MEETING AMERICAN MATHEMATICAL SOCIETY AND MATHEMATICAL ASSOCIATION OF AMERICA

Mathematicians Offer Answers To Everyday Conundrums

The San Antonio Riverwalk was overrun by nearly 4000 mathematicians from 13 to 16 January at this year's joint meetings of the American Mathematical Society and the Mathematical Association of America. More than 1000 scheduled talks ranged from pure math to some vividly real-world applications, highlighted here.

Shooting the Virtual Rapids

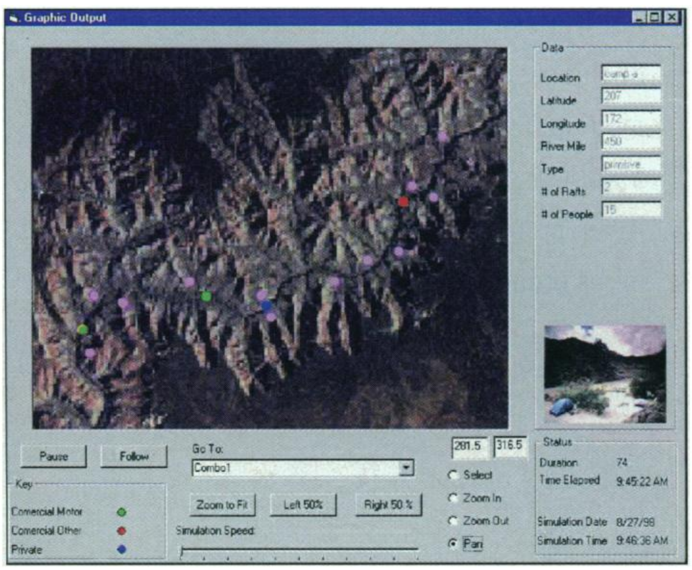
Picture this. You've just paid several thousand dollars for a weeklong wilderness trip down the Colorado River through the Grand Canyon. You're getting along great with your fellow passengers, and the guide is doing a marvelous job of pointing out magnificent views and picking campsites along the way. But you keep running into other groups. The campsites are crowded, and the crush of people at some of the scenic spots is threatening to spill over and damage the natural beauty. This isn't what you paid for. You're not a happy camper.

Help is on the way, in the form of a mathematical model based on a set of rules called fuzzy logic. As part of a policy review on recreational use of the Colorado River, the National Park Service has sought the help of mathematicians at Northern Arizona University (NAU) in Flagstaff and resource management experts at the University of Arizona (UA) to analyze traffic patterns on the 400-kilometer stretch of the Colorado River from Lees Ferry to Diamond Creek. Catherine Roberts, a mathematician at Northern Arizona, described the group's model at the math meetings.

Every year, the Park Service issues permits for approximately 1200 commercial and 300 private launches. During the peak summer months, up to eight boats a day depart, so that upward of 100 groups are somewhere on the river at any given time. If the boat traffic doesn't flow smoothly, a wilderness experience can feel more like a trip to Disneyland, and crowds spreading out to sightsee or camp can damage ecosystems and archaeological relics.

To minimize these problems, the Park Service has to settle on a schedule of launches and a timetable for opening or closing sites along the river, along with other rules of "acceptable use." For decades the schedule has been set by rules of thumb and rough estimates of what the river could bear. The Park Service hoped the mathematicians could come up with something better.

The Grand Canyon River Trip Simulator, as the NAU-UA project is called, combines a detailed computerized map of the Colorado River, broken into 90-meter segments, with a database of actual trip diaries that indicated, among other things, when and where boaters typically stopped for lunch, sightseeing, and camping. The computer then generates dozens or hundreds of imaginary launches, follows the boats along the digital river, and reports on the simulated visitors' experiences.



The river wired. Simulated boats run the Grand Canyon in the river-trip model.

CREDIT: J. MEITNER/UNIVERSITY OF ARIZONA

The model gives each simulated boat on the river independent authority to decide where to stop and for how long (within the bounds of park rules). A simulated guide's decisions take into account factors such as time of day, distance to the next campsite, and the presence of other groups. But because of the fuzzy logic, which softens the cutoff between Yes and No with a gradient of Maybes, the guides in the model decide what to do based on probabilities, not inflexible rules.

