round random-hashing verification scheme with a probability of at least  $2^{-r}$  (where  $m \gg r > 0$ . From Eq. 8, upon passing the test, the conditional fidelity of the *N* pairs as *N* singlets is at least  $1 - 2^{-(m-r)}$ . From (28), this implies that Eve's information is exponentially small in m - r, more precisely,  $2^{-c} + O(2^{-2(m-r)})$ , where  $c = m - r - \log_2[2(N - m) + m - r + 1/(\log_e 2)]$ .

- R. B. Griffiths and C.-S. Niu, *Phys. Rev. Lett.* **76**, 3228 (1996).
- 38. Our classical argument applies to the N-Bell basis, whose basis vectors are highly entangled. It is perhaps surprising at first that the coarse-grained probabilities of a quantum mechanical experiment involving only local operations and classical communication can have a classical interpretation with respect

to such a highly nonlocal basis. Put in another way, given the lesson from the EPR paradox, it is perhaps surprising that classical arguments can still be used to demonstrate that two distantly separated quantum subsystems are, in fact, highly quantum (that is, highly entangled).

- 39. C. A. Fuchs, *Fortschr. Phys.* **46**, 535 (1998), and references cited therein.
- 40. Incidentally, our result also proves the security of quantum money proposed by Wiesner (1). Indeed, the proof for our second example can be used to derive a probabilistic bound on the entropy of the combined system consisting of the quantum banknote and the bank. Consequently, any double-spending strategy will almost surely fail in the verification step (as in BB84) done by the bank because this entropy will no longer be close to zero.
- 41. J. Smolin, personal communication.
- 42. H.-K. Lo particularly thanks A. Ekert for pressing him to investigate the security of QKD. We thank numerous colleagues, including C. H. Bennett, G. Brassard, I. Chuang, D. P. DiVincenzo, C. A. Fuchs, N. Gisin, D. Cottesman, E. Knill, D. W. C. Leung, N. Lütkenhaus, D. Mayers, S. Popescu, J. Preskill, J. Smolin, T. Spiller, A. Steane, and A. C.-C. Yao for invaluable conversations and suggestions. Many helpful suggestions from an anonymous referee are gratefully acknowledged. H. F. Chau is supported by Hong Kong Government RGC grant HKU 7095/97P.

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# The Effect of Spin Splitting on the Metallic Behavior of a Two-Dimensional System

THE PORTS

(6)

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Experiments on a constant-density two-dimensional hole system in a gallium arsenide quantum well revealed that the metallic behavior observed in the zero-magnetic-field temperature dependence of the resistivity depends on the symmetry of the confinement potential and the resulting spin splitting of the valence band.

For many years, it was accepted that there could be no metallic phase in a disordered two-dimensional (2D) carrier system. This was due to the scaling arguments of Abrahams *et al.* (1) and the support of subsequent experiments (2). In the past few years, however, experiments on high-quality 2D systems have provided us with reason to revisit the question of whether or not a metallic phase in a 2D system can exist (3–9). Early temperature-dependence data from highmobility Si metal oxide semiconductor fieldeffect transistors (MOSFETs) showed a decrease in resistivity as the temperature T was reduced below 2 K (3). This metallic behavior is the opposite of the expected insulating behavior in which the resistivity should become infinite as T approaches zero. In addition, the behavior was not only metallic in a certain electron density range but also scaled with a single parameter as the density was reduced and as the sample became insulating, suggesting a true metal-to-insulator phase transition (3).

Since these experiments, the metallic behavior has been observed in Si MOSFETS (3, 4), SiGe quantum wells (5, 6), GaAs/AlGaAs heterostructures (7, 8), and AlAs quantum wells (9), demonstrating that there are still some unsolved puzzles in the fundamental nature of 2D carrier systems. Multiple mechanisms, including electron-electron interaction (10), spin splitting (11), and T dependent

dence of traps (12), have been proposed as causes of the metallic behavior, but no clear model that fully describes this sizeable body of experimental data has emerged. The experiments reported here add to our understanding by demonstrating a correlation between the zero-magnetic-field spin splitting and the metallic behavior.

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Spin splitting of carriers in a 2D system at a zero magnetic field occurs when there is spin-orbit interaction and an inversion asymmetry of the potential in which the carriers move (13). The energy bands are split into two spin subbands, which have different populations because their energies at any nonzero wave vector  $\vec{k}$  are slightly different. The existence of these spin subbands has been well established both experimentally and theoretically (13–18).

Our experiments were performed on highmobility 2D hole systems in GaAs quantum wells (QWs), which were chosen because they have a large intercarrier separation  $r_{c}$ (19), they have already shown metallic behavior (7, 8), and they exhibit a large and tunable spin splitting (17). In GaAs, the spin splitting arises from the inversion asymmetries of the zinc-blende crystal structure and of the potential used to confine the electrons to two dimensions. The asymmetry of the crystal structure is fixed, but the asymmetry of the confining potential, and therefore the spin splitting, can be changed by applying an  $\vec{E}$  perpendicular to the 2D plane (E<sub>1</sub>) using gates (17). We found that the spin splitting can be tuned while the density is kept constant and that the metallic behavior of the 2D

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