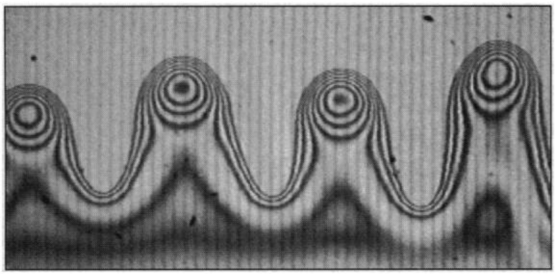
The decision to use fuzzy logic was a crucial step in the design of the model, Roberts says. "We initially thought we could model this with a standard differential-equation type of approach" that ignored decision-making altogether and viewed the boats as making a more or less random walk along the river, she says. "We discarded that quite quickly when we realized that it's a very complex system."

The model should let the Park Service explore the effects of different launch schedules and rules. But officials will have to use some fuzzy logic of their own in basing decisions such as the ratio of commercial to noncommercial boaters on the computer output. "Policy decisions that may please one group of interested parties may upset someone else," Roberts notes. Linda Jalbert, the project leader for the National Park Service's Colorado River Management Plan, says the river-trip simulator "may not provide 'the' solution, but it will allow us to look at more alternatives." And she thinks the mathematical scheduling will win broad support—especially if it enables the park to increases the number of launches.

Shocking News on Fingerless Flows

Anyone who has ever done a sloppy job of painting a kitchen wall knows that certain splotches of paint will outpace their counterparts, creating a wavy pattern that gets increasingly exaggerated until the paint dries, resulting in a permanent source of marital discord. For experts in thin film physics, there's no mystery to this phenomenon, called fingering: The competition between surface tension and gravity (or whatever force is driving the flow) encourages small perturbations to grow rather than shrink. What these researchers find remarkable are situations when fluid flows don't finger.

A group of mathematicians has now come up with a new theory that explains at least some cases of these strangely orderly flows—but invokes another oddity to do so: a kind of shock wave called an undercompressive shock, which develops at the advancing edge of the fluid. "Undercompressive shock" sounds like an oxymoron, as ordinary shock waves develop in flows that are being compressed, like the airflow past a supersonic plane. But the new computer work shows that undercompressive shocks exist in some films, where they smooth out the perturbations that would otherwise grow into fingers.



Fingers of fluid. The surprise is when they don't appear.

At the mathematics meetings, Andrea Bertozzi, a Duke University mathematician and physicist who is an expert on fluid-flow equations, described how she, Duke colleague Andreas Münch, and Michael Shearer of North Carolina State University in Raleigh analyzed fingerless flows observed by experimentalists at the College de France in Paris. The French physicists, Xavier Fanton and Anne Marie Cazabat, had used heat to drive a thin film of an oily liquid known as poly-dimethylsiloxane up an inclined plane. They found that the thinnest films, obtained with the plane held vertically, produced the usual fingers, but slightly thicker films, climbing at shallower angles, were fingerless. What was causing the fluid to suddenly knuckle under?

Early last year, Fanton and Cazabat faxed Bertozzi their experimental data, hoping she and her colleagues could help. The mathematicians first set out to simulate the fluid flow on a computer, Bertozzi says, explaining that "you have to understand the underlying dynamics of the front before you can even address the question of fingering." She and her colleagues devised an equation describing how variations in the film's thickness are related to the speed of flow. Simplified, one-dimensional simulations based on the equation predicted that the flowing fluid would develop a pronounced bump at its advancing front, called a capillary ridge—a feature that Fanton and Cazabat had also seen in their experiment.

But for a certain range of initial thicknesses, the simulations also showed an unexpected behavior: The ridge thickens to a certain maximum height and then begins to widen. "What's happening, really, is there are two waves that are separating," Bertozzi says. Each one turned out to be a shock wave—a boundary at which a fluid's velocity changes suddenly—but the faster, leading one is highly unusual. In ordinary, compressive shocks, such as those that occur in sonic booms, sound waves squeeze both sides of the boundary. An undercompressive shock, in contrast, outruns the pressure waves in the fluid behind it: In effect, it's the proverbial sound of one hand clapping.

When the mathematicians added a second dimension to their simulation and looked at how flow responded to perturbations, they saw the same fingerless flow the French physicists had fixed on, as the two groups reported in the 7 December *Physical Review Letters*. Apparently, the undercompressive shock somehow keeps the fluid flowing smoothly in the thicker films.

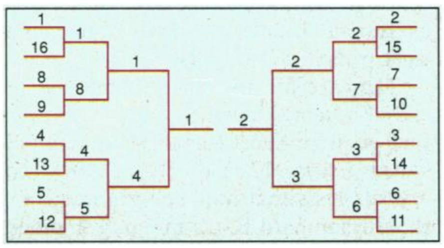
Cazabat is delighted, calling the theorists' model "the first mathematically correct treatment of the problem." Her group plans to do new experiments to test the predictions of the model. The mathematicians, meanwhile, are trying to turn the link between orderly flows and odd shocks into a rigorous mathematical proof.

CREDIT: (TOP) BERTOZZI, MÜNCH, FANTON, AND CAZABAT, *PHYSICAL REVIEW LETTERS* 81 (23), P. 5169 (1998)

The Good Seed

March Madness is not far off. The National Collegiate Athletic Association (NCAA) will soon be "seeding" its annual 64-team basketball tournament. But at the meeting, Allen Schwenk, a mathematician at Western Michigan University in Kalamazoo, had news for the college sports moguls and anyone else who organizes playoff competitions: The NCAA isn't doing it right. What the system needs, he says, is a dose of randomness.

The scheme that the NCAA and many other tournament organizers now follow to place the teams in a tournament "tree" tends to give the top-ranked teams an inordinately easy schedule, Schwenk says. In seeding the 16 basketball teams in each of four regional playoffs, for example, the NCAA always pits the bottom-ranked teams against the very best teams in the first round:

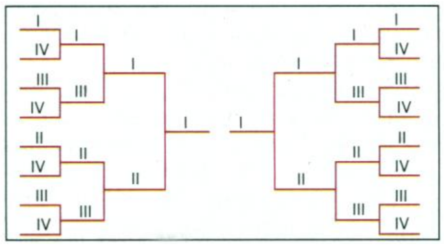


"In the long run, it's still the best teams that win," says Schwenk. "But they should win based on the quality of their skills, not on the fact that you give them such an easy schedule that they can hardly miss."

Oddly, the system can also favor the bottom-ranked team over the teams ranked right above it. To become a Cinderella champion, team 16 has to start by beating team 1, but after that, its most likely opponents are teams 8, 4, and 2. Team number 15, on the other hand, starts with team 2, but then is likely to face teams 7, 3, and 1, and team 14's likely opponents are 3, 6, 2, and 1. The only advantage team 14 enjoys is a slightly better chance of surviving the first round.

After being seeded near the bottom of a bridge tournament, Schwenk found himself wondering if there was a fairer way. "Part of my thinking was making the whole process a little more appealing to the bottom teams," he says. He decided that a good seeding system should adhere to three criteria. First, like the standard system, it should delay confrontation between the top-ranked teams: Teams 1 and 2 should never meet before the final round, none of the top four should ever meet before the semifinals, and so forth. Second, it should not tempt a team to seek a lower ranking because of the advantage that can bring. Finally, it should minimize the favoritism that eases the top-ranked teams' route to the championship.

To his surprise, Schwenk could find just one seeding system that meets all three criteria. His system eschews precise rankings. Instead, it only requires identifying the top two teams, the next best two, the next best four, the next best eight, and so forth. These tiers, say I, II, III, etc., are then assigned positions in the tournament tree so as to satisfy the criterion of delayed confrontation:



Finally, the teams in each tier are placed at random in their tier's assigned positions. The randomness eliminates small advantages like the one enjoyed by the bottom-ranked team in the conventional system, and it also reduces favoritism by making it less likely that the very best and worst teams will face each other in the first round.

Schwenk has not shown his proof to the NCAA, which is happy with its current system. "The [tournament] committee is not really interested in changing things," says William Hancock, director of Division I Men's Basketball at the NCAA. Hancock says he gets a dozen or so (unsolicited) alternative seeding plans every year. "We get every possible scenario." Still, sportswriter Dennis Brackin of the Minneapolis *Star Tribune* says the smaller conferences, whose teams tend to get ranked at the bottom, might welcome the proposal. "They would love to do something more random," he says, but "my guess is the big schools would fight it."

–**BARRY CIPRA